x_1
x_2
$\vdots$
x_N

$$\bar{x} = \frac{1}{N}\sum_{i=1}^{N} x_i$$

平均
mean

average (代表値)

$$V(x) = \frac{1}{N}\sum_{i=1}^{N} (x_i - \bar{x})^2$$

variance
分散

回帰分析.

準備

内積

$$\vec{a} = \begin{pmatrix} a_1 \\ a_2 \\ \vdots \\ a_N \end{pmatrix}, \quad \vec{b} = \begin{pmatrix} b_1 \\ b_2 \\ \vdots \\ b_N \end{pmatrix} \in \mathbb{R}^N$$

$$(\vec{a}, \vec{b}) = \sum_{i=1}^{N} a_i b_i$$

$$\|\vec{a}\|^2 = \sum_{i=1}^{N} a_i^2 = (\vec{a}, \vec{a})$$

$$\vec{x} = \frac{1}{\sqrt{N}}\begin{pmatrix} x_1 - \bar{x} \\ x_2 - \bar{x} \\ \vdots \\ x_N - \bar{x} \end{pmatrix}, \quad \vec{y} = \frac{1}{\sqrt{N}}\begin{pmatrix} y_1 - \bar{y} \\ y_2 - \bar{y} \\ \vdots \\ y_N - \bar{y} \end{pmatrix}$$

$$\|\vec{x}\|^2 = V(x)$$

$$(\vec{x}, \vec{y}) = \frac{1}{N}\sum_{i=1}^{N} (x_i - \bar{x})(y_i - \bar{y})$$

$$= \mathrm{Cov}(x, y)$$ 共分散.

$$\rho_{xy} = \frac{Cov(x,y)}{\sqrt{V(x)}\sqrt{V(y)}} = \frac{Cov(x,y)}{\sigma_x \sigma_y}$$

σ_x: xの標準偏差.

$$= \frac{(\vec{x}, \vec{y})}{\|\vec{x}\| \cdot \|\vec{y}\|}$$

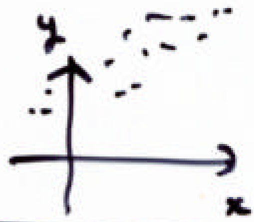
- Cauchy-Schwartzの不等式

$\vec{a}, \vec{b} \in \mathbb{R}^N$ に対して

$$|(\vec{a}, \vec{b})| \leqq \|\vec{a}\| \cdot \|\vec{b}\|$$

Cauchy-Schwartz不等式から

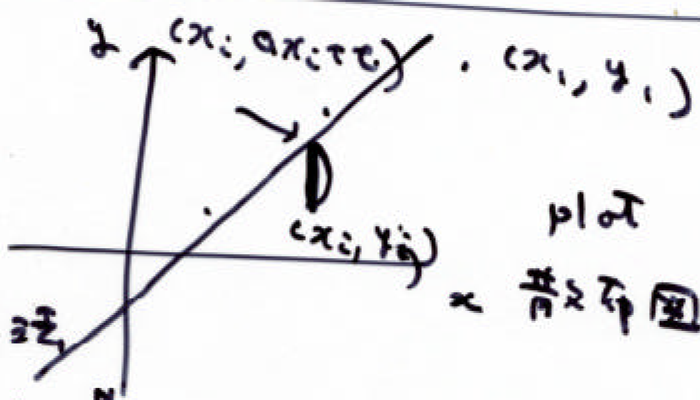
$\underbrace{-1 \leqq \rho_{xy} \leqq 1}_{\text{相関係数}}$ が従う.



$|\rho_{xy}| \to 1$ のとき データは直線状に分布する.

