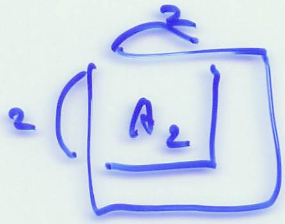
$$A = \begin{pmatrix} 2 & 1 & 1 \\ 1 & 2 & 1 \\ 1 & 1 & 2 \end{pmatrix}$$



$(A\vec{x}, \vec{x})$ が正定値.

$\Longleftrightarrow \ (A\vec{x}, \vec{x}) > 0 \quad (\vec{x} \neq \vec{0})$

$\Longleftrightarrow \ A_1 = (a_{11}) > 0$

$\det(A_2) > 0, \ \det(A) > 0$

$A_1 = 2 > 0, \quad A_2 = \begin{vmatrix} 2 & 1 \\ 1 & 2 \end{vmatrix} = 4 - 1 = 3 > 0.$

$$|A| = \begin{vmatrix} 2 & 1 & 1 \\ 1 & 2 & 1 \\ 1 & 1 & 2 \end{vmatrix} = - \begin{vmatrix} 1 & 1 & 2 \\ 1 & 2 & 1 \\ 2 & 1 & 1 \end{vmatrix}$$

1列の展開

$$= - \begin{vmatrix} 1 & 1 & 2 \\ 0 & 1 & -1 \\ 0 & -1 & -3 \end{vmatrix}$$

$$\begin{array}{rrrr} & 2 & 1 & 1 \\ -) & 2 & 2 & 4 \\ \hline & 0 & -1 & -3 \end{array}$$

$$= - \begin{vmatrix} 1 & -1 \\ -1 & -3 \end{vmatrix} = -\left((-3) - 1\right) = 4 > 0$$

$A_1 > 0$, $\det(A_2) > 0$, $\det(A) > 0$

$\Longrightarrow$ $(A\vec{x}, \vec{x})$ 正定値.

- $n=2$のとき. $A: 2\times 2$. 対称.

  $A_1 > 0$, $\det(A) \Longleftrightarrow (A\vec{x}, \vec{x})$: 正定値.

$$T = \begin{pmatrix} 1 & 0 & t_1 \\ 0 & 1 & t_2 \\ 0 & 0 & 1 \end{pmatrix} \qquad \begin{pmatrix} t_1 \\ t_2 \end{pmatrix} = \vec{t}$$

正則.

$\vec{x} = T\vec{w}$

$\boxed{\vec{x} \neq \vec{0} \Longleftrightarrow \vec{w} \neq \vec{0}}$

$$(A\vec{x}, \vec{x}) = (AT\vec{w}, T\vec{w})$$

$$= ({}^tTAT\vec{w}, \vec{w}) \underbrace{> 0 \quad (全ての\vec{w} \neq \vec{0})}_{示すべきこと.}$$

$$A = \left(\begin{array}{c|c} A_2 & \vec{a} \\ \hline {}^t\vec{a} & a_{33} \end{array}\right)$$

$${}^tTAT = \left(\begin{array}{c|c} A_2 & \boxed{A_2\vec{t} + \vec{a}} \\ \hline {}^t(A_2\vec{t} + \vec{a}) & \delta \end{array}\right)$$

$\overset{\parallel}{\vec{0}}$

$\vec{t} = A_2^{-1}\vec{a}$

$\det(A_2) > 0$

A_2 : 正則.

$$\delta = {}^t\vec{t}\, A_2 \vec{t} + a_{33}.$$

$\det(A_1) > 0$

$\det(A_2) > 0$

$$= \left(\begin{array}{cc|c} \multicolumn{2}{c|}{A_2} & \begin{matrix} 0 \\ 0 \end{matrix} \\ \hline 0 & 0 & \delta \end{array}\right)$$

$\vec{y} = \begin{pmatrix} w_1 \\ w_2 \end{pmatrix}$

$({}^tTAT\vec{w}, \vec{w})$

$y \neq \vec{0}$に対して > 0

$$= \boxed{(A_2\vec{y}, \vec{y})} + \delta |w_3|^2$$

δ > 0 を示す.

