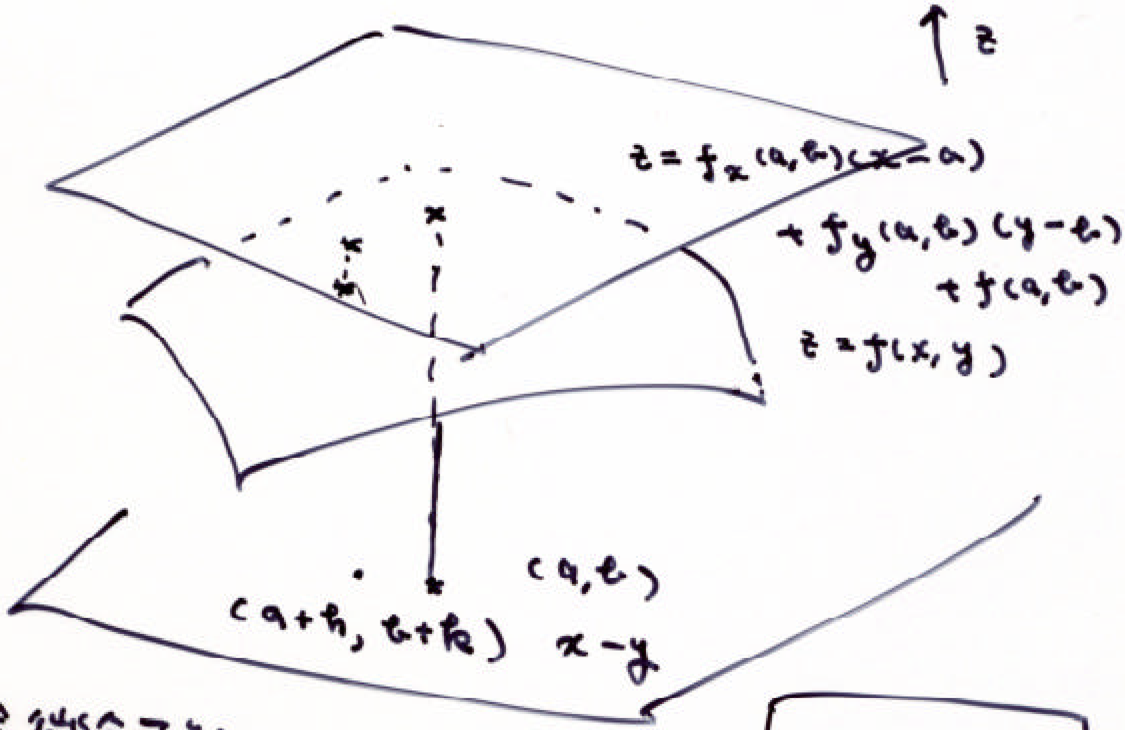
復習

$z = f(x, y)$ の (a, b) における接平面

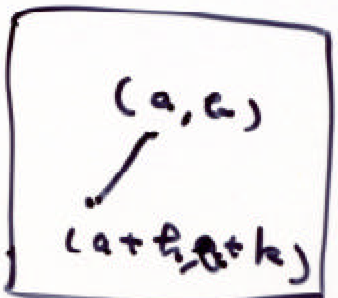
$$\rightsquigarrow \quad z = f_x(a,b)(x-a) + f_y(a,b)(y-b) + f(a,b)$$

問題 $z(t) = f(x(t), y(t))$ に対して

$$z'(t) = ?$$



全微分可能



$$\Delta := f(a+h, b+k) - f_x(a,b)h - f_y(a,b)k - f(a,b)$$

$$f(x(t), y(t)) = f_x(a,b)(x(t)-a) + f_y(a,b)(y(t)-b) + f(a,b) + \Delta$$

● $f(x,y)$ が (a,b) で全微分可能

$$\frac{\Delta}{\boxed{\sqrt{h^2+k^2}}} \longrightarrow 0 \quad ((h,k)\to(0,0))$$

(a,b) と $(a+h, b+k)$ の距離.

