I $A = \begin{pmatrix} 2 & 0 & 2 \\ 1 & -1 & -2 \\ -1 & 3 & 5 \end{pmatrix}$

$$A\vec{v} = \lambda I_3 \vec{v}$$

$$(\lambda I_3 - A)\vec{v} = \vec{0}$$

$\boxed{A\vec{v} = \lambda \vec{v}}$, $\vec{v} \neq \vec{0}$ をみたす $\vec{v}$ が存在

$\Longleftrightarrow$ $|\lambda I_3 - A| = 0$

固有多項式

B : n次の正方行列.

$\left(B\vec{x} = \vec{0} \Rightarrow \vec{x} = \vec{0} \right) \Longleftrightarrow \det(B) \neq 0$

$\Longleftrightarrow$ B : 正則

$\left(B\vec{x} = \vec{0},\ \vec{x} \neq \vec{0}\ \text{をみたす}\ \vec{x}\ \text{あり} \right) \Longleftrightarrow \det(B) = 0$

$$|\lambda I_3 - A| = (\lambda - 1)(\lambda - 2)(\lambda - 3)$$

$\underline{\lambda = 1}$ $\vec{p_1} = \begin{pmatrix} -2 \\ -2 \\ 1 \end{pmatrix}$.

$\underline{\lambda = 2}$ $\vec{p_2} = \begin{pmatrix} 3 \\ 1 \\ 0 \end{pmatrix}$ $\underline{\lambda = 3}$ $\vec{p_3} = \begin{pmatrix} 2 \\ 0 \\ 1 \end{pmatrix}$

$$P = (\vec{p_1}\ \vec{p_2}\ \vec{p_3})$$

$$AP = (A\vec{p_1}\ A\vec{p_2}\ A\vec{p_3}) = (\vec{p_1}\ 2\vec{p_2}\ 3\vec{p_3})$$

$$= (\vec{p_1}\ \vec{p_2}\ \vec{p_3}) \begin{pmatrix} 1 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 3 \end{pmatrix} = P \begin{pmatrix} 1 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 3 \end{pmatrix}$$

より

$$(\vec{a_1}\ \vec{a_2} \cdots \vec{a_\ell}) \begin{pmatrix} c_1 \\ c_2 \\ \vdots \\ c_\ell \end{pmatrix} = c_1\vec{a_1} + c_2\vec{a_2} + \cdots + c_\ell \vec{a_\ell}$$

$$|P| = \begin{vmatrix} -2 & 3 & 2 \\ -2 & 1 & 0 \\ 1 & 0 & 1 \end{vmatrix} = \begin{vmatrix} 0 & 3 & 4 \\ 0 & 1 & 2 \\ 1 & 0 & 1 \end{vmatrix} = \begin{vmatrix} 3 & 4 \\ 1 & 2 \end{vmatrix} = 6-4 = 2 \neq 0$$

$\to$ P：正則

定理 A：n次正方行列

$\alpha_1, \cdots, \alpha_k$：固有値

$\alpha_i \neq \alpha_j \quad (i \neq j)$

$A\vec{v}_i = \alpha_i \vec{v}_i \quad (i = 1, \cdots, k)$

$$\vec{v}_1 + \cdots + \vec{v}_k = \vec{0} \Rightarrow \vec{v}_1 = \cdots = \vec{v}_k = \vec{0}$$

$\cdots$ ①

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kの帰納法，$k=1$ 明らか．

$$\begin{array}{ll} ①\times\alpha_k & \alpha_k\vec{v}_1 + \alpha_k\vec{v}_2 + \cdots + \alpha_k\vec{v}_{k-1} + \cancel{\alpha_k\vec{v}_k} = \vec{0} \\ A\cdot① & \alpha_1\vec{v}_1 + \alpha_2\vec{v}_2 + \cdots + \alpha_{k-1}\vec{v}_{k-1} + \cancel{\alpha_k\vec{v}_k} = \vec{0} \end{array}$$

$$\boxed{(\alpha_k - \alpha_1)\vec{v}_1} + (\alpha_k - \alpha_2)\vec{v}_2 + \cdots + (\alpha_k - \alpha_{k-1})\vec{v}_{k-1} = \vec{0}$$

$$A(\vec{v}_1 + \cdots + \vec{v}_k) = A\vec{v}_1 + A\vec{v}_2 + \cdots + A\vec{v}_k$$
$$= \alpha_1\vec{v}_1 + \alpha_2\vec{v}_2 + \cdots + \alpha_k\vec{v}_k$$

$$A\boxed{(\alpha_k - \alpha_1)\vec{v}_1} = (\alpha_k - \alpha_1)A\vec{v}_1$$
$$= (\alpha_k - \alpha_1)\alpha_1\vec{v}_1$$
$$= \alpha_1\boxed{(\alpha_k - \alpha_1)\vec{v}_1}$$

