$$\underbrace{\frac{d}{dt}\begin{pmatrix} y_1(t) \\ y_2(t) \end{pmatrix}}_{\begin{pmatrix} y_1'(t) \\ y_2'(t) \end{pmatrix}} = A \begin{pmatrix} y_1(t) \\ y_2(t) \end{pmatrix}$$

$$A = \begin{pmatrix} 1 & 12 \\ 3 & 1 \end{pmatrix}$$

固有値が複素数の場合に重要な例あり.

⤳ ・景気じゅんかん.

・均こう点に近づく.

固有値が実で, $\alpha \neq \beta$.

$$|\lambda I_2 - A| = \begin{vmatrix} \lambda - 1 & -12 \\ -3 & \lambda - 1 \end{vmatrix}$$

$$= (\lambda - 1)^2 - 6^2 = 0$$

$\lambda = 1 \pm 6$ 　 $7, -5$ が固有値.

<u>$\lambda = 7$</u>

$$\begin{pmatrix} 6 & -12 \\ -3 & 6 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = \vec{0} \iff x - 2y = 0.$$

$$\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 2y \\ y \end{pmatrix} = y\begin{pmatrix} 2 \\ 1 \end{pmatrix} \qquad \vec{v_1} = \begin{pmatrix} 2 \\ 1 \end{pmatrix}$$

$\lambda = -5$

$$\begin{pmatrix} -6 & -12 \\ -3 & -6 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = \vec{0} \iff x + 2y = 0$$

$$\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} -2y \\ y \end{pmatrix} = y\begin{pmatrix} -2 \\ 1 \end{pmatrix}$$

$$\vec{v_2} = \begin{pmatrix} -2 \\ 1 \end{pmatrix}$$

$$P = (\vec{v_1}\ \vec{v_2}) \qquad AP = P\begin{pmatrix} 7 & 0 \\ 0 & -5 \end{pmatrix}$$

$$\boxed{\frac{d}{dt}\vec{y}(t) = A\,\vec{y}(t)}$$

$P P^{-1}$

$$AP = P\begin{pmatrix} 7 & 0 \\ 0 & -5 \end{pmatrix}$$

$\vec{z}(t) = P^{-1}\vec{y}(t)$ とおく.

$\frac{d}{dt}\vec{z}(t) = \cdots\cdots$

どうするか.

$A(t) : m \times n$

$= (a_{ij}(t))$

i行j列

公式

$$\boxed{\begin{aligned} &\frac{d}{dt}\left(A(t)B(t)\right) \\ &= A'(t)B(t) \\ &\quad + A(t)B'(t) \end{aligned}}$$

証

$\frac{d}{dt}A(t)$

$= \left(a'_{ij}(t)\right)$

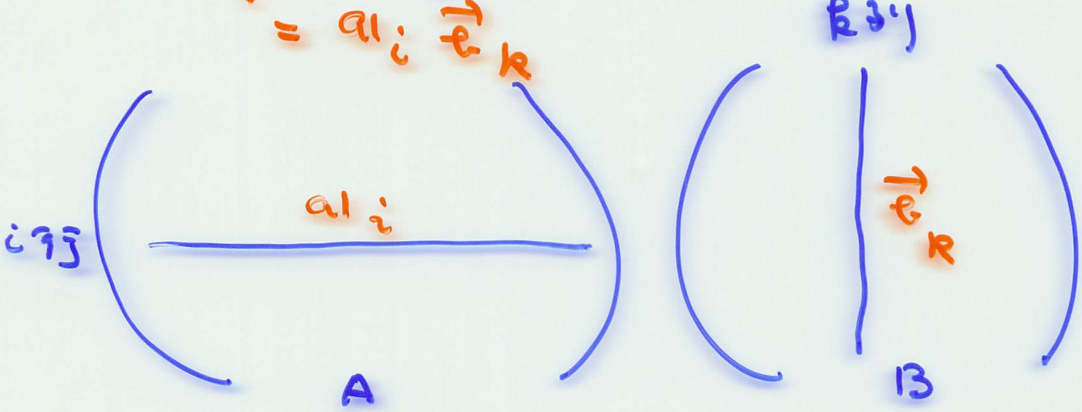
$B(t) : n \times l$

$= \left(b_{jk}(t)\right)$

$A(t)B(t) = \left(c_{ik}(t)\right)$

$$c_{ik}(t) = \sum_{j=1}^{n} a_{ij}(t)\, b_{jk}(t)$$

$= \vec{a}_i \cdot \vec{b}_k$



$$\boxed{(fg)' = f'g + fg'}$$

Leibnitzの公式

$$c'_{ik}(t) = \sum_{j=1}^{n} a'_{ij}(t)\, b_{jk}(t) + \sum_{j=1}^{n} a_{ij}(t)\, b'_{jk}(t)$$