定理　f: C^1 級

（f, f_x, f_y が各点で存在し
各点で連続）

$\Longrightarrow$ f: 全微分可能.

$$\frac{z(t)-z(c)}{t-c} = \frac{f(x(t),y(t)) - f(a,b)}{t-c}$$

$\boxed{x(c)=a,\ y(c)=b}$　　$t\to c$

$$= f_x(a,b)\boxed{\frac{x(t)-x(c)}{t-c}} + f_y(a,b)\boxed{\frac{y(t)-y(c)}{t-c}} + \frac{\Delta}{t-c}$$

$x'(c)$　　$y'(c)$

0　　$\sqrt{x'(c)^2+y'(c)^2}$

$$\boxed{\left|\frac{\Delta}{t-c}\right|} = \boxed{\left|\frac{\Delta}{\sqrt{(x(t)-a)^2+(y(t)-b)^2}}\right|} \times \sqrt{\left(\frac{x(t)-x(c)}{t-c}\right)^2 + \left(\frac{y(t)-y(c)}{t-c}\right)^2}$$

$t\to c$　$\to 0$（全微分可能）

$(x(t)-a,\ y(t)-b) \longrightarrow (0,0)$

$\qquad \| $

$\quad (h, k)$

結論

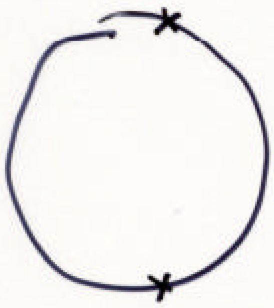
$$z(t) = f(x(t), y(t))$$

$$z'(c) = f_x(x(c), y(c))\, x'(c) + f_y(x(c), y(c))\, y'(c)$$

$$z'(t) = f_x \cdot x'(t) + f_y\, y'(t)$$

陰関数

implicit $\longleftrightarrow$ explicit.