$\vec{v}_1$と思う

帰納法 $\underbrace{(\alpha_k - \alpha_1)}_{\neq 0}\vec{v}_1 = \cdots = \underbrace{(\alpha_k - \alpha_{k-1})}_{\neq 0}\vec{v}_{k-1} = \vec{0}$

$\leadsto \vec{v}_1 = \vec{v}_2 = \cdots = \vec{v}_{k-1} = \vec{0} \leadsto \vec{v}_k = \vec{0}$

$\vec{p}_1 , \vec{p}_2 , \vec{p}_3$

$A\vec{p}_1 = \vec{p}_1 , \quad A\vec{p}_2 = 2\vec{p}_2 , \quad A\vec{p}_3 = 3\vec{p}_3$

$$(\vec{p}_1\ \vec{p}_2\ \vec{p}_3)\begin{pmatrix}c_1\\c_2\\c_3\end{pmatrix} = \underbrace{c_1\vec{p}_1}_{\vec{v}_1} + \underbrace{c_2\vec{p}_2}_{\vec{v}_2} + \overbrace{c_3\vec{p}_3}^{\vec{v}_3} = \vec{0}$$

とする.

$\alpha_1 = 1, \quad \alpha_2 = 2, \quad \alpha_3 = 3$

$$\boxed{\begin{aligned} A\vec{v}_3 &= A c_3 \vec{p}_3 \\ &= c_3 A\vec{p}_3 \\ &= 3c_3\vec{p}_3 = 3\vec{v}_3 \end{aligned}}$$

$\to \ c_1\vec{p}_1 = c_2\vec{p}_2 = c_3\vec{p}_3 = \vec{0} \ \rightsquigarrow \ c_1 = c_2 = c_3 = 0$

$\vec{p}_1 \neq \vec{0}, \ \vec{p}_2 \neq \vec{0}, \ \vec{p}_3 \neq \vec{0}$

- $\left(P\begin{pmatrix}c_1\\c_2\\c_3\end{pmatrix} = \vec{0} \Rightarrow \vec{c} = \vec{0} \right)$

$\Longleftrightarrow \det(P) \neq 0 \Longleftrightarrow P$: 正則.

- $\Phi_A(\lambda) = \lambda^n - (a_{11} + a_{22} + \cdots + a_{nn})\lambda^{n-1}$

$+ \cdots + (-1)^n \det(A) = 0.$

$\| \quad |\lambda I_n - A|$

$\mathrm{tr}(A) = a_{11} + a_{22} + \cdots + a_{nn}$

$A = (a_{ij})$ 　 Aのtrace，跡．

代数学の基本定理．

$$f(\lambda) = \lambda^n + a_{n-1}\lambda^{n-1} + \cdots + a_1\lambda + a_0$$

$$\rightsquigarrow \quad f(\lambda) = (\lambda - \alpha_1)\cdots(\lambda - \alpha_n)$$

$\alpha_1, \cdots, \alpha_n \in \mathbb{C}$

$\lambda^2 + 1 = (\lambda - i)(\lambda + i)$

A：n次正方行列．

$$\Phi_A(\lambda) = (\lambda - \alpha_1)\cdots(\lambda - \alpha_n)$$

$\alpha_1, \alpha_2, \cdots, \alpha_n \in \mathbb{C}$

固有値．

定理　A：n次正方行列

$\alpha_1, \cdots, \alpha_n$：固有値．

$\alpha_i \neq \alpha_j \quad (i \neq j)$

$\Phi_A(\lambda)$が単根

$\Rightarrow A$：対角化可能．

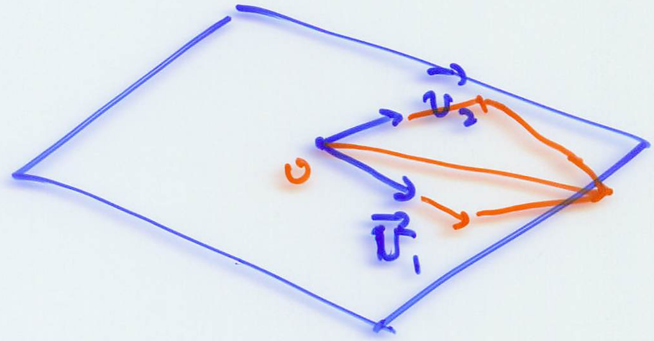
$A\vec{p_i} = \alpha_i \vec{p_i} \quad (\vec{p_i} \neq \vec{0})$

