$$\Phi_A(\lambda) = (\lambda-2)^2(\lambda-8)$$

$$A = \begin{pmatrix} 3 & -1 & -2 \\ -1 & 3 & 2 \\ -2 & 2 & 6 \end{pmatrix}$$

$V(2) := \mathrm{Ker}(2I_3 - A)$

$$\begin{pmatrix} 2 & & \\ & 2 & \\ & & 2 \end{pmatrix} - \begin{pmatrix} 3 & -1 & -2 \\ -1 & 3 & 2 \\ -2 & 2 & 6 \end{pmatrix} = \begin{pmatrix} -1 & 1 & +2 \\ 1 & -1 & -2 \\ 2 & -2 & -4 \end{pmatrix}$$

$$\downarrow$$

$$\begin{pmatrix} 1 & -1 & -2 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

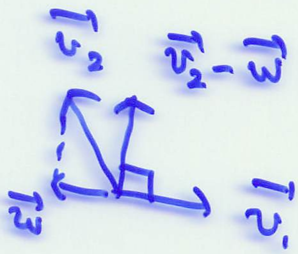
$$x - y - 2z = 0$$

$$x = y + 2z$$

$$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} y + 2z \\ y \\ z \end{pmatrix} = y\begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix} + z\begin{pmatrix} +2 \\ 0 \\ 1 \end{pmatrix}$$

$$\vec{v_1} = \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix}, \quad \vec{v_2} = \begin{pmatrix} +2 \\ 0 \\ 1 \end{pmatrix}$$

$$\vec{p_1} = \frac{1}{\|\vec{v_1}\|}\vec{v_1} = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix}$$



$\vec{v_2}$ の $\vec{p_1}$ 方向への正射影を $\vec{w}$

$$\vec{w} = (\vec{p_1}, \vec{v_2})\vec{p_1} = \left(\frac{1}{\sqrt{2}}\begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix}, \begin{pmatrix} +2 \\ 0 \\ 1 \end{pmatrix}\right) \times \frac{1}{\sqrt{2}}\begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix}$$

$$= +\begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix} = \vec{v_1}$$

$$\vec{v_2} - \vec{w} = \vec{v_2} - \vec{v_1} = \begin{pmatrix} 2 \\ 0 \\ 1 \end{pmatrix} - \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix} = \begin{pmatrix} 1 \\ -1 \\ 1 \end{pmatrix}$$

$$\vec{p_2} = \frac{1}{\sqrt{3}}\begin{pmatrix} 1 \\ -1 \\ 1 \end{pmatrix}$$

$\underline{V(8) = \mathrm{Ker}(8I_3 - A)}$

$$\begin{pmatrix} 8 & & \\ & 8 & \\ & & 8 \end{pmatrix} - \begin{pmatrix} 3 & -1 & -2 \\ -1 & 3 & 2 \\ -2 & 2 & 6 \end{pmatrix}$$

$$= \begin{pmatrix} 5 & 1 & 2 \\ 1 & 5 & -2 \\ 2 & -2 & 2 \end{pmatrix} \longrightarrow \begin{pmatrix} 5 & 1 & 2 \\ 1 & 5 & -2 \\ 1 & -1 & 1 \end{pmatrix}$$

$$\downarrow$$

$$\begin{pmatrix} 1 & -1 & 1 \\ 0 & 6 & -3 \\ 0 & 6 & -3 \end{pmatrix} \longleftarrow \begin{pmatrix} 1 & -1 & 1 \\ 1 & 5 & -2 \\ 5 & 1 & 2 \end{pmatrix}$$

$$\cancel{\begin{array}{rrr} 5 & 1 & 2 \\ -)\ 5 & & \\ \hline 0 & -24 & \end{array}}$$

$$\begin{array}{rrr} 5 & 1 & 2 \\ -)\ 5 & -5 & 5 \\ \hline 0 & 6 & -3 \end{array}$$

$$\downarrow$$

$$\begin{pmatrix} 1 & -1 & 1 \\ 0 & 1 & -\frac{1}{2} \\ 0 & 0 & 0 \end{pmatrix} \longrightarrow \begin{pmatrix} 1 & 0 & \frac{3}{2} \\ 0 & 1 & -\frac{1}{2} \\ 0 & 0 & 0 \end{pmatrix}$$

$$\begin{pmatrix} x \\ y \\ z \end{pmatrix} \in V(8) \iff \begin{cases} x = -\frac{3}{2} z \\ y = \frac{1}{2} z \end{cases}$$

$$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} -\frac{3}{2} z \\ \frac{1}{2} z \\ z \end{pmatrix} = \frac{z}{2} \begin{pmatrix} -3 \\ 1 \\ 2 \end{pmatrix} \qquad \vec{p}_3 = \frac{1}{\sqrt{14}} \begin{pmatrix} -3 \\ 1 \\ 2 \end{pmatrix}$$