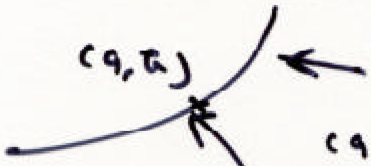


$g(x, y) = x^2 + y^2 - 1 = 0$

$y = \sqrt{1-x^2}$

$y = -\sqrt{1-x^2}$

問題



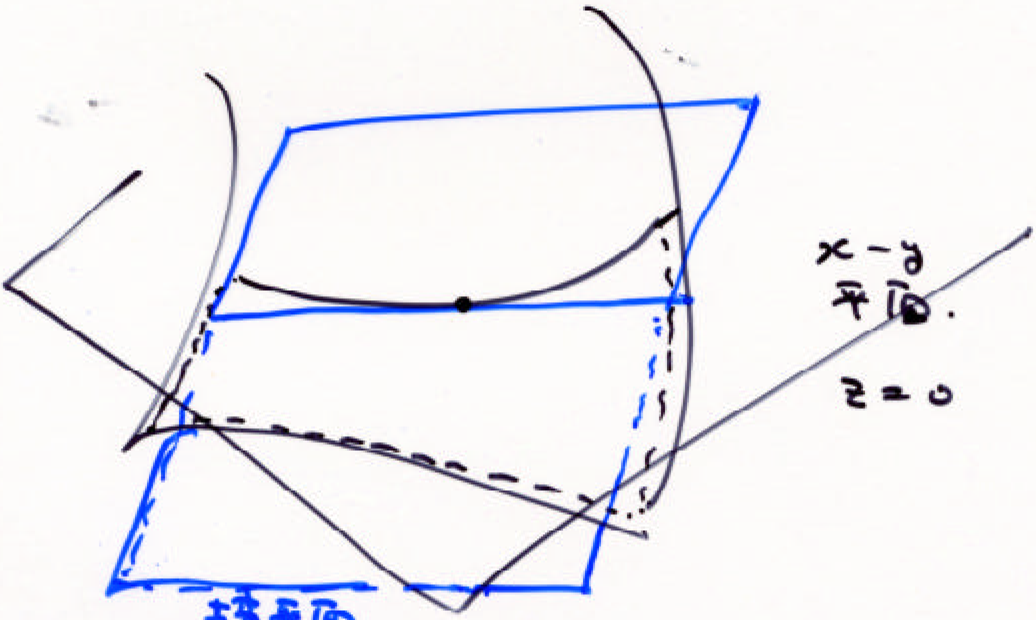
$g(x, y) = 0$

(a, b) の近くで

$y = \varphi(x)$

$$\boxed{\varphi'(a) = \ ?}$$

$z = g(x, y)$



接平面

$$z = g_x(a,b)(x-a) + g_y(a,b)(y-b) + \underbrace{g(a,b)}_{=0}$$

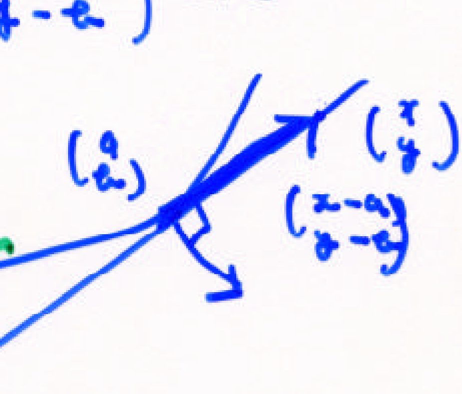
$\boxed{\text{接平面の } z=0 \text{ の断面}}$

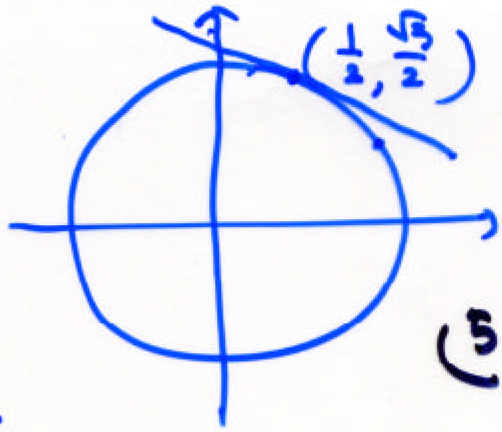
$$g_x(a,b)(x-a) + g_y(a,b)(y-b) = 0$$

接線の方程式

$$\begin{pmatrix} g_x(a,b) \\ g_y(a,b) \end{pmatrix} \cdot \begin{pmatrix} x-a \\ y-b \end{pmatrix} = 0$$

法線方向
normal direction





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$$g(x, y) = x^2 + y^2 - 1 = 0$$

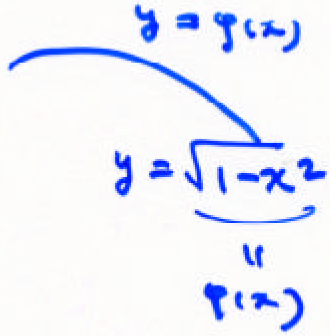
$$g_x = 2x$$

$$g_y = 2y$$

$$1 \cdot \left(x - \frac{1}{2}\right) + \sqrt{3}\left(y - \frac{\sqrt{3}}{2}\right) = 0$$

接線

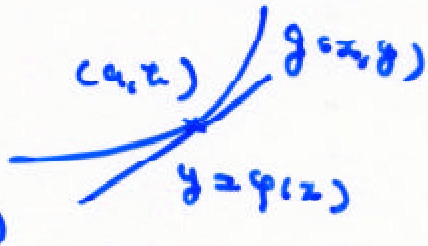
$$y - \frac{\sqrt{3}}{2} = -\frac{1}{\sqrt{3}}\left(x - \frac{1}{2}\right)$$



傾き $= -\dfrac{1}{\sqrt{3}}$.

$$\varphi'\left(\frac{1}{2}\right) = -\frac{1}{\sqrt{3}}$$

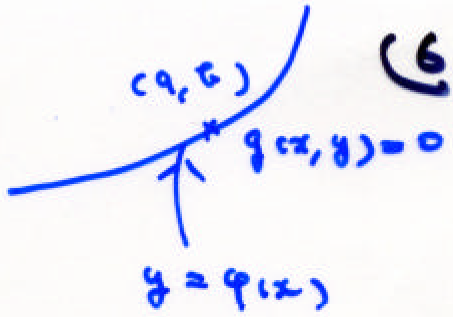
一般的には　$g_y(a, b) \neq 0$



$$y - b = -\frac{g_x(a, b)}{g_y(a, b)}(x - a)$$

$$\boxed{\varphi'(a) = -\frac{g_x(a, b)}{g_y(a, b)}}$$

$$g_x(a, b)(x - a) + \underbrace{g_y(a, b)}_{\neq 0}(y - b) = 0$$



$g(t, \varphi(t)) \equiv 0$

($t = a$ の近くで)

$$g_x(t, \varphi(t)) \cdot 1 + g_y(t, \varphi(t))\, \varphi'(t) \equiv 0$$

$g_y \neq 0$ $\quad \varphi'(t) = -\dfrac{g_x(t, \varphi(t))}{g_y(t, \varphi(t))}$

$$\boxed{\frac{d}{dt} f(x(t), y(t)) = f_x\, x'(t) + f_y \cdot y'(t)}$$

→ Lagrange の未定乗数法.

停留点がいつ極大・極小になるか?