モデル

$y = ax + b$



plot

× 散布図

最小2乗法

$$S = \frac{1}{N}\sum_{i=1}^{N}(y_i - ax_i - b)^2$$ を最小

とする (a, b) を求める.

x：誤差を含まない量と考える.

$$\varepsilon_i = y_i - (a x_i + b)$$

$$= (\text{実測値}) - (\text{理論値})$$

$$= \text{誤差}$$

$$\boxed{\varepsilon\text{の分散が最小} \Longleftrightarrow \text{最小自乗法}.}$$

$$S = \frac{1}{N}\sum_{i=1}^{N}(y_i - a x_i - b)^2$$

$$\frac{\partial S}{\partial b} = \frac{1}{N}\sum_{i=1}^{N} 2(y_i - a x_i - b)\times(-1) = 0$$

$$= (-2)(\bar{y} - a\bar{x} - b) = 0$$

$$\boxed{b = \bar{y} - a\bar{x}}$$

$$S = \frac{1}{N}\sum_{i=1}^{N}\left\{(y_i - a x_i) - (\bar{y} - a\bar{x})\right\}^2$$

$$= \frac{1}{N}\sum_{i=1}^{N}\left\{(y_i - \bar{y}) - a(x_i - \bar{x})\right\}^2$$

$$= \| \underbrace{\vec{y} - a\vec{x}}_{\text{第}i\text{成分 } \frac{1}{\sqrt{N}}\left\{(y_i - \bar{y}) - a(x_i - \bar{x})\right\}} \|^2$$

準備 $\|\vec{a} \pm \vec{b}\|^2 = \|\vec{a}\|^2 \pm 2(\vec{a}, \vec{b}) + \|\vec{b}\|^2$

$$\|\vec{a} \pm \vec{b}\|^2 = (\vec{a} \pm \vec{b}, \vec{a} \pm \vec{b})$$

$$S = \|\vec{y} - a\vec{x}\|^2$$

$$= \|\vec{y}\|^2 - 2a(\vec{x}, \vec{y}) + a^2\|\vec{x}\|^2$$

$$= \|\vec{x}\|^2 \left(a - \frac{(\vec{x}, \vec{y})}{\|\vec{x}\|^2}\right)^2 + \|\vec{y}\|^2 - \frac{(\vec{x}, \vec{y})^2}{\|\vec{x}\|^2}$$

$$\begin{cases} a = \dfrac{(\vec{x}, \vec{y})}{\|\vec{x}\|^2} \\ b = \bar{y} - a\bar{x} \end{cases}$$

の時，Sは最小．

$$a = \frac{\mathrm{Cov}(x, y)}{V(x)}$$

$$y = ax + b$$

回帰直線

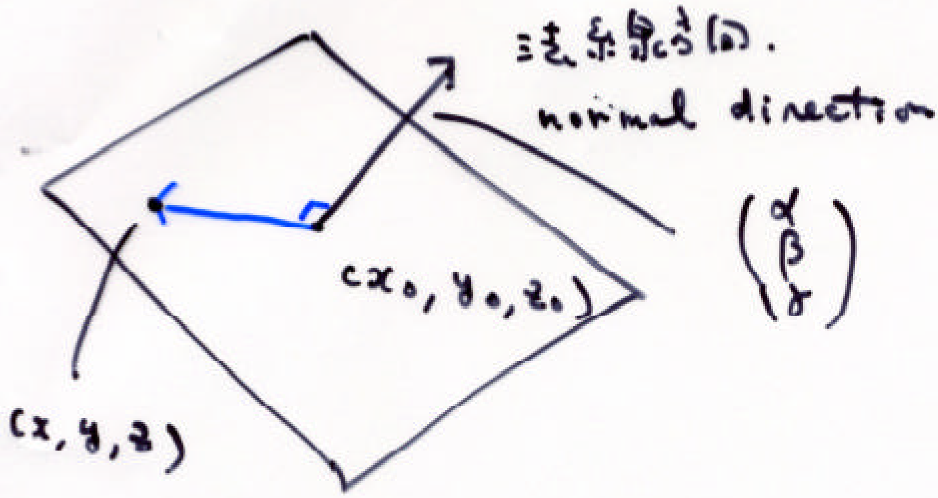
$$S = \|\vec{y}\|^2 - \frac{(\vec{x}, \vec{y})^2}{\|\vec{x}\|^2}$$