$$\det({}^tTAT) = \det\left(\begin{array}{c|c} A_2 & \begin{matrix}0\\0\end{matrix} \\ \hline 0\ 0 & \delta \end{array}\right) = \boxed{\det(A_2)}\cdot\delta$$

$\det(A_2) > 0$ 仮定

$$\det({}^tTAT) = \det({}^tT)\det(A)\det(T)$$

$\det({}^tT) = 1$, $\det(T) = 1$

$$\rightsquigarrow \det(A) = \det(A_2)\cdot\delta$$

$\det(A) > 0$ 仮定　　$\det(A_2) > 0$ 仮定.

$$\rightsquigarrow \delta > 0$$

$$({}^tTAT\vec{w}, \vec{w}) = (A_2\vec{y}, \vec{y}) + \underbrace{\delta w_3^2}_{\geqq 0} \geqq 0$$

$$\vec{y} = \begin{pmatrix} w_1 \\ w_2 \end{pmatrix}$$

$(A_2\vec{y}, \vec{y}) \geqq 0$, $= 0 \iff w_1 = w_2 = 0$

$\delta w_3^2 \geqq 0$, $= 0 \iff w_3 = 0$

$({}^tTAT\vec{w}, \vec{w}) = 0 \iff w_1 = w_2 = w_3 = 0$

$$\vec{w} \neq \vec{0} \Rightarrow ({}^tTAT\vec{w}, \vec{w}) > 0$$

$$({}^tTAT\vec{w}, \vec{w}) = (A\vec{x}, \vec{x})$$

A : n次正方行列, 実対称.

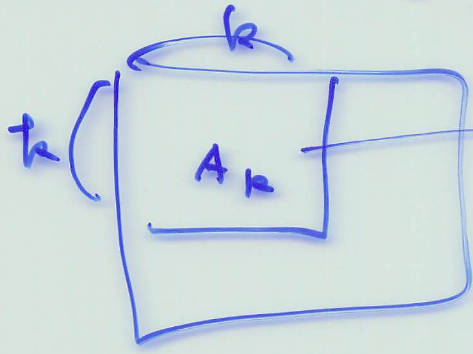
$(A\vec{x}, \vec{x}) > 0 \quad (x \neq \vec{0})$

A の固有値 $\alpha_1, \cdots, \alpha_n \in \mathbb{R}$

実対称.

$\Longleftrightarrow \alpha_1, \cdots, \alpha_n > 0$

$\Longleftrightarrow \det(A_1), \det(A_2), \cdots, (-1)^n \det(A_n) > 0$



← k次の主座小行列.

$(-1)^k \det(A_k) > 0$

$(k = 1, 2, \cdots, n)$

$$f = x^3 + y^3 + z^3 - 3xz - 3y^2.$$

$$\begin{cases} f_x = 3x^2 - 3z = 0 \dashrightarrow z = x^2 \\ f_y = 3y^2 - 6y = 0 \dashrightarrow y = 0, 2 \\ f_z = 3z^2 - 3x = 0 \dashrightarrow x = z^2 \end{cases}$$

$$\Rightarrow \quad x = x^4 \iff x = 0, 1.$$

$(0,0,0)$, $(1,0,1)$, $(0,2,0)$, $(1,2,1)$

が停留点

$$H = \begin{pmatrix} 6x & 0 & -3 \\ 0 & 6(y-1) & 0 \\ -3 & 0 & 6z \end{pmatrix}$$

$F(x,z) = f(x,0,z)$

$\underline{(0,0,0)}$

$$\begin{vmatrix} 6x & -3 \\ -3 & 6z \end{vmatrix} = \begin{vmatrix} 0 & -3 \\ -3 & 0 \end{vmatrix} = -9 < 0.$$

