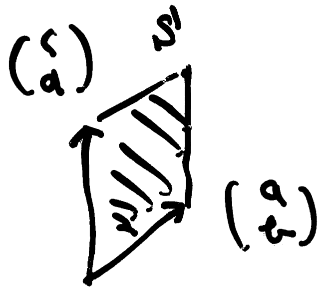
$$\begin{vmatrix} a & b \\ c & d \end{vmatrix} = ad - bc$$



- $S = |ad - bc|$

  $= \left| \begin{vmatrix} a & c \\ b & d \end{vmatrix} \right|$ 行列式の幾何的な意味.

- $$\begin{cases} ax + by = \alpha \\ cx + dy = \beta \end{cases}$$

  $\begin{vmatrix} a & b \\ c & d \end{vmatrix} \neq 0$ のとき

  $$x = \frac{\begin{vmatrix} \alpha & b \\ \beta & d \end{vmatrix}}{\begin{vmatrix} a & b \\ c & d \end{vmatrix}} \qquad y = \frac{\begin{vmatrix} a & \alpha \\ c & \beta \end{vmatrix}}{\begin{vmatrix} a & b \\ c & d \end{vmatrix}}$$

  クラメール の公式.

- $A = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$

$$\left(A \begin{pmatrix} x \\ y \end{pmatrix} = \vec{0} \Rightarrow \begin{pmatrix} x \\ y \end{pmatrix} = \vec{0} \right) \iff |A| \neq 0$$

$$\begin{cases} a_{11}x + a_{12}y + a_{13}z = \alpha_1 & \cdots (1) \\ a_{21}x + a_{22}y + a_{23}z = \alpha_2 & \cdots (2) \\ a_{31}x + a_{32}y + a_{33}z = \alpha_3 & \cdots (3) \end{cases}$$

zを消去、

やり直し

$$\begin{cases} a_1 x + b_1 y + c_1 z = \alpha_1 & \cdots (1) \\ a_2 x + b_2 y + c_2 z = \alpha_2 & \cdots (2) \\ a_3 x + b_3 y + c_3 z = \alpha_3 & \cdots (3) \end{cases}$$

zを消去.

$(1) \times c_2 - (2) \times c_1$

$$(a_1 c_2 - a_2 c_1) x + (b_1 c_2 - b_2 c_1) y = \alpha_1 c_2 - \alpha_2 c_1$$

$$\begin{vmatrix} a_1 & c_1 \\ a_2 & c_2 \end{vmatrix} x + \begin{vmatrix} b_1 & c_1 \\ b_2 & c_2 \end{vmatrix} y = \begin{vmatrix} \alpha_1 & c_1 \\ \alpha_2 & c_2 \end{vmatrix} \quad \text{I}$$

同様に

$$\begin{vmatrix} a_1 & c_1 \\ a_3 & c_3 \end{vmatrix} x + \begin{vmatrix} b_1 & c_1 \\ b_3 & c_3 \end{vmatrix} y = \begin{vmatrix} \alpha_1 & c_1 \\ \alpha_3 & c_3 \end{vmatrix} \quad \text{II}$$

$$\begin{vmatrix} a_2 & c_2 \\ a_3 & c_3 \end{vmatrix} x + \begin{vmatrix} b_2 & c_2 \\ b_3 & c_3 \end{vmatrix} y = \begin{vmatrix} \alpha_2 & c_2 \\ \alpha_3 & c_3 \end{vmatrix} \quad \text{III}$$

$(\text{I}) \times b_3 - (\text{II}) \times b_2 + (\text{III}) \times b_1$

xの係数.

$$\left(b_1 \begin{vmatrix} a_2 & c_2 \\ a_3 & c_3 \end{vmatrix} - b_2 \begin{vmatrix} a_1 & c_1 \\ a_3 & c_3 \end{vmatrix} + b_3 \begin{vmatrix} a_1 & c_1 \\ a_2 & c_2 \end{vmatrix} \right)$$

(I) 2つ目の線形性

$$\left| \vec{a} \quad \lambda\vec{b}+\mu\vec{c} \quad \vec{d} \right|$$

$$= \lambda \left| \vec{a} \quad \vec{b} \quad \vec{d} \right| + \mu \left| \vec{a} \quad \vec{c} \quad \vec{d} \right|$$