<u>定理</u> A : 対称, $\alpha \neq \beta$ 固有値

$$\left.\begin{array}{l} A\vec{v}_1 = \alpha \vec{v}_1 \\ A\vec{v}_2 = \beta \vec{v}_2 \end{array}\right\} \longrightarrow (\vec{v}_1, \vec{v}_2) = 0$$

$$(\vec{p}_1, \vec{p}_3) = (\vec{p}_2, \vec{p}_3) = 0$$

$P = (\vec{p}_1\ \vec{p}_2\ \vec{p}_3)$ 直交.

$${}^tPP = P\,{}^tP = I_3$$

$$AP = P \begin{pmatrix} 2 & & \\ & 2 & \\ & & 8 \end{pmatrix}$$

$${}^tPAP = \begin{pmatrix} 2 & & \\ & 2 & \\ & & 8 \end{pmatrix}$$

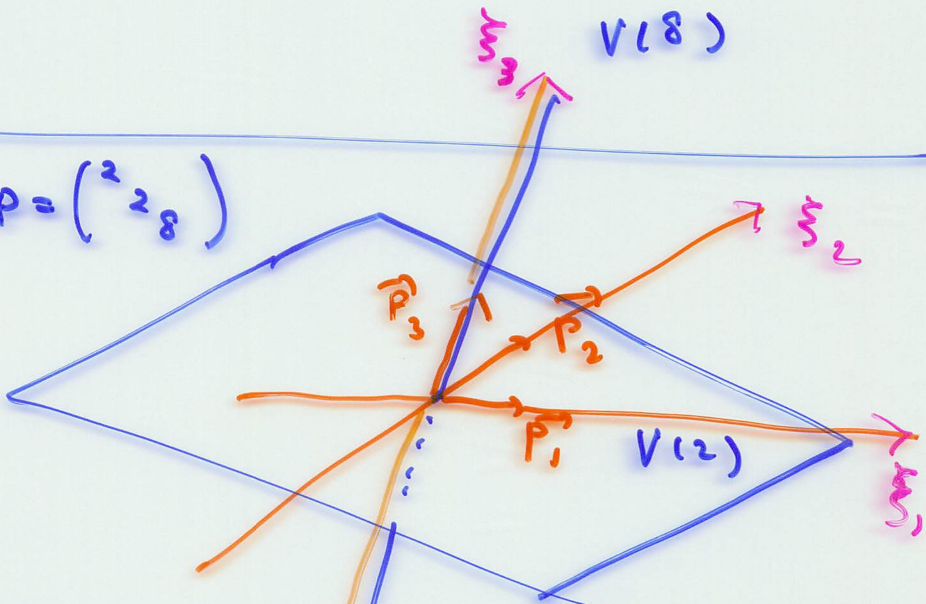
2次形式

$$(A\vec{x}, \vec{x}) = 3\alpha^2 + 3\beta^2 + 6\gamma^2 - 2\alpha\beta - 4\alpha\gamma + 4\beta\gamma.$$

$$A = \begin{pmatrix} 3 & -1 & -2 \\ -1 & 3 & 2 \\ -2 & 2 & 6 \end{pmatrix}$$

$$\vec{x} = \begin{pmatrix} \alpha \\ \beta \\ \gamma \end{pmatrix}$$

$${}^tPAP = \begin{pmatrix} 2 & & \\ & 2 & \\ & & 8 \end{pmatrix}$$



$$= (A\overbrace{P\,{}^tP}^{I_3}\vec{x}, \underbrace{P\,{}^tP}_{I_3}\vec{x})$$

$$= ({}^tPAP\,\underbrace{{}^tP\vec{x}}_{\vec{\xi}}, \underbrace{{}^tP\vec{x}}_{\vec{\xi}})$$

$$\vec{\xi} = \begin{pmatrix} \xi_1 \\ \xi_2 \\ \xi_3 \end{pmatrix}$$

$$= \left(\begin{pmatrix} 2 & & \\ & 2 & \\ & & 8 \end{pmatrix} \begin{pmatrix} \xi_1 \\ \xi_2 \\ \xi_3 \end{pmatrix}, \begin{pmatrix} \xi_1 \\ \xi_2 \\ \xi_3 \end{pmatrix} \right)$$

$$= 2\xi_1^2 + 2\xi_2^2 + 8\xi_3^2$$

$${}^tP\vec{x} = \vec{\xi} \rightarrow \vec{x} = P\vec{\xi} = (\vec{p}_1, \vec{p}_2, \vec{p}_3)\begin{pmatrix} \xi_1 \\ \xi_2 \\ \xi_3 \end{pmatrix}$$