$\rightsquigarrow$ f は $(0,0,0)$ で極大でも極小でもない

<u>定理</u>

$g_x(a,b) = g_y(a,b) = 0$

$$\begin{vmatrix} g_{xx} & g_{xy} \\ g_{yx} & g_{yy} \end{vmatrix} < 0 \text{ at } (a,b)$$

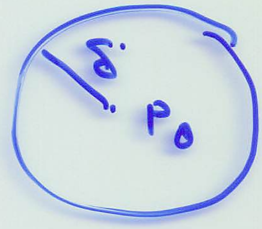
H. 固有値 α, β

$\longrightarrow$ 極小でも極大でもない

$$\underset{\overset{\wedge}{0}}{\det(H)} = \alpha\beta \rightsquigarrow \alpha > 0,\ \beta < 0.$$

$P_0(a,b,c)$ が f の 極小値.

弱い意味で



$$\left(\begin{array}{l} f(a,b,c) \leqq f(x,y,z) \\ \qquad\qquad\quad \geqq \\ \qquad\qquad (x,y,z) \in B_\delta(P_0) \end{array} \right.$$

をみたす $\delta > 0$ あり.

$F(t) = f(t,b,c)$ $t = a$ で弱い意味で 極小.

$$\Longrightarrow F'(a) = 0$$

$\leadsto$ $f_x(a,b,c) = f_y(a,b,c) = f_z(a,b,c) = 0.$

P_0 は f の停留点.

<u>定理</u>

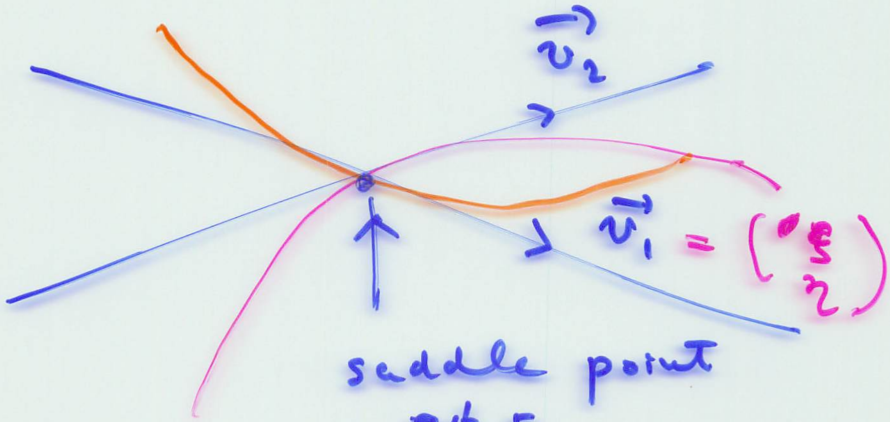
$$H = \begin{pmatrix} f_{xx} & f_{xy} & f_{xz} \\ f_{yx} & f_{yy} & f_{yz} \\ f_{zx} & f_{zy} & f_{zz} \end{pmatrix} (P_0)$$

$(H\vec{\xi}, \vec{\xi})$ が 正定値 $\Longrightarrow$ f は P_0 で <u>強い</u> 意味で 極小.

