$$A = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$$

$$\Phi_A(\lambda) = \lambda^2 - (a+d)\lambda + \det(A) = 0$$

の 2解 α, β が 虚解 虚

imaginary

$\alpha = a + ib, \quad \beta = a - ib.$

$A\vec{v}_1 = \alpha \vec{v}_1 \qquad \vec{v}_1 = \vec{w}_1 + i\vec{w}_2 \neq \vec{0}$

$A\vec{v}_2 = \beta \vec{v}_2 \longleftarrow \vec{v}_2 = \vec{w}_1 - i\vec{w}_2$ 実.

$P = (\vec{w}_1\ \vec{w}_2)$ 正則.

$$AP = P \begin{pmatrix} a & b \\ -b & a \end{pmatrix}$$

$$\frac{d}{dt}\vec{x}(t) = A\vec{x}(t)$$

$$\vec{z}(t) = P^{-1}\vec{x}(t) \rightsquigarrow \frac{d}{dt}\vec{z}(t) = \begin{pmatrix} a & b \\ -b & a \end{pmatrix}\vec{z}(t)$$

$$X_0(t) = \begin{pmatrix} \cos bt & \sin bt \\ -\sin bt & \cos bt \end{pmatrix}$$

$$\frac{d}{dt} X_0(t) = \begin{pmatrix} -b \sin bt & b \cos bt \\ -b \cos bt & -b \sin bt \end{pmatrix}$$

$$= \begin{pmatrix} \cos bt & \sin bt \\ -\sin bt & \cos bt \end{pmatrix} \begin{pmatrix} 0 & b \\ -b & 0 \end{pmatrix}$$

$$= \begin{pmatrix} 0 & b \\ -b & 0 \end{pmatrix} \begin{pmatrix} X_0(t) \end{pmatrix}$$

<u>まとめ</u> $\quad \frac{d}{dt} X_0(t) = X_0(t) \begin{pmatrix} 0 & b \\ -b & 0 \end{pmatrix}$

$$= \begin{pmatrix} 0 & b \\ -b & 0 \end{pmatrix} X_0(t)$$

$$X(t) = e^{at} X_0(t) \qquad aI_2$$

$$\frac{d}{dt} X(t) = a e^{at} X_0(t) + e^{at} X_0(t) \begin{pmatrix} 0 & b \\ -b & 0 \end{pmatrix}$$

$$= \underbrace{e^{at} X_0(t)}_{\substack{\shortparallel \\ X(t)}} aI_2 + \underbrace{e^{at} X_0(t)}_{\substack{\shortparallel \\ X(t)}} \begin{pmatrix} 0 & b \\ -b & 0 \end{pmatrix}$$

$$= X(t) \begin{pmatrix} a & b \\ -b & a \end{pmatrix}$$

$$= \underbrace{\begin{pmatrix} a & b \\ -b & a \end{pmatrix}}_{\substack{\shortparallel \\ A_0}} X(t)$$

$$\boxed{\frac{d}{dt}\vec{z}(t) = \underbrace{\boxed{\begin{pmatrix} a & b \\ -b & a \end{pmatrix}}}_{= A_0}\vec{z}(t).}$$

$$\Big(X(\overset{s}{\overset{\shortparallel}{-t}})\,\vec{z}(t) \Big)'$$

$$= X(-t)\,A_0\,(-1)\,\vec{z}(t) + X(\overset{-t}{\cancel{}})\,A_0\,\vec{z}(t)$$

$$= X(-t)\left\{ -A_0 + A_0 \right\}\vec{z}(t) = \vec{0}$$

$$X(-t)\,\vec{z}(t) = X(0)\,\vec{z}(0)$$

$$= I_2\,\vec{z}(0) = \vec{z}(0)$$

$$X_0(t)\,X_0(s)$$

$$= \begin{pmatrix} \cos bt & \sin bt \\ -\sin bt & \cos bt \end{pmatrix}\begin{pmatrix} \cos bs & \sin bs \\ -\sin bs & \cos bs \end{pmatrix}$$