$A'(t)B(t)$ の (i,k) 成分

$A(t)B'(t)$ の (i,k) 成分

$$\frac{d}{dt}\vec{y}(t) = \underbrace{A}_{PP^{-1}}\vec{y}(t) \qquad AP = P\begin{pmatrix} 7 & 0 \\ 0 & -5 \end{pmatrix}$$

$$\vec{z}(t) = P^{-1}\vec{y}(t)$$

$$\frac{d}{dt}\vec{z}(t) = \underbrace{(P^{-1})'}_{= O_2}\vec{y}(t) + P^{-1}\frac{d}{dt}\vec{y}(t)$$

$$= P^{-1}\frac{d}{dt}\vec{y}(t)$$

$$= P^{-1}AP \cdot P^{-1}\vec{y}(t)$$

$$= \begin{pmatrix} 7 & 0 \\ 0 & -5 \end{pmatrix}\vec{z}(t)$$

$$\boxed{\frac{d}{dt}\vec{z}(t) = \begin{pmatrix} 7 & 0 \\ 0 & -5 \end{pmatrix}\vec{z}(t)}$$

$\vec{z}(t) = \begin{pmatrix} z_1(t) \\ z_2(t) \end{pmatrix}$ とおく.

$$\begin{cases} z_1'(t) = 7 z_1(t) \\ z_2'(t) = -5 z_2(t) \end{cases}$$

基本

$a \in \mathbb{R}$

$f'(t) = a f(t) \rightsquigarrow f(t) = e^{at} f(0)$

$$(f(t)e^{-at})' = f'(t)e^{-at} + f(t)(-a)e^{-at}$$

$$= a f(t) e^{-at} - a f(t) e^{-at}$$

$$\equiv 0$$

$\rightsquigarrow f(t)e^{-at} \equiv C$ 定数.

$t=0$ とすれば $f(0) \equiv C$

$\rightsquigarrow f(t) = f(0)e^{at}$

$$z_1(t) = z_1(0)e^{7t}$$

$$z_2(t) = z_2(0)e^{-5t}$$

$$\vec{z}(t) = \begin{pmatrix} e^{7t} & 0 \\ 0 & e^{-5t} \end{pmatrix} \underbrace{\begin{pmatrix} z_1(0) \\ z_2(0) \end{pmatrix}}_{= P^{-1}\vec{y}(0)}$$

$$\vec{z}(t) = P^{-1}\vec{y}(t)$$

$$\longrightarrow \quad \vec{y}(t) = P \begin{pmatrix} e^{7t} & 0 \\ 0 & e^{-5t} \end{pmatrix} P^{-1} \vec{y}(0)$$

A : 2×2 行列　$\alpha, \beta \in \mathbb{R}$, $\alpha \neq \beta$ 固有値

$\alpha \rightsquigarrow \vec{v}_1$

$\beta \rightsquigarrow \vec{v}_2$

$P = (\vec{v}_1, \vec{v}_2)$ 正則.

$$\frac{d}{dt}\vec{y}(t) = A\vec{y}(t)$$

$\vec{z}(t) = P^{-1}\vec{y}(t)$ とおくと ....

$$\vec{z}(t) = \begin{pmatrix} e^{\alpha t} & 0 \\ 0 & e^{\beta t} \end{pmatrix} \begin{pmatrix} z_1(0) \\ z_2(0) \end{pmatrix}$$

$$\rightsquigarrow \quad \vec{y}(t) = P \begin{pmatrix} e^{\alpha t} & 0 \\ 0 & e^{\beta t} \end{pmatrix} P^{-1}\vec{y}(0)$$

まとめ

A : n次正方行列

$$\operatorname{rank}(A) = n \iff \det(A) \neq 0$$
$$\iff A : \text{正則}$$
$$\iff \operatorname{Ker}(A) = \{\vec{0}\}$$
$$\left(\iff \left(A\vec{x} = \vec{0} \Rightarrow \vec{x} = \vec{0} \right) \right)$$

$$\det \begin{pmatrix} a & 1 & 1 \\ 1 & a & 1 \\ 1 & 1 & a \end{pmatrix} = - \begin{vmatrix} 1 & a & 1 \\ a & 1 & 1 \\ 1 & 1 & a \end{vmatrix} = - \begin{vmatrix} 1 & a & 1 \\ 0 & 1-a^2 & 1-a \\ 0 & 1-a & a-1 \end{vmatrix}$$

$$= - \begin{vmatrix} 1-a^2 & 1-a \\ 1-a & a-1 \end{vmatrix}$$

$$= -(1-a)^2 \begin{vmatrix} 1+a & 1 \\ 1 & -1 \end{vmatrix}$$

$$= (a-1)^2 (a+2)$$

$a \neq 1, -2$ $\quad \operatorname{rank}(A) = 3.$

$a = 1, -2 \longrightarrow$ check する.