$\Rightarrow$

$$AP = P\begin{pmatrix} \alpha_1 & & \\ & \ddots & \\ & & \alpha_n \end{pmatrix}, \quad P = (\vec{p_1} \cdots \vec{p_n})$$

正則．

$$V(2)=\left\{\begin{pmatrix}x\\y\\z\end{pmatrix};\ A\begin{pmatrix}x\\y\\z\end{pmatrix}=2\begin{pmatrix}x\\y\\z\end{pmatrix}\right\}$$

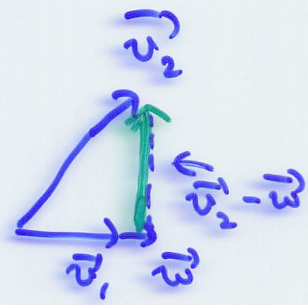


$$=\left\{\begin{pmatrix}x\\y\\z\end{pmatrix};\ x-y-2z=0\right\}$$

$$\vec{v}_1=\begin{pmatrix}1\\1\\0\end{pmatrix},\quad \vec{v}_2=\begin{pmatrix}2\\0\\1\end{pmatrix}$$

$$=\mathbb{R}\vec{v}_1+\mathbb{R}\vec{v}_2$$

$$=\{c_1\vec{v}_1+c_2\vec{v}_2;\ c_1, c_2\in\mathbb{R}\}$$



$$\vec{v}_1 \rightsquigarrow \vec{p}_1=\frac{1}{\|\vec{v}_1\|}\vec{v}_1=\frac{1}{\sqrt{2}}\vec{v}_1=\frac{1}{\sqrt{2}}\begin{pmatrix}1\\1\\0\end{pmatrix}$$

$$\vec{w}=\frac{(\vec{v}_1,\vec{v}_2)}{\|\vec{v}_1\|^2}\cdot\vec{v}_1=\frac{2}{2}\vec{v}_1=\begin{pmatrix}1\\1\\0\end{pmatrix}$$

$$\vec{v}_2-\vec{w}=\begin{pmatrix}2\\0\\1\end{pmatrix}-\begin{pmatrix}1\\1\\0\end{pmatrix}=\begin{pmatrix}1\\-1\\1\end{pmatrix}$$

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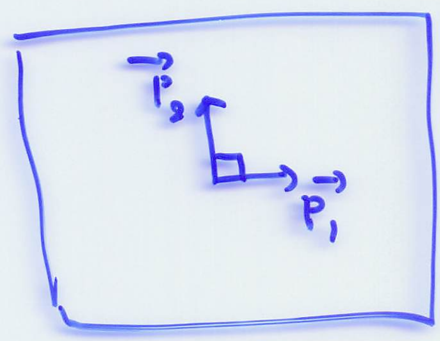
$$\vec{p}_2=\frac{1}{\sqrt{3}}\begin{pmatrix}1\\-1\\1\end{pmatrix}$$

$V(2)$



$$\boxed{\begin{cases}A\vec{p}_1=2\vec{p}_1\\A\vec{p}_2=2\vec{p}_2\end{cases}}$$

$$\|\vec{p}_1\|=\|\vec{p}_2\|=1$$

$$(\vec{p}_1,\vec{p}_2)=0$$

部分空間を考える.

P: $\mathbb{C}$の系数とする.　　　$\lambda^2+1=(\lambda-i)(\lambda+i)$

注　A: 実, $\alpha_1, \cdots, \alpha_n \in \mathbb{R} \Rightarrow P$: 実.

$$A=\begin{pmatrix} 3 & -1 & -2 \\ -1 & 3 & 2 \\ -2 & 2 & 6 \end{pmatrix}$$

${}^tA = A$

A: 対称..

