$g = x + y - 1 = 0$ ⋯⓪

$f = x^{\frac{1}{2}} y^{\frac{1}{2}}$ $(x, y > 0)$

$g_x = g_y = 1$

$f_x = \frac{1}{2} x^{-\frac{1}{2}} y^{\frac{1}{2}}$, $f_y = \frac{1}{2} x^{\frac{1}{2}} y^{-\frac{1}{2}}$.

である.

$$\begin{cases} \frac{1}{2}\left(\frac{y}{x}\right)^{\frac{1}{2}} = \lambda & \cdots ① \\ \frac{1}{2}\left(\frac{x}{y}\right)^{\frac{1}{2}} = \lambda & \cdots ② \end{cases}$$

$\frac{y}{x} = \frac{x}{y} = 4\lambda^2$ より $x = y$ となる.

⓪に代入して $x = y = \frac{1}{2}$ となる. ①に代入して

$$\lambda = \frac{1}{2}$$

を得る.

$$(x, y) = \left(\frac{1}{2}, \frac{1}{2}\right)$$

が極大か極小かを判定しよう.

$$L = f - \lambda g$$

とおく

$$L_{xx} = f_{xx} - \frac{1}{2} g_{xx}$$

$$= -\frac{1}{4} x^{-\frac{3}{2}} y^{\frac{1}{2}} = -\frac{1}{4} \cdot x^{-1}\left(\frac{y}{x}\right)^{\frac{1}{2}}$$

$$L_{xy} = +\frac{1}{4} x^{-\frac{1}{2}} y^{-\frac{1}{2}}$$

$$L_{yy} = -\frac{1}{4} x^{\frac{1}{2}} y^{-\frac{3}{2}} = -\frac{1}{4} y^{-1}\left(\frac{x}{y}\right)^{\frac{1}{2}}$$

$\left(\frac{1}{2}, \frac{1}{2}\right)$ において

$$L_{xx} = -\frac{1}{4} \cdot 2 = -\frac{1}{2}$$

$$L_{yy} = -\frac{1}{4} \cdot 2 = -\frac{1}{2}$$

$$L_{xy} = L_{yx} = -\frac{1}{4} \cdot 2 = +\frac{1}{2}.$$

$$B = \begin{vmatrix} 0 & 1 & 1 \\ 1 & -\frac{1}{2} & +\frac{1}{2} \\ 1 & +\frac{1}{2} & -\frac{1}{2} \end{vmatrix}$$

$$= \begin{vmatrix} 0 & 1 & 1 \\ 1 & -\frac{1}{2} & \frac{1}{2} \\ 2 & 0 & 0 \end{vmatrix} = 2 \begin{vmatrix} 1 & 1 \\ -\frac{1}{2} & \frac{1}{2} \end{vmatrix} = 2 > 0$$

より正定

3行目について展開