動力学系　dynamical system　力学系

$$A = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$$　実行列　$a, b, c, d \in \mathbb{R}$

$$\Phi_A(\lambda) = \lambda^2 - (a+d)\lambda + \det(A) = 0$$

判別式

$D > 0$ の場合は．既習．　déjà vu．

固有値 α, β　$\alpha \neq \beta$, $\alpha, \beta \in \mathbb{R}$

$$A\vec{v}_1 = \alpha \vec{v}_1, \quad A\vec{v}_2 = \beta \vec{v}_2 \quad (\vec{v}_j \neq \vec{0})$$

$$P = (\vec{v}_1 \ \vec{v}_2)$$

$$\boxed{\frac{d}{dt}\vec{x}(t) = A\vec{x}(t)}$$

$$\vec{x}(t) = P \begin{pmatrix} e^{\alpha t} & 0 \\ 0 & e^{\beta t} \end{pmatrix} P^{-1} \vec{x}(0)$$ が解．

$D = 0$　省略．

$D < 0$

$$\lambda^2 + p\lambda + q = 0 \quad (p, q \in \mathbb{R})$$

$$\lambda = \frac{-p \pm \sqrt{D}}{2} = \frac{-p \pm i\sqrt{-D}}{2}$$

$$\alpha = \frac{-p + i\sqrt{-D}}{2}, \quad \beta = \frac{-p - i\sqrt{-D}}{2}$$

$$\boxed{\beta = \overline{\alpha}}$$ が互いに複素共役

$$\Phi_A(\lambda) = \lambda^2 - (a+b)\lambda + \det(A) = 0$$

の判別式 $D < 0$

$$\rightsquigarrow \frac{d}{dt}\vec{x}(t) = A\vec{x}(t) \text{ の解法?}$$

α, β：固有値.

$A\vec{v}_1 = \alpha\vec{v}_1$ $\qquad \vec{v}_1 \in \mathbb{C}^2$

$\vec{v}_1 = \vec{w}_1 + i\vec{w}_2$

$\overline{\alpha} = \beta$

複素共役

$\vec{w}_1 \in \mathbb{R}^2, \vec{w}_2 \in \mathbb{R}^2$

$\vec{v}_2 = \vec{w}_1 - i\vec{w}_2$

$A\vec{v}_2 = \beta\vec{v}_2$

$$\begin{pmatrix} z_1 \\ z_2 \end{pmatrix} = \begin{pmatrix} x_1 + i y_1 \\ x_2 + i y_2 \end{pmatrix} = \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} + i \begin{pmatrix} y_1 \\ y_2 \end{pmatrix}$$

$$A = \begin{pmatrix} 0 & -2 \\ 1 & 2 \end{pmatrix}$$

$$\Phi_A(\lambda) = \begin{vmatrix} \lambda & 2 \\ -1 & \lambda - 2 \end{vmatrix}$$

$$= \lambda(\lambda - 2) + 2$$

$$= \lambda^2 - 2\lambda + 2$$

$$= (\lambda - 1)^2 + 1 = 0$$

$$(\lambda - 1)^2 = -1$$

$\alpha = 1 + i$

$\beta = 1 - i$.

が固有値.

$\underline{\alpha = 1+i}$

$$\begin{pmatrix} 1+i & 2 \\ \boxed{-1 \quad -1+i} \end{pmatrix} \begin{pmatrix} z_1 \\ z_2 \end{pmatrix} = \vec{0}$$

$$(1+i)\cdot(1 \quad 1-i) = (1+i \quad (1-i)(1+i))$$

$$= (1+i \quad 2)$$

$$z_1 + (1-i) z_2 = 0$$

$$z_2 = 1, \ z_1 = -1 + i$$

$$\vec{v_1} = \begin{pmatrix} -1+i \\ 1 \end{pmatrix} = \begin{pmatrix} -1 \\ 1 \end{pmatrix} + i \begin{pmatrix} 1 \\ 0 \end{pmatrix}$$

$$\boxed{A\vec{v_1} = (1+i)\vec{v_1}.}$$

$$\vec{v_2} = \begin{pmatrix} -1-i \\ 1 \end{pmatrix} = \begin{pmatrix} -1 \\ 1 \end{pmatrix} - i \begin{pmatrix} 1 \\ 0 \end{pmatrix}$$

$$A\vec{v_2} = (1-i)\vec{v_2}$$

$$\vec{w_1} = \begin{pmatrix} -1 \\ 1 \end{pmatrix}, \ \vec{w_2} = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$$

$$\begin{cases} A\vec{w_1} = \vec{w_1} - \vec{w_2} \\ A\vec{w_2} = \vec{w_1} + \vec{w_2} \end{cases}$$

$$A(\vec{w_1} \ \vec{w_2}) = (\vec{w_1} \ \vec{w_2}) \begin{pmatrix} 1 & 1 \\ -1 & 1 \end{pmatrix}$$

