

$$\frac{d}{dt} \begin{pmatrix} y_1(t) \\ y_2(t) \end{pmatrix} = A \begin{pmatrix} y_1(t) \\ y_2(t) \end{pmatrix}$$

$$\equiv \begin{pmatrix} y_1'(t) \\ y_2'(t) \end{pmatrix}$$

$$A = \begin{pmatrix} 1 & 12 \\ 3 & 1 \end{pmatrix}$$

固有値の重複度は2,
±6 共に重複度は1あり。

→ 異なる2方向へ
±6 = 2 方向へ近づく。

固有値が異なり、 $\alpha \neq \beta$ 。

$$|\lambda I_2 - A| = \begin{vmatrix} \lambda - 1 & -12 \\ -3 & \lambda - 1 \end{vmatrix} \\ = (\lambda - 1)^2 - 6^2 = 0$$

$$\lambda = 1 \pm 6 \quad \underline{7, -5} \text{ の固有値。}$$

$$\underline{\lambda = 7} \quad \begin{pmatrix} 6 & -12 \\ -3 & 6 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = \vec{0} \Leftrightarrow x - 2y = 0.$$

$$\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 2y \\ y \end{pmatrix} = y \begin{pmatrix} 2 \\ 1 \end{pmatrix} \quad \underline{\vec{v}_1} = \begin{pmatrix} 2 \\ 1 \end{pmatrix}$$

$$\lambda = -5 \quad \begin{pmatrix} -6 & -12 \\ -3 & -6 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = \vec{0} \Leftrightarrow x + 2y = 0$$

$$\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} -2y \\ y \end{pmatrix} = y \begin{pmatrix} -2 \\ 1 \end{pmatrix} \\ \underline{\vec{v}_2} = \begin{pmatrix} -2 \\ 1 \end{pmatrix}$$

$$P = (\vec{v}_1 \ \vec{v}_2) \quad AP = P \begin{pmatrix} 7 & 0 \\ 0 & -5 \end{pmatrix}$$

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$$\frac{d}{dt} \begin{pmatrix} y_1 \\ y_2 \end{pmatrix} = A \begin{pmatrix} y_1 \\ y_2 \end{pmatrix}$$

$$\vec{y}(t) = P^{-1} \vec{y}(t) e^{tC}$$

$$\frac{d}{dt} \vec{z}(t) = \dots$$

$$A(t) : m \times n$$

$$\frac{d}{dt} A(t) = (a'_{ij}(t))$$

$$B(t) : n \times l$$

$$A(t)B(t) = (c_{jk}(t))$$

$$c_{jk}(t) = \sum_{i=1}^n a_{ij}(t) b_{ik}(t)$$

$$\begin{pmatrix} a_{11} & \dots & a_{1n} \\ \vdots & \ddots & \vdots \\ a_{m1} & \dots & a_{mn} \end{pmatrix}$$

$$(fg)' = f'g + fg'$$

$$c'_{jk}(t) = \sum_{i=1}^n a'_{ij}(t) b_{ik}(t)$$

$$+ \sum_{i=1}^n a_{ij}(t) b'_{ik}(t)$$

$$A(t)B'(t) + B(t)A'(t)$$

$$\frac{d}{dt} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = A \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} \quad A = P \begin{pmatrix} 2 & 0 \\ 0 & -5 \end{pmatrix} P^{-1}$$

$$\begin{aligned} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} &= P^{-1} y \\ \frac{d}{dt} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} &= (P^{-1})' y + P^{-1} \frac{d}{dt} y \end{aligned}$$

$$\begin{aligned} &= P^{-1} \frac{d}{dt} y \\ &= P^{-1} A P \cdot P^{-1} y \\ &= \begin{pmatrix} 2 & 0 \\ 0 & -5 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} \end{aligned}$$

$$\frac{d}{dt} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} 2 & 0 \\ 0 & -5 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix}$$

$$\begin{pmatrix} x_1(t) \\ x_2(t) \end{pmatrix} = \begin{pmatrix} z_1(t) \\ z_2(t) \end{pmatrix} C$$

$$\begin{cases} z_1'(t) = 2z_1(t) \\ z_2'(t) = -5z_2(t) \end{cases}$$

$\frac{z}{t}$

$$a \in \mathbb{R} \quad \begin{cases} f'(t) = a f(t) \\ f(t) = e^{at} f(0) \end{cases}$$