$$= \|\vec{y}\|^2 \left(1 - \frac{(\vec{x}, \vec{y})^2}{\|\vec{y}\|^2 \|\vec{x}\|^2}\right)$$

$$= V(y)\,(1 - \rho_{xy}^2)$$

これから $\rho_{xy} \to \pm 1 \quad S \to 0$

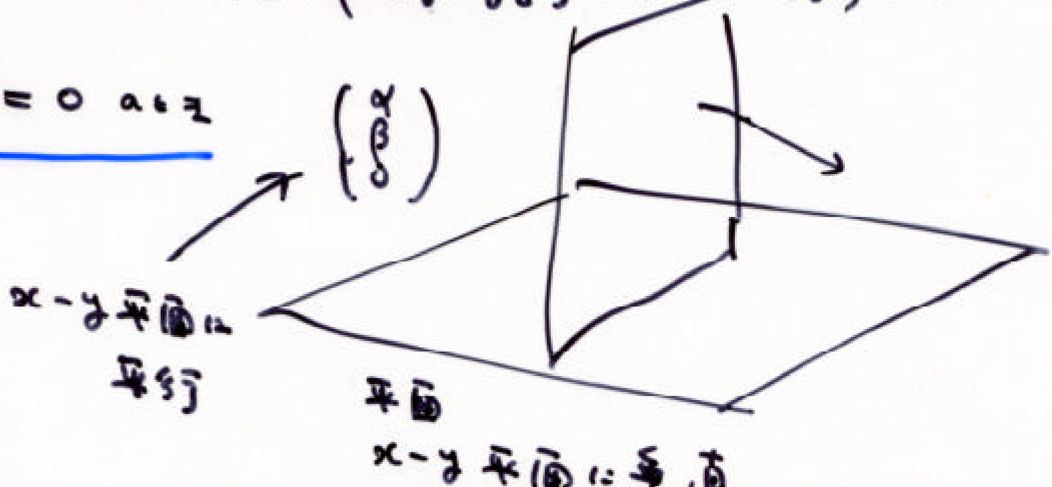
準備　空間中の平面



$$\begin{pmatrix} x-x_0 \\ y-y_0 \\ z-z_0 \end{pmatrix} \cdot \begin{pmatrix} \alpha \\ \beta \\ \gamma \end{pmatrix} = 0$$