$$A=\begin{pmatrix} a & c \\ c & b \end{pmatrix}$$

${}^tA = A$

P: 回転
$$= \begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix}$$

$$AP = P\begin{pmatrix} \alpha & 0 \\ 0 & \beta \end{pmatrix}$$

$$\left(A\begin{pmatrix} x \\ y \end{pmatrix}, \begin{pmatrix} x \\ y \end{pmatrix}\right) = ax^2 + 2cxy + by^2$$
$$= \alpha\xi^2 + \beta\eta^2$$

$$|\lambda I_3 - A| = (\lambda-2)^2 \times (\lambda-8)$$

$\underline{\lambda = 2}$

$$2I_2 - A = \begin{pmatrix} -1 & 1 & 2 \\ 1 & -1 & -2 \\ 2 & -2 & -4 \end{pmatrix} \longrightarrow \cdots \longrightarrow \begin{pmatrix} 1 & -1 & -2 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

$$\begin{pmatrix} x \\ y \\ z \end{pmatrix} : (2I_2 - A)\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \vec{0} \Longleftrightarrow x - y - 2z = 0$$

$$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} +y+2z \\ y \\ z \end{pmatrix} = y\begin{pmatrix} +1 \\ 1 \\ 0 \end{pmatrix} + z\begin{pmatrix} +2 \\ 0 \\ 1 \end{pmatrix}$$

$\underline{\lambda = 8}$

$$8I_3 - A \longrightarrow \cdots \longrightarrow \begin{pmatrix} 1 & 0 & \frac{1}{2} \\ 0 & 1 & -\frac{1}{2} \\ 0 & 0 & 0 \end{pmatrix}$$

$$\begin{pmatrix} x \\ y \\ z \end{pmatrix} : \begin{cases} x + \frac{1}{2}z = 0 \\ y - \frac{1}{2}z = 0 \end{cases}$$

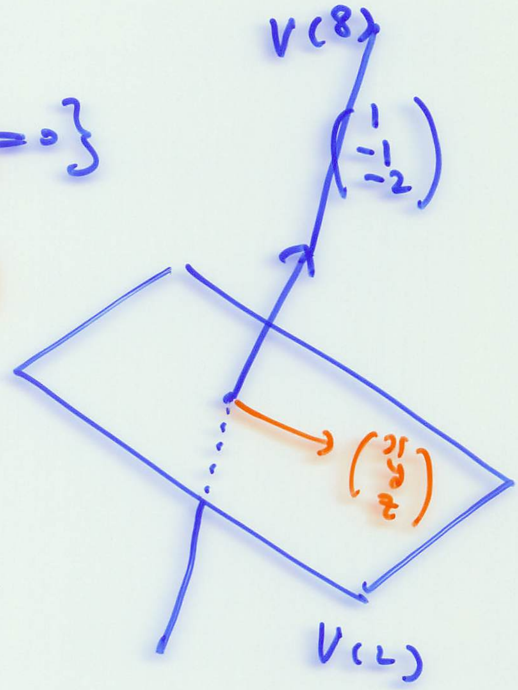
$$= \begin{pmatrix} -\frac{1}{2}z \\ \frac{1}{2}z \\ z \end{pmatrix} = \frac{1}{2}z\begin{pmatrix} -1 \\ 1 \\ 2 \end{pmatrix}$$

$$V(2) = \{ \vec{v} \,;\ A\vec{v} = 2\vec{v} \}$$

$$= \left\{ \begin{pmatrix} x \\ y \\ z \end{pmatrix} ;\ \boxed{x - y - 2z} = 0 \right\}$$

$$\qquad\qquad\qquad\qquad \begin{pmatrix} x \\ y \\ z \end{pmatrix} \cdot \begin{pmatrix} 1 \\ -1 \\ -2 \end{pmatrix}$$

$$V(8) = \mathbb{R}\,\vec{v}_3$$



$$\vec{v}_3 = \begin{pmatrix} -1 \\ 1 \\ 2 \end{pmatrix}$$

$$\vec{p}_3 = \frac{1}{\|\vec{v}_3\|}\vec{v}_3$$

$$= \frac{1}{\sqrt{6}} \begin{pmatrix} -1 \\ 1 \\ 2 \end{pmatrix}$$

$$\|\vec{p}_i\| = 1 \quad (i = 1, 2, 3)$$

$$(\vec{p}_i, \vec{p}_j) = 0 \quad (i \neq j)$$

$$P = (\vec{p}_1\ \vec{p}_2\ \vec{p}_3)$$ 直交行列. → 来週

$$AP = P \begin{pmatrix} 2 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 8 \end{pmatrix}$$

$$\left(A \begin{pmatrix} x \\ y \\ z \end{pmatrix}, \begin{pmatrix} x \\ y \\ z \end{pmatrix} \right) = 2\xi_1^2 + 2\xi_2^2 + 8\xi_3^2$$

$$A = \begin{pmatrix} 1 & 2 & 2 & 0 & -3 \\ 2 & 4 & 4 & 3 & 0 \\ -1 & -1 & 1 & 0 & 3 \\ 0 & 1 & 3 & 2 & 4 \end{pmatrix}$$

$$A \begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \end{pmatrix} = \vec{0}$$ を解け.