加法定理

$$= \begin{pmatrix} \cos b(s+t) & \sin b(s+t) \\ -\sin b(s+t) & \cos b(s+t) \end{pmatrix}$$

$$= X_0(t+s)$$

まとめ $\quad X_0(t)\,X_0(s) = X_0(t+s)$

$$X(t)\,X(s) = e^{at}X_0(t)\cdot e^{as}X_0(s)$$

$$= e^{a(t+s)}X_0(t+s)$$

$$= X(t+s)$$

- $\frac{d}{dt}\vec{z}(t) = A_0 \vec{z}(t)$

- $X(-t)\vec{z}(t) = \vec{z}(0)$

- $X(s)X(t) = X(s+t)$

$\longrightarrow X(-t)X(t) = X(0) = I_2$

$X(t)$は正則 $(X(t))^{-1} = X(-t)$

$\longrightarrow (X(t))^{-1}\vec{z}(t) = \vec{z}(0)$

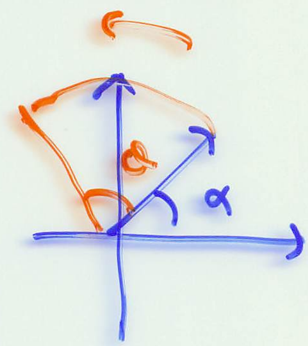
$\longrightarrow \vec{z}(t) = X(t)\vec{z}(0)$

$$\vec{z}(t) = e^{at}\underbrace{\begin{pmatrix} \cos bt & \sin bt \\ -\sin bt & \cos bt \end{pmatrix}}_{= R(-bt)}\vec{z}(0)$$

$= R(-bt)$ θ回転.

$$\underset{\parallel}{R(\theta)}$$

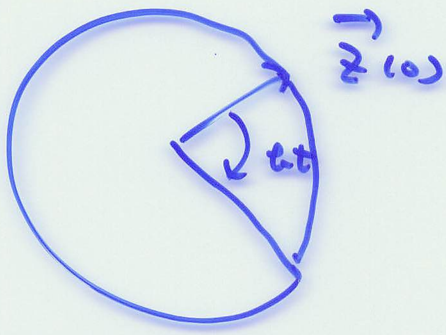
$$\boxed{\begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix}}\begin{pmatrix} \cos\alpha \\ \sin\alpha \end{pmatrix}$$



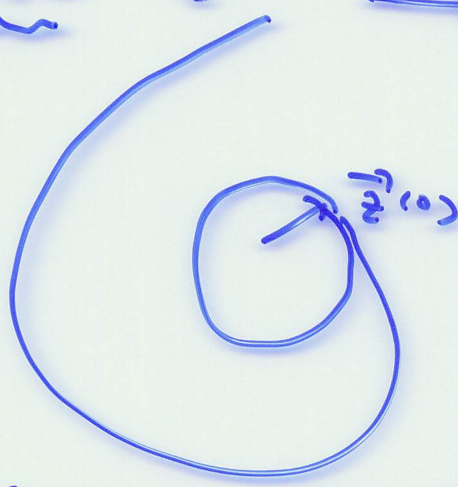
$$= \begin{pmatrix} \cos\theta\cos\alpha - \sin\theta\sin\alpha \\ \sin\theta\cos\alpha + \cos\theta\sin\alpha \end{pmatrix}$$

$$= \begin{pmatrix} \cos(\theta+\alpha) \\ \sin(\theta+\alpha) \end{pmatrix}$$

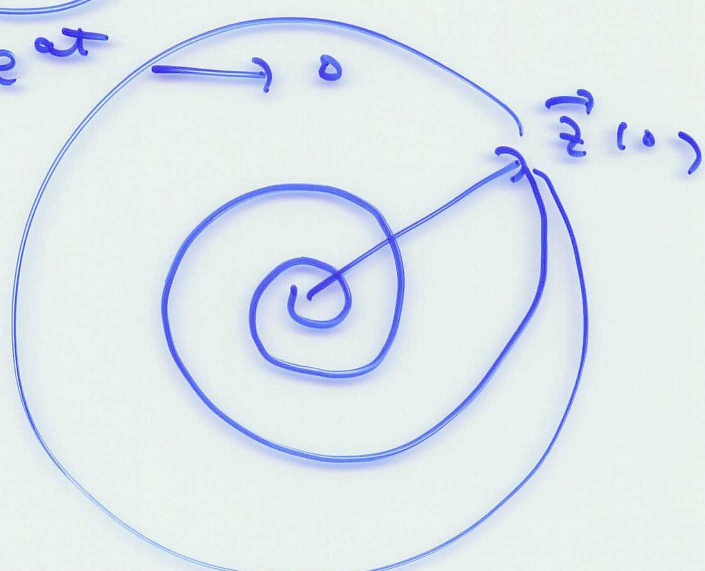
$$\vec{z}(t) = e^{at} \, R(-bt) \, \vec{z}(0)$$