(II) $\left| \vec{a} \quad \vec{a} \quad \vec{b} \right| = 0$

どれか2つが等しいと0

(III) $|I_3| = 1$

(IV) $\left| \vec{a} \quad \vec{b} \quad \vec{c} \right| = - \left| \vec{b} \quad \vec{a} \quad \vec{c} \right|$

$$\begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix}$$

余因子展開

$$= a_1 \begin{vmatrix} b_2 & c_2 \\ b_3 & c_3 \end{vmatrix} - a_2 \begin{vmatrix} b_1 & c_1 \\ b_3 & c_3 \end{vmatrix} + a_3 \begin{vmatrix} b_1 & c_1 \\ b_2 & c_2 \end{vmatrix}$$

$$(\mathrm{I}) \times b_3 - (\mathrm{II}) \times b_2 + (\mathrm{III}) \times b_1$$

$$\begin{vmatrix} b_1 & a_1 & c_1 \\ b_2 & a_2 & c_2 \\ b_3 & a_3 & c_3 \end{vmatrix} x + \overbrace{\begin{vmatrix} b_1 & b_1 & c_1 \\ b_2 & b_2 & c_2 \\ b_3 & b_3 & c_3 \end{vmatrix}}^{=0} y = \begin{vmatrix} \alpha_1 & b_1 & c_1 \\ \alpha_2 & b_2 & c_2 \\ \alpha_3 & b_3 & c_3 \end{vmatrix}$$

$$\begin{vmatrix} b_1 & a_1 & c_1 \\ b_2 & a_2 & c_2 \\ b_3 & a_3 & c_3 \end{vmatrix} = -\left| \vec{a}\ \vec{b}\ \vec{c} \right|$$

$$\begin{vmatrix} \alpha_1 & b_1 & c_1 \\ \alpha_2 & b_2 & c_2 \\ \alpha_3 & b_3 & c_3 \end{vmatrix} = -\left| \vec{\alpha}\ \vec{b}\ \vec{c} \right|$$

$$\left| \vec{a}\ \vec{b}\ \vec{c} \right| x = \left| \vec{\alpha}\ \vec{b}\ \vec{d} \right|$$

定理 $\left| \vec{a}\ \vec{b}\ \vec{c} \right| \neq 0$ なら

$$x = \frac{\left| \vec{\alpha}\ \vec{b}\ \vec{c} \right|}{\left| \vec{a}\ \vec{b}\ \vec{c} \right|}$$

クラメールの'公式'

$$y = \frac{\left| \vec{a}\ \vec{\alpha}\ \vec{c} \right|}{\left| \vec{a}\ \vec{b}\ \vec{c} \right|}, \quad z = \frac{\left| \vec{a}\ \vec{b}\ \vec{\alpha} \right|}{\left| \vec{a}\ \vec{b}\ \vec{c} \right|}$$

行基本変形

行 row

$$\begin{cases} x + y + z + w = 1 & \cdots ① \\ 5x + 6y + 3z + 7w = 1 & \cdots ② \\ 2x + y + 4z = 6 & \cdots ③ \end{cases}$$

$$\Updownarrow$$

$$\left(\begin{array}{cccc|c} 1 & 1 & 1 & 1 & 1 \\ 5 & 6 & 3 & 7 & 1 \\ 2 & 1 & 4 & 0 & 6 \end{array}\right) \longrightarrow \left(\begin{array}{cccc|c} 1 & 1 & 1 & 1 & 1 \\ 0 & 1 & -2 & 2 & -4 \\ 0 & -1 & 2 & -2 & 4 \end{array}\right)$$

拡大行列

$\boxed{2r に 1r \times (-5) を足す}$

3r に 1r × (−2) を足す

① × (−5) + ②

$$\begin{array}{rrrrr} 5 & 6 & 3 & 7 & 1 \\ -)\ 5 & 5 & 5 & 5 & 5 \\ \hline 0 & 1 & -2 & 2 & -4 \end{array} \qquad \begin{array}{rrrrr} 2 & 1 & 4 & 0 & 6 \\ -)\ 2 & 2 & 2 & 2 & 2 \\ \hline 0 & -1 & 2 & -2 & 4 \end{array}$$

3r に 2r × (−1) を足す

階段行列

$$\left(\begin{array}{cccc|c} 1 & 1 & 1 & 1 & 1 \\ 0 & 1 & -2 & 2 & -4 \\ 0 & 0 & 0 & 0 & 0 \end{array}\right)$$

○ pivot

2r × (−1) を 1r に足す

$$\begin{array}{rrrrr} 1 & 1 & 1 & 1 & 1 \\ -)\ 0 & 1 & -2 & 2 & -4 \\ \hline 1 & 0 & 3 & -1 & 5 \end{array}$$

$$\begin{array}{ccccc} x & y & z & w & \\ \end{array}$$
$$\left(\begin{array}{cccc|c} 1 & 0 & 3 & -1 & 5 \\ 0 & 1 & -2 & 2 & -4 \\ 0 & 0 & 0 & 0 & 0 \end{array}\right)$$

$$\begin{cases} x + 3z - w = 5 \\ y - 2z + 2w = -4 \\ \end{cases}$$

$0x + 0y + 0z + 0w = 0$

$z = \alpha,\ w = \beta$ とおく $\longrightarrow$

$$\begin{cases} x = -3\alpha + \beta + 5 \\ y = 2\alpha - 2\beta - 4 \\ z = \alpha \\ w = \beta \end{cases}$$

$$A = \begin{pmatrix} 1 & 1 & 1 & 1 \\ 5 & 6 & 3 & 7 \\ 2 & 1 & 4 & 0 \end{pmatrix} \quad \vec{b} = \begin{pmatrix} 1 \\ 1 \\ 6 \end{pmatrix}$$

rank $A = 2$, 変数 4個.

pivotの数. $\rightsquigarrow$ パラメータ $4-2$個.