負. 大

$\alpha > 0 \quad \vec{v}_1, \ v \qquad \beta < 0 \quad \vec{v}_2$

$$H\vec{v}_1 = \alpha \vec{v}_1 \qquad H\vec{v}_2 = \beta \vec{v}_2$$



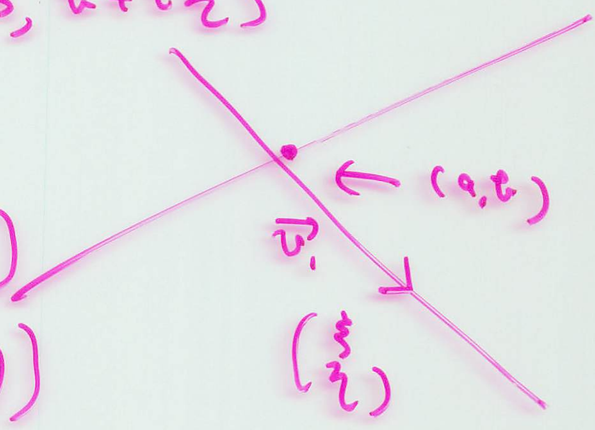
$G(t) = g(a + t\xi, \ b + t\eta)$

$G'(0) = 0$

$$G''(0) = \left(H \begin{pmatrix} \xi \\ \eta \end{pmatrix}, \begin{pmatrix} \xi \\ \eta \end{pmatrix} \right)$$

$$= \left(\alpha \begin{pmatrix} \xi \\ \eta \end{pmatrix}, \begin{pmatrix} \xi \\ \eta \end{pmatrix} \right)$$

$$= \alpha \| \vec{v}_1 \|^2 > 0$$



$f(t) = \cos t$, $f'(t) = -\sin t$, $f''(t) = -\cos t$,

$f^{(3)}(t) = \sin t$

$f^{(4)}(t) = \cos t$.

$$\boxed{\begin{aligned} &\underline{\text{公式}}\quad (\cos t)' = -\sin t \\ &\qquad\quad (\sin t)' = \cos t \end{aligned}}$$

$f^{(4k)}(t) = \cos t$ $\qquad$ $f^{(4k)}(0) = 1$

$f^{(4k+1)}(t) = -\sin t$ $\qquad$ $f^{(4k+1)}(0) = f^{(4k+3)}(0) = 0$

$f^{(4k+2)}(t) = -\cos t$ $\qquad$ $f^{(4k+2)}(0) = -1$.

$f^{(4k+3)}(t) = \sin t$.

$$\cos t = 1 - \frac{1}{2!}x^2 + \frac{1}{4!}x^4 - + \cdots\cdots$$

$$\sin t = x - \frac{1}{3!}x^3 + \frac{1}{5!}x^5 - + \cdots$$

$$e^t = 1 + t + \frac{t^2}{2!} + \frac{1}{3!}t^3 + \cdots\cdots$$

$$e^{it} = 1 + it - \frac{1}{2!}t^2 - \frac{1}{3!}it^3 + \frac{1}{4!}t^4 \cdots\cdots$$

$$= \left(1 - \frac{1}{2!}t^2 + \frac{1}{4!}t^4 - + \cdots\right) + i\left(t - \frac{1}{3!}t^3 + \frac{1}{5!}t^5 - + \cdots\right)$$

$$= \cos t + i \sin t$$

Euler の定理

$\alpha = a + ib.$ $(a, b \in \mathbb{R})$

$$e^{\alpha t} := e^{at}(\cos bt + i \sin bt)$$

定義

$$\boxed{(e^{\alpha t})' = \alpha e^{\alpha t}}$$

$$F(t) = f_1(t) + i f_2(t)$$

実数値

$$\boxed{F'(t) = f_1'(t) + i f_2'(t)}$$

$$(\cos bt + i \sin bt)'$$

$$= -b \sin bt + i b \cos bt$$

$$= ib(\cos bt + i \sin bt)$$

$$\boxed{(e^{ibt})' = ib\, e^{ibt}}$$

F, G 複素数値

$$(FG)' = F'G + FG'$$

$$(e^{\alpha t})' = (e^{at})' e^{ibt} + e^{at}(e^{ibt})'$$

$$= a e^{at} e^{ibt} + e^{at}\, ib\, e^{ibt}$$

$$= (a + ib) e^{at} e^{ibt}$$

$$= \alpha e^{\alpha t}$$

$$\boxed{(e^{\alpha t})' = \alpha e^{\alpha t}}$$

(1) $(1, 2, 1)$ で極小を示せ

(2) $(1, 0, 1)$, $(0, 2, 0)$ では極小・極大でない

x, y について考える

x と z.

$H_1 > 0$, $\det(H_2) > 0$, $\det(H) > 0$