$$= \xi_1\vec{p}_1 + \xi_2\vec{p}_2 + \xi_3\vec{p}_3$$

$z = f(x, y)$

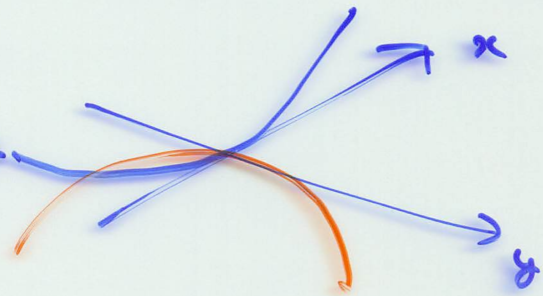
$$f_x(a,b) = f_y(a,b) = 0$$

i.e. (a,b) は f の停留点である.

$z = x^2 - y^2$ $\quad z_x = 2x$

$\qquad\qquad\qquad\quad z_y = -2y$

$(0,0)$: 停留点



$$H = H(f) = \begin{pmatrix} f_{xx} & f_{xy} \\ f_{yx} & f_{yy} \end{pmatrix} \text{ at } (a,b)$$

Hesse 行列.

$f_{xy} = f_{yx}$. 対称.

$(H\vec{v}, \vec{v})$ は 正定値.
負

定義

(i) $\boxed{(H\vec{v}, \vec{v}) > 0 \quad \vec{v} \neq \vec{0}}$ $<$

$\Longrightarrow$ (a,b) は極小.
大.

H の固有値 α, β.

$\left(\begin{array}{l} \alpha, \beta > 0 \\ \alpha, \beta < 0 \end{array} \right) \iff f_{xx}(a,b) > 0$ ($<$)

かつ

$\det(H) > 0$

B : 2次実対称.

$(B\vec{x}, \vec{x}) > 0 \quad (\vec{x} \neq \vec{0})$

$\iff B_1 = (b_{11}) > 0$

かつ $\det B > 0$.

A : 3次実対称.　　　　A : $\alpha, \beta, \gamma \in \mathbb{R}$ 固有値.

$(A\vec{x}, \vec{x})$ が… 正定値

負

$\Longleftrightarrow \quad (A\vec{x}, \vec{x}) > 0 \quad (\vec{x} \neq \vec{0})$

$<$

$\Longleftrightarrow \quad \alpha, \beta, \gamma > 0.$

$<$

$\Longleftrightarrow \quad A_1 > 0, \ \det(A_2) > 0,$

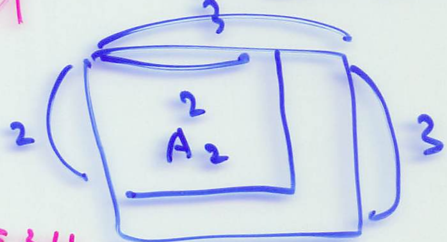
$<$

$\det(A) > 0$

$<$

$A_1 = (a_{11})$

$A_2 = \begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{pmatrix}$



Aの主座小行列

直交行列 P が存在し

$${}^tPAP = \begin{pmatrix} \alpha & & \\ & \beta & \\ & & \gamma \end{pmatrix} \qquad {}^tP\vec{x} = \vec{\xi}, \quad P\vec{\xi} = \vec{x}$$

$$(A\vec{x}, \vec{x}) = \alpha\xi_1^2 + \beta\xi_2^2 + \gamma\xi_3^2$$

$$\boxed{\vec{x} \neq \vec{0} \Longleftrightarrow \vec{\xi} \neq \vec{0}}$$

P : 正則.

$(\vec{x} \neq \vec{0} \Rightarrow (A\vec{x}, \vec{x}) > 0)$　　// 正

$\Longleftrightarrow \left(\vec{\xi} \neq \vec{0} \Rightarrow \alpha\xi_1^2 + \beta\xi_2^2 + \gamma\xi_3^2 > 0\right)$

$\alpha, \beta, \gamma > 0 \Longrightarrow$ 正　　　　正 $\Rightarrow \alpha > 0$

$\alpha \leqq 0$ とする. $\vec{\xi} = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}$　　$\alpha\xi_1^2 + \beta\xi_2^2 + \gamma\xi_3^2 = \alpha \leqq 0$

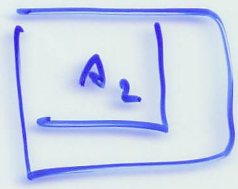
• $(A\vec{x},\vec{x}) > 0 \quad (\vec{x} \neq \vec{0}) \Leftrightarrow \alpha, \beta, \gamma > 0$ 固有値.