$$\alpha(x-x_0) + \beta(y-y_0) + \gamma(z-z_0) = 0$$

$\gamma = 0$ のとき



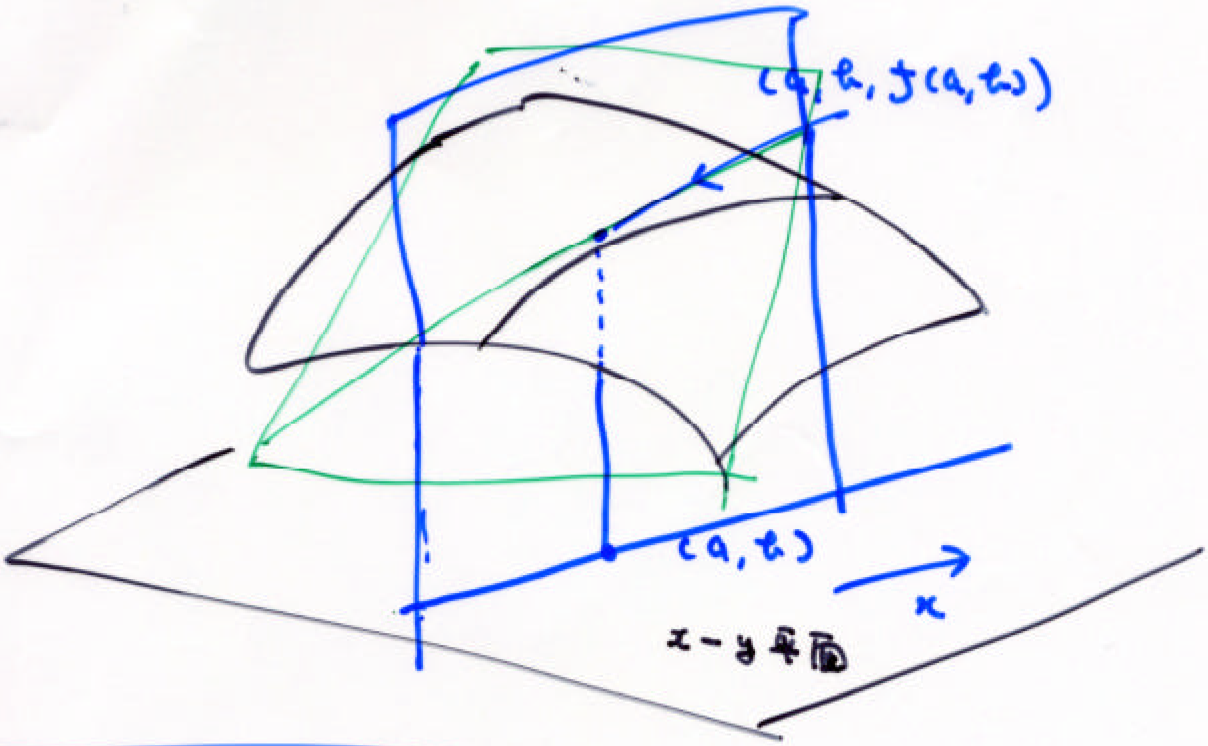
$\gamma \neq 0$

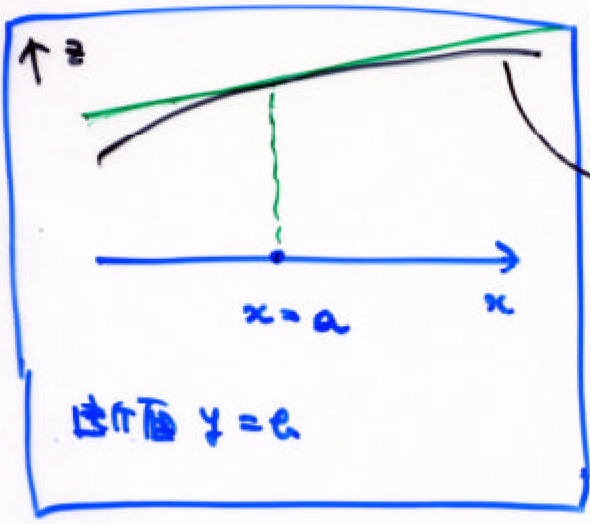
$$z = -\frac{\alpha}{\gamma}(x-x_0) - \frac{\beta}{\gamma}(y-y_0) + z_0$$

定数とおきなおして

$$z = A(x-x_0) + B(y-y_0) + z_0$$

$z = f(x, y)$ 曲面





$z = f(x, b) = F(x)$

接平面

$$z = A(x-a) + B(y-b) + f(a, b)$$

$y = b$

$$z = A(x-a) + f(a, b)$$

接線の [redacted] 傾き

$$F'(a) = \frac{\partial f}{\partial x}(a, b)$$

$$\rightsquigarrow \boxed{A = \frac{\partial f}{\partial x}(a, b)}$$

まとめ

$z = f(x, y)$ の

$(a, b, f(a, b))$ における接平面

$$z = f_x(a, b)(x-a) + f_y(a, b)(y-b) + f(a, b)$$

〜→ 全微分につながる.

変数変換 etc.

小テスト

I. 次の曲面の接平面を求めよ.

(1) $z = x^2 - 3xy + y^2$

at $(1, 1, -1)$

(2) $z = x^3 - 3xy + y^3$

at $(1, 2, 3)$

II.

$$(x\ y\ z)\begin{pmatrix} 1 & 1 & 1 \\ 1 & 2 & 1 \\ 1 & 1 & 3 \end{pmatrix}\begin{pmatrix} x \\ y \\ z \end{pmatrix}$$

III $z = x^2 + y^2 + (1 - x - y)^2$

の停留点を求めよ.