$a > 0 \qquad e^{at} \longrightarrow +\infty$



$a < 0 \qquad e^{at} \longrightarrow 0$



$P^{-1}\vec{x}(t) = \vec{z}(t) \qquad \vec{x}(t) = P\vec{z}(t) \qquad \vec{z}(0) = P^{-1}\vec{x}(0)$

$$\cancel{\vec{z}(t) = P\,\vec{x}(t)} \qquad = P \cdot e^{at} X_0(t) \overbrace{P^{-1}\vec{x}(0)}$$

A : 2次正方行列.

$$e^{tA} = I_2 + tA + \frac{1}{2!}t^2A^2 + \frac{1}{3!}t^3A^3 + \cdots$$

$$e^t = 1 + t + \frac{1}{2!}t^2 + \cdots$$

$$\to \quad x(t) = e^{tA}.$$

$$A = \begin{pmatrix} 5 & 2 \\ -3 & 0 \end{pmatrix} \qquad P = \begin{pmatrix} -1 & -2 \\ 1 & 3 \end{pmatrix}$$

$$\longleftarrow \quad AP = P\begin{pmatrix} 3 & 0 \\ 0 & 2 \end{pmatrix}$$

$$A = P\begin{pmatrix} 3 & 0 \\ 0 & 2 \end{pmatrix}P^{-1}$$

$$A^n = P\begin{pmatrix} 3^n & 0 \\ 0 & 2^n \end{pmatrix}P^{-1}.$$

$$P I_2 P^{-1} + Pt\begin{pmatrix} 3 & 0 \\ 0 & 2 \end{pmatrix}P^{-1} + P\frac{t^2}{2!}\begin{pmatrix} 3^2 & 0 \\ 0 & 2^2 \end{pmatrix}P^{-1}$$

$$+ P\frac{t^3}{3!}\begin{pmatrix} 3^3 & 0 \\ 0 & 2^n \end{pmatrix}P^{-1} + \cdots\cdots$$

$$+ P\frac{t^n}{n!}\begin{pmatrix} 3^n & 0 \\ 0 & 2^n \end{pmatrix}P^{-1}$$

$$= P\begin{pmatrix} \sum_{k=0}^{n} \frac{(3t)^k}{k!} & 0 \\ 0 & \sum_{k=0}^{n} \frac{(2t)^k}{k!} \end{pmatrix}P^{-1}$$

$$\sum_{k=0}^{n \to +\infty} \frac{t^k}{k!} P \begin{pmatrix} 3^k & 0 \\ 0 & 2^k \end{pmatrix} P^{-1}$$

$$= P \begin{pmatrix} \sum_{k=0}^{n} \frac{(3t)^k}{k!} & 0 \\ 0 & \sum_{k=0}^{n} \frac{(2t)^k}{k!} \end{pmatrix} P^{-1}$$

$$\to P \begin{pmatrix} e^{3t} & 0 \\ 0 & e^{2t} \end{pmatrix} P^{-1}.$$

$$e^{tA} = P \begin{pmatrix} e^{3t} & 0 \\ 0 & e^{2t} \end{pmatrix} P^{-1}$$

$$\frac{d}{dt} \vec{x}(t) = A \vec{x}(t)$$

$$\vec{x}(t) = \underbrace{\boxed{P \begin{pmatrix} e^{3t} & 0 \\ 0 & e^{2t} \end{pmatrix} P^{-1}}}_{= e^{tA}} \vec{x}(0)$$

$$A = P \begin{pmatrix} a & b \\ -b & a \end{pmatrix} P^{-1} \quad a \in \mathbb{R}$$

$$e^{tA} = P \underset{\sim}{e^{ta}} \begin{pmatrix} \cos bt & \sin bt \\ -\sin bt & \cos bt \end{pmatrix} P^{-1}$$

$$e^{ta}$$

$$\begin{cases} \cos\theta = 1 - \frac{1}{2!}\theta^2 + \frac{1}{4!}\theta^4 - \frac{1}{6!}\theta^6 + - \cdots \\ \sin\theta = \theta - \frac{1}{3!}\theta^3 + \frac{1}{5!}\theta^5 - + \cdots \end{cases}$$

$$e^{tA} = I_2 + tA + \frac{1}{2!} t^2 A^2 + \frac{1}{3!} t^3 A^3 + \cdots$$

- $e^{tA} \cdot e^{sA} = e^{(s+t)A}$

- $\frac{d}{dt} e^{tA} = A e^{tA}$
  $= e^{tA} A.$