↑

正則となる

$\alpha = a + ib$, $\beta = a - ib$ $\quad a, b \in \mathbb{R}$.

$$A\vec{v}_1 = \alpha \vec{v}_1 \qquad \vec{v}_1 = \vec{w}_1 + i\vec{w}_2 \quad \cdots 1$$

$$A\vec{v}_2 = \beta \vec{v}_2 \qquad \vec{v}_2 = \vec{w}_1 - i\vec{w}_2 \quad \cdots 2$$

$\downarrow$

$$\vec{w}_1 = \frac{1}{2}(\vec{v}_1 + \vec{v}_2)$$

$$\vec{w}_2 = \frac{1}{2i}(\vec{v}_1 - \vec{v}_2)$$

$$A\vec{w}_1 = A\cdot\left\{\frac{1}{2}(\vec{v}_1 + \vec{v}_2)\right\}$$

$$= \frac{1}{2}(A\vec{v}_1 + A\vec{v}_2)$$

$$= \frac{1}{2}(\alpha\vec{v}_1 + \beta\vec{v}_2)$$

$$= \frac{1}{2}\left\{(a+ib)(\vec{w}_1 + i\vec{w}_2) + (a-ib)(\vec{w}_1 - i\vec{w}_2)\right\}$$

$$= a\vec{w}_1 - b\vec{w}_2.$$

$$A\vec{w}_2 = b\vec{w}_1 + a\vec{w}_2$$ 同様。

$$A\underbrace{(\vec{w}_1\ \vec{w}_2)}_{=P} = (\vec{w}_1\ \vec{w}_2)\begin{pmatrix} a & b \\ -b & a \end{pmatrix}$$

P 正則.

$$AP = P\begin{pmatrix} a & b \\ -b & a \end{pmatrix}$$

$$\frac{d}{dt}\vec{x}(t) = A\vec{x}(t) \xrightarrow{\vec{z}(t) = P^{-1}\vec{x}(t)} \frac{d}{dt}\vec{z}(t) = \begin{pmatrix} a & b \\ -b & a \end{pmatrix}\vec{z}(t)$$

$$AP = P \begin{pmatrix} a & b \\ -b & a \end{pmatrix}$$

$$\frac{d}{dt}\vec{x}(t) = A\vec{x}(t) \qquad \vec{z}(t) = P^{-1}\vec{x}(t)$$

$$\begin{aligned} \frac{d}{dt}\vec{z}(t) &= P^{-1}\frac{d}{dt}\vec{x}(t) \\ &= P^{-1}A\vec{x}(t) \\ &= P^{-1}AP \cdot P^{-1}\vec{x}(t) \\ &= \begin{pmatrix} a & b \\ -b & a \end{pmatrix}\vec{z}(t) \end{aligned}$$

<u>134</u> $\frac{d}{dt}\vec{x}(t) = \begin{pmatrix} 0 & -2 \\ 1 & 2 \end{pmatrix}\vec{x}(t)$

$\vec{z}(t) = P^{-1}\vec{x}(t)$ とおくと $\quad P = (\vec{w}_1\ \vec{w}_2)$

$\frac{d}{dt}\vec{z}(t) = \begin{pmatrix} 1 & 1 \\ -1 & 1 \end{pmatrix}\vec{z}(t) \quad = \begin{pmatrix} -1 & 1 \\ 1 & 0 \end{pmatrix}$

$$X(t) = \begin{pmatrix} \cos \omega t & \sin \omega t \\ -\sin \omega t & \cos \omega t \end{pmatrix}$$

$$\frac{d}{dt} X(t) = X(t) \,\square$$

$\square$ 2×2行列

$$= \square \, X(t)$$

$$(\cos t)' = -\sin t$$

$$(\sin t)' = \cos t$$

$$(\sin \omega t)' = \omega \cos \omega t$$

固有値

$$X'(t) = \begin{pmatrix} -\omega \sin \omega t & \omega \cos \omega t \\ -\omega \cos \omega t & -\omega \sin \omega t \end{pmatrix}$$

$$= \begin{pmatrix} \cos \omega t & \sin \omega t \\ -\sin \omega t & \cos \omega t \end{pmatrix} \left(\begin{bmatrix} *_1 \\ *_2 \end{bmatrix} \; \square \right)$$

$$*_1 \cos \omega t + *_2 \sin \omega t = -\omega \sin \omega t$$