(解空間が2次元)

行基本変形

(I) $i \neq j$ i行とj行を交換.

(II) $i \neq j$ i行のλ倍をj行に加える.

(III) $\lambda \neq 0$ i行をλ倍.

$\rightsquigarrow$ 可逆な変形.

$(\mathrm{I})^{-1}$ i行とj行を交換.

$(\mathrm{II})^{-1}$ i行の$(-\lambda)$倍をj行に加える

$(\mathrm{III})^{-1}$ i行を$\frac{1}{\lambda}$倍.

- 行基本変形 $\longleftrightarrow$ 基本行列を左からかける.
- 基本行列は逆行列あり

C言語

pivot

$$\left(\begin{array}{cccc|c} 1 & 2 & 2 & 0 & -3 \\ 2 & 4 & 4 & 3 & 0 \\ -1 & -1 & 1 & 0 & 3 \\ 0 & 1 & 3 & 2 & 4 \end{array}\right) \longrightarrow \left(\begin{array}{cccc|c} 1 & 2 & 2 & 0 & -3 \\ 0 & 0 & 0 & 3 & +6 \\ 0 & 1 & 3 & 0 & 0 \\ 0 & 1 & 3 & 2 & 4 \end{array}\right)$$

2rに 1r×(−2) を+

3rに 1r×1 を+

1列に着目.

$$\begin{array}{rrrrr} 2 & 4 & 4 & 3 & 0 \\ -)\ 2 & 4 & 4 & 0 & -6 \\ \hline & & & 3 & +6 \end{array} \qquad \begin{array}{rrrrr} -1 & -1 & 1 & 0 & 3 \\ +)\ 1 & 2 & 2 & 0 & -3 \\ \hline 0 & 1 & 3 & 0 & 0 \end{array}$$

↓ 2rと3rを交換

$$\left(\begin{array}{cccc|c} 1 & 2 & 2 & 0 & -3 \\ 0 & 1 & 3 & 0 & 0 \\ 0 & 0 & 0 & 3 & +6 \\ 0 & 1 & 3 & 2 & 4 \end{array}\right)$$

2r×(−1) を 3rに+

$$\left(\begin{array}{cccc|c} 1 & 2 & 2 & 0 & -3 \\ 0 & 1 & 3 & 0 & 0 \\ 0 & 0 & 0 & 3 & +6 \\ 0 & 0 & 0 & 2 & 4 \end{array}\right) \xrightarrow[\frac{1}{3}\text{倍}]{3r\text{を}} \left(\begin{array}{cccc|c} 1 & 2 & 2 & 0 & -3 \\ 0 & 1 & 3 & 0 & 0 \\ 0 & 0 & 0 & 1 & +2 \\ 0 & 0 & 0 & 2 & 4 \end{array}\right)$$

34をずらす

$$\left(\begin{array}{cccc|c} 1 & 0 & -4 & 0 & -3 \\ 0 & 1 & 3 & 0 & 0 \\ 0 & 0 & 1 & 2 & 2 \\ 0 & 0 & 0 & 0 & 0 \end{array}\right) \longleftarrow \left(\begin{array}{cccc|c} 1 & 2 & 2 & 0 & -3 \\ 0 & 1 & 3 & 0 & 0 \\ 0 & 0 & 0 & 1 & 2 \\ 0 & 0 & 0 & 0 & 0 \end{array}\right)$$

$$\begin{array}{rrrrr} 1 & 2 & 2 & 0 & -3 \\ 0 & 2 & 6 & 0 & 0 \\ \hline 1 & 0 & -4 & 0 & -3 \end{array}$$

w = α とおく

$$\cancel{\begin{cases} x \quad -4z = -3 \\ y + 3z = 0 \\ z + 2w = \end{cases}}$$

$$\left(\begin{array}{cccc|c} 1 & 0 & 0 & 8 & -3 \\ 0 & 1 & 0 & -6 & 0 \\ 0 & 0 & 1 & 2 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{array}\right)$$

$$\cancel{\begin{cases} x \quad +8w = -3 \\ y \quad -6w = 0 \\ z + 2w = \end{cases}}$$

$$\begin{cases} x - 4z = -3 \\ y + 3z = 0 \\ w = +2 \end{cases}$$

$z = \alpha$

$$\begin{cases} x = 4\alpha - 3 \\ y = -3\alpha \\ z = \alpha \\ w = +2 \end{cases}$$

変数 4個

rank$(A) = 3$

になる.

小テスト

$$\begin{array}{c} \begin{array}{ccc} x & y & z \end{array} \\ \left(\begin{array}{ccc|c} 1 & 2 & -1 & 3 \\ 1 & 4 & -5 & 1 \\ 3 & 7 & -5 & 8 \end{array}\right) \end{array}$$