$\Downarrow$

• $A_1 > 0,\ \det(A_2) > 0,\ \det(A_3) > 0$

↑は平凡. $\vec{x} = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}$ $(A\vec{x},\vec{x}) = a_{11} > 0$

$\vec{x} = \begin{pmatrix} x_1 \\ x_2 \\ 0 \end{pmatrix}$ $\vec{y} = \begin{pmatrix} x_1 \\ x_2 \end{pmatrix}$



$(A\vec{x},\vec{x}) = (A_2\vec{y},\vec{y})$

$(A_2\vec{y},\vec{y}) > 0 \quad (\vec{y} \neq 0)$ が必要

$n=2$ の結果から $\det(A_2) > 0$ が分かる.

$\det(A) > 0$ が必要.

$${}^tPAP = \begin{pmatrix} \alpha & & \\ & \beta & \\ & & \gamma \end{pmatrix}$$

$$\det({}^tPAP) = \alpha\beta\gamma$$
$$\parallel$$
$$\det({}^tP)\det(A)\det(P)$$

$$\det({}^tPP) = \det(I_3) = 1$$

$$\det(A) = \alpha\beta\gamma > 0$$

$n=2$　　H : 2次対称.　固有値 α, β.

$(H\vec{x}, \vec{x}) > 0 \quad (\vec{x} \neq 0)$

$\Longleftrightarrow \quad \alpha, \beta > 0 \quad \Longleftrightarrow \quad H_1 = (h_{11}) > 0$

$\det(H) > 0$

$\Rightarrow$ は同様.

$\Leftarrow$

ok

$n=3$と同じ.

$$H = \begin{pmatrix} a & c \\ c & b \end{pmatrix}$$

$$\Phi_H(\lambda) = \lambda^2 - (a+b)\lambda + (ab - c^2)$$
$$= (\lambda - \alpha)(\lambda - \beta)$$

仮定　$ab - c^2 > 0$, $a > 0$.

$$\cancel{\Phi_{{}^tPHP}(\lambda) = \Phi_H(\lambda) = (\lambda - \alpha)(\lambda - \beta)}$$

$$\begin{cases} \alpha + \beta = a + b \\ \alpha\beta = ab - c^2 > 0 \end{cases}$$

$\uparrow$ 仮定.　　仮定

$$ab > c^2 \geqq 0, \quad a > 0$$

$$b > 0$$

$$\left.\begin{array}{l} \alpha + \beta = a + b > 0 \\ \alpha\beta > 0 \end{array}\right\} \rightarrow \underline{\alpha, \beta > 0}$$

$$A = \begin{pmatrix} 3 & -1 & -2 \\ -1 & 3 & 2 \\ -2 & 2 & 6 \end{pmatrix}$$

固有値 2, 2, 8

$$3 > 0, \quad \begin{vmatrix} 3 & -1 \\ -1 & 3 \end{vmatrix} = 9 - 1 = 8 > 0$$

$$\begin{vmatrix} 3 & -1 & -2 \\ -1 & 3 & 2 \\ -2 & 2 & 6 \end{vmatrix} = - \begin{vmatrix} -1 & 3 & 2 \\ 3 & -1 & -2 \\ -2 & 2 & 6 \end{vmatrix} = \begin{vmatrix} 1 & -3 & -2 \\ 3 & -1 & -2 \\ -2 & 2 & 6 \end{vmatrix}$$

$$\begin{array}{rrrr} & 3 & -1 & -2 \\ - & 3 & -9 & -6 \\ \hline & 0 & 8 & 4 \end{array} \qquad \begin{array}{rrrr} & -2 & 2 & 6 \\ + & 2 & -6 & -4 \\ \hline & 0 & -4 & 2 \end{array}$$

$$= \begin{vmatrix} \boxed{\begin{matrix} 1 \\ 0 \\ 0 \end{matrix}} & \begin{matrix} 3 & -2 \\ 8 & 4 \\ -4 & 2 \end{matrix} \end{vmatrix}$$

$$\parallel$$

$$\begin{vmatrix} 8 & 4 \\ -4 & 2 \end{vmatrix} = 32 > 0$$

Ⅰ $A = \begin{pmatrix} 2 & 1 & 1 \\ 1 & 2 & 1 \\ 1 & 1 & 2 \end{pmatrix}$

$(A\vec{x}, \vec{x})$ が正定値を示せ

Ⅱ

$2 > 0 \qquad \begin{vmatrix} 2 & 1 \\ 1 & 2 \end{vmatrix} = 3 > 0$

$\begin{vmatrix} 2 & 1 & 1 \\ 1 & 2 & 1 \\ 1 & 1 & 2 \end{vmatrix} =$