$$\begin{aligned} (f(t) e^{-at})' &= f'(t) e^{-at} + f(t) (-a) e^{-at} \\ &= a f(t) e^{-at} - a f(t) e^{-at} \\ &= 0 \end{aligned}$$

$$\begin{aligned} f(t) e^{-at} &\equiv C \text{ 常数} \\ t=0 \text{ 时 } f(0) &\equiv C \end{aligned}$$

$$f(t+1) = f(0) e^{at}$$

$$\begin{aligned} z_1(t+1) &= z_1(0) e^{2t} \\ z_2(t+1) &= z_2(0) e^{-5t} \end{aligned}$$

$$\begin{aligned} \vec{z}(t) &= \begin{pmatrix} e^{7t} & 0 \\ 0 & e^{-5t} \end{pmatrix} \underbrace{\begin{pmatrix} z_1(0) \\ z_2(0) \end{pmatrix}}_{P^{-1} \vec{y}(0)} \\ &\stackrel{||}{=} P^{-1} \vec{y}(t) \end{aligned}$$

$$\longrightarrow \vec{y}(t) = P \begin{pmatrix} e^{7t} & 0 \\ 0 & e^{-5t} \end{pmatrix} P^{-1} \vec{y}(0)$$

$A: 2 \times 2$ 'सिद्ध' $\alpha, \beta \in \mathbb{R}, \alpha \neq \beta$

1) व स

$$\begin{aligned} \alpha &\leadsto v_1 \\ \beta &\leadsto v_2 \end{aligned}$$

$$P = (v_1, v_2) \text{ है।}$$

$$\frac{d}{dt} \vec{y}(t) = A \vec{y}(t)$$

$$\vec{z}(t) = P^{-1} \vec{y}(t) \quad t \geq 0 \dots$$

$$\vec{z}(t) = \begin{pmatrix} e^{\alpha t} & 0 \\ 0 & e^{\beta t} \end{pmatrix} \begin{pmatrix} z_1(0) \\ z_2(0) \end{pmatrix}$$

$$\leadsto \vec{y}(t) = P \begin{pmatrix} e^{\alpha t} & 0 \\ 0 & e^{\beta t} \end{pmatrix} P^{-1} \vec{y}(0)$$

$$\underline{C=1}$$

$$A: n \times \mathbb{R} \begin{pmatrix} 1 & 1 & 1 \\ 1 & a & 1 \\ 1 & 1 & a \end{pmatrix}$$

$$\left(\begin{aligned} \text{rank}(A) = 4 &\Leftrightarrow \det(A) \neq 0 \\ &\Leftrightarrow A: \mathbb{R}^3 \\ &\Leftrightarrow \ker(A) = \{\vec{0}\} \\ &(\Leftrightarrow (A\vec{x} = \vec{0}) \Rightarrow \vec{x} = \vec{0}) \end{aligned} \right.$$

$$\begin{aligned} \det \begin{pmatrix} a & 1 & 1 \\ 1 & a & 1 \\ 1 & 1 & a \end{pmatrix} &= - \begin{vmatrix} 1 & a & 1 \\ a & 1 & 1 \\ 1 & 1 & a \end{vmatrix} = - \begin{vmatrix} 1 & a & 1 \\ 0 & 1-a^2 & 1-a \\ 0 & 1-a & a-1 \end{vmatrix} \\ &= - \begin{vmatrix} 1-a^2 & 1-a \\ 1-a & a-1 \end{vmatrix} \\ &= -(1-a)^2 \begin{vmatrix} 1+a & 1 \\ 1 & -1 \end{vmatrix} \\ &= (a-1)^2 (a+2) \end{aligned}$$

$$a \neq 1, -2 \quad \text{rank}(A) = 3.$$

$$\underline{a=1, -2} \rightarrow \text{check } ?$$

$$A = \begin{pmatrix} 2 & 1 & 1 \\ 1 & 2 & 1 \\ 1 & 1 & 2 \end{pmatrix}$$

$$\Phi_A(\lambda) = (\lambda - 1)^2 (\lambda - 4)$$

$$P = \frac{1}{\sqrt{6}} \begin{pmatrix} \sqrt{3} & 1 & \sqrt{2} \\ -\sqrt{3} & 1 & \sqrt{2} \\ 0 & -2 & \sqrt{2} \end{pmatrix} \in \mathbb{R}^{3 \times 3}$$