$$A = \begin{pmatrix} 2 & 1 & 1 \\ 1 & 2 & 1 \\ 1 & 1 & 2 \end{pmatrix}$$

$$\Phi_A(\lambda) = (\lambda - 1)^2 (\lambda - 4)$$

$$P = \frac{1}{\sqrt{6}} \begin{pmatrix} \sqrt{3} & 1 & \sqrt{2} \\ -\sqrt{3} & 1 & \sqrt{2} \\ 0 & -2 & \sqrt{2} \end{pmatrix}$$ ← 直交行列.

$$AP = P \begin{pmatrix} 1 & & \\ & 1 & \\ & & 4 \end{pmatrix}$$

$\boxed{\|\vec{x}\| = 1 \text{ の下で}}$

$$(A\vec{x}, \vec{x}) = (AP \cdot {}^tP\vec{x},\ P\,{}^tP\vec{x})$$

$$= ({}^tPAP \cdot {}^tP\vec{x},\ {}^tP\vec{x}) \qquad \vec{\xi} = {}^tP\vec{x}.$$

$$= \left(\begin{pmatrix} 1 & & \\ & 1 & \\ & & 4 \end{pmatrix} \vec{\xi},\ \vec{\xi} \right)$$

$$= \xi_1^2 + \xi_2^2 + 4\xi_3^2$$

${}^tP$ も直交

$$\|{}^tP\vec{x}\| = \|\vec{\xi}\| = 1$$

$$\boxed{\begin{aligned} P: \text{直交} \quad {}^tPP = I_n \iff & (P\vec{x}, P\vec{y}) = (\vec{x}, \vec{y}) \\ \iff & \|P\vec{x}\| = \|\vec{x}\| \end{aligned}}$$

$\xi_1^2+\xi_2^2+\xi_3^2=1$ の下で

$$y=\xi_1^2+\xi_2^2+4\xi_3^2$$

$$= \xi_1^2+\xi_2^2+4(1-\xi_1^2-\xi_2^2)$$

$$= 4-3\xi_1^2-3\xi_2^2 \leq 4.$$

attain.

$\xi_1=\xi_2=0,\ \xi_3=\pm 1.$

$$\vec{\xi}=\pm\begin{pmatrix}0\\0\\1\end{pmatrix} \rightsquigarrow$$

$$\vec{\xi}={}^tP\vec{x}$$

$$\vec{x}=P\vec{\xi}$$

$$=\pm P\begin{pmatrix}0\\0\\1\end{pmatrix}$$

$$=\pm\vec{p}_3$$

$$\xi_1^2+\xi_2^2=1-\xi_3^2$$

$$= 1-\xi_3^2+4\xi_3^2 = 1+3\xi_3^2 \geq 1$$

$\xi_3=0 \quad \xi_1^2+\xi_2^2=1$

$$\vec{x}=P\begin{pmatrix}\xi_1\\\xi_2\\0\end{pmatrix}$$

$$=\xi_1\vec{p}_1+\xi_2\vec{p}_2$$

$$\xi_1^2+\xi_2^2=1$$

最小値

1

レポート　期末

I

$$A = \begin{pmatrix} -2 & 2 & 1 \\ 2 & -5 & 1 \\ 1 & 1 & -3 \end{pmatrix}$$

$(A\vec{x}, \vec{x})$ が負定値を示せ.

II

$$A = \begin{pmatrix} 1 & 0 & 0 \\ 1 & 2 & 0 \\ 1 & 0 & -1 \end{pmatrix}$$

$$\frac{d\vec{x}(t)}{dt} = A\vec{x}(t)$$

を解け.

III　$w = xyz - x - y - z$ の極値

1月に出と書く問.　　期限 2/7, 8.

ボックスもあり.

$$A = \begin{pmatrix} 1 & 1 \\ 0 & 2 \end{pmatrix}$$

$$\frac{d}{dt}\vec{y}(t) = A\,\vec{y}(t)$$

[illegible]