$$AP = P \begin{pmatrix} 1 & & \\ & 1 & \\ & & 4 \end{pmatrix} \quad \boxed{\|\vec{x}\| = 1 \text{ a.k.a.}}$$

$$\begin{aligned} (A\vec{x}, \vec{x}) &= (AP \cdot {}^t P \vec{x}, P^t P \vec{x}) \\ &= ({}^t P A P \cdot {}^t P \vec{x}, {}^t P \vec{x}) \quad \vec{\xi} = {}^t P \vec{x} \\ &= \left(\begin{pmatrix} 1 & & \\ & 1 & \\ & & 4 \end{pmatrix} \vec{\xi}, \vec{\xi} \right) \\ &= \xi_1^2 + \xi_2^2 + 4\xi_3^2 \end{aligned}$$

$${}^t P \in \mathbb{R}^3$$

$$\|{}^t P \vec{x}\| = \|\vec{\xi}\| = 1$$

$$\begin{aligned} P: \mathbb{R}^3 \quad {}^t P P &= I_n \Leftrightarrow (P\vec{x}, P\vec{y}) \\ &= (\vec{x}, \vec{y}) \\ \Leftrightarrow \|P\vec{x}\| &= \|\vec{x}\| \end{aligned}$$

$$x_1^2 + x_2^2 + x_3^2 = 1 \quad \text{a.k.a.}$$

$$y = x_1^2 + x_2^2 + 4x_3^2$$

$$= x_1^2 + x_2^2 + 4(1 - x_1^2 - x_2^2)$$

$$= 4 - 3x_1^2 - 3x_2^2 \leq 4.$$

attain.

$$x_1 = x_2 = 0, \quad x_3 = \pm 1.$$

$$T = \pm \begin{pmatrix} 0 & 0 \\ 0 & 0 \\ -1 & 0 \end{pmatrix} \rightsquigarrow$$

$$\begin{aligned} T &= \pm p \begin{pmatrix} 0 & 0 \\ 0 & 0 \\ -1 & 0 \end{pmatrix} \\ x &= p T \\ &= \pm p \begin{pmatrix} 0 & 0 \\ 0 & 0 \\ -1 & 0 \end{pmatrix} \\ &= \pm \begin{pmatrix} 0 & 0 \\ 0 & 0 \\ -1 & 0 \end{pmatrix} \end{aligned}$$

$$x_1^2 + x_2^2 = 1 - x_3^2$$

$$= 1 - x_3^2 + 4x_3^2 = 1 + 3x_3^2 \leq 1$$

$$x_3 = 0 \quad x_1^2 + x_2^2 = 1$$

$$x = p \begin{pmatrix} x_1 \\ x_2 \\ 0 \end{pmatrix}$$

$$= x_1 p_1 + x_2 p_2$$

$$x_1^2 + x_2^2 = 1$$

$$\begin{pmatrix} 0 & 0 \\ 0 & 0 \\ 1 & 0 \end{pmatrix}$$

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2. 例題 (1) 例題

I

$$A = \begin{pmatrix} -2 & 2 & 1 \\ 2 & -5 & 1 \\ 1 & 1 & -3 \end{pmatrix}$$

$(A\vec{x}, \vec{x})$ の値を求めよ。

II

$$A = \begin{pmatrix} 1 & 0 & 0 \\ 1 & 2 & 0 \\ 1 & 0 & -1 \end{pmatrix}$$

$$\frac{d\vec{x}(t)}{dt} = A\vec{x}(t)$$

を解け。

III

$$w = xyz - x - y - z \quad \text{の極値を求めよ}$$

1) 極値を求めよ。

2) H.P.C. 2(7, 8).

3) 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100.

$$A = \begin{pmatrix} 1 & 1 \\ 0 & 2 \end{pmatrix}$$

$$\frac{d}{dt} \vec{y}(t) = A \vec{y}(t)$$

$$\Sigma \text{ is } \mathbb{R}^2 \text{ to } \mathbb{R}^2.$$
