On va commencer ?

calc

$\left(\frac{1}{u^4}\right)' = -\frac{4}{u^{4+1}}$

小テスト解説.

(1) $y = (1-2x)^5 = u^5$ (但し $u = 1-2x$)

$\frac{dy}{dx} = \frac{dy}{du} \cdot \frac{du}{dx} = 5u^4 \times (-2) = -10(1-2x)^4$

(2) $y = \frac{1}{(3x-2)^5} = \frac{1}{u^5}$ (但し $u = 3x-2$)

$\frac{dy}{dx} = \frac{dy}{du} \cdot \frac{du}{dx} = -5u^{-6} \cdot 3$

$= -15\frac{1}{(3x-2)^6}$

$\left(\frac{1}{x}\right)' = -\frac{1}{x^2}$

$u' = \frac{1}{x^2}$

$\int = 1 - \frac{1}{x}$

(3) $y = \left(\frac{x-1}{x}\right)^4 = u^4$ (但し $u = \frac{x-1}{x}$)

$\frac{dy}{dx} = \frac{dy}{du} \times \frac{du}{dx} = 4u^3 \times \frac{(x-1)'x - (x-1)(x)'}{x^2}$

$\boxed{\left(x^{\frac{1}{n}}\right)' = \frac{1}{n}x^{\frac{1}{n}-1}}$

$= 4\left(\frac{x-1}{x}\right)^3 \times \frac{x-(x-1)}{x^2} = 4\left(\frac{x-1}{x}\right)^3 \times \frac{1}{x^2}$

(4) $y = x^{\frac{2}{3}} = \left(x^{\frac{1}{3}}\right)^2 = u^2$ (但し $u = x^{\frac{1}{3}}$)

$\frac{dy}{dx} = \frac{dy}{du} \cdot \frac{du}{dx} = 2u \times \frac{1}{3}x^{-\frac{2}{3}}$

$\boxed{u = x^{\frac{1}{3}}}$

$= \frac{2}{3}x^{\frac{1}{3}} \times x^{-\frac{2}{3}} = \frac{2}{3}x^{-\frac{1}{3}}$

(5) $y = \frac{1}{\sqrt[4]{x}}$ $\quad \frac{dy}{dx} = \frac{-\left(\sqrt[4]{x}\right)'}{\left(\sqrt[4]{x}\right)^2} = \frac{-\frac{1}{4}x^{-\frac{3}{4}}}{x^{\frac{1}{2}}}$

$\left(x^{\frac{1}{4}}\right)^2 = x^{\frac{1}{4}\times 2} = x^{\frac{1}{2}}$

$= -\frac{1}{4}x^{-\frac{3}{4}} \cdot x^{-\frac{1}{2}} = -\frac{1}{4}x^{-\frac{5}{4}}$

$\left(-\frac{3}{4}\right) + \left(-\frac{1}{2}\right) = -\frac{5}{4}$

(4) $\left(x^{\frac{2}{3}}\right)' = \frac{2}{3} x^{-\frac{1}{3}}$

$\frac{2}{3} - 1 = -\frac{1}{3}$

$$\left(\sqrt[n]{x}\right)' = \frac{1}{n} x^{\frac{1}{n}-1} = \frac{1}{n} \frac{\sqrt[n]{x}}{x}$$

$$\boxed{\left(x^{\alpha}\right)' = \alpha x^{\alpha - 1}}$$

$\left(x^{\frac{1}{4}}\right)' = \frac{1}{4} x^{-\frac{3}{4}}$

$\frac{1}{4} - 1 = -\frac{3}{4}$

(5) $\left(\frac{1}{f}\right)' = -\frac{f'}{f^2}$

$\left(x^{-\frac{1}{4}}\right)' = -\frac{1}{4} x^{-\frac{5}{4}}$

$-\frac{1}{4} - 1 = -\frac{5}{4}$

補足: $\left(x^{\frac{1}{n}}\right)' = \frac{1}{n} x^{\frac{1}{n} - 1}$ $(n = 1, 2, 3, \cdots)$

déjà montré!

$\frac{m}{n}$: 正の有理数. déjà-vu.

$y = \left(x^{\frac{m}{n}}\right) = u^m$

$u = x^{\frac{1}{n}}$

$m = 1, 2, 3, \cdots$

自然数

$\mathbb{N}$ 自然数全体

natural numbers

$$\frac{dy}{dx} = \frac{dy}{du} \cdot \frac{du}{dx}$$

$$= m u^{m-1} \cdot \frac{1}{n} x^{\frac{1}{n} - 1}$$

$$u^{m-1} = \left(x^{\frac{1}{n}}\right)^{m-1} = x^{\frac{m}{n} - \frac{1}{n}}$$

$$= \frac{m}{n} x^{\frac{m}{n} - \frac{1}{n}} \cdot x^{\frac{1}{n} - 1}$$

$$= \frac{m}{n} x^{\frac{m}{n} - 1}$$

$\alpha = \frac{m}{n}$ として

$$\left(x^{\frac{m}{n}}\right)' = \frac{m}{n} x^{\frac{m}{n} - 1}$$

$$\boxed{\left(x^{\alpha}\right)' = \alpha x^{\alpha - 1}}$$

有理数 rational numbers

$\frac{m}{n}$ 　 n：自然数

m：整数.

無理数 irrational

$\sqrt{2}, \sqrt{3}, \cdots\cdots \pi, e.$ etc.

$\sqrt{3}$ が無理数であることを示す.

$\sqrt{3} = \frac{m}{n}$ とおいて 矛盾を出す.

$\frac{1}{4}$年 四半期

$1 \longmapsto 1+r$ 　　　　 $(1+r)^4$

$(1+r)^4 - 1$ 年率を表す.

$\fallingdotseq 1+4r$

e　Napier の数　　$\frac{1}{\sqrt{2\pi}} e^{-\frac{x^2}{2}}$　正規分布

100% 年利率.　　元金 + 利子
interest

単利　$1 \longmapsto 1+1$　元利合計

半年複利　$1 \longmapsto \left(1+\frac{1}{2}\right)^2$

$\frac{1}{3}$年　$1 \longmapsto \left(1+\frac{1}{3}\right)^3$

$\vdots$

$\frac{1}{n}$年複利　$1 \longmapsto \left(1+\frac{1}{n}\right)^n$

$\boxed{a_n = \left(1+\frac{1}{n}\right)^n}$ とおく.

$\{a_n\}$ は収束する.　　$a_n \to e$.

(I) 単調増加　$a_1 < a_2 < a_3 < \cdots$

$a_n = \left(1+\frac{1}{n}\right)^n$

2項定理

$$(1+\theta)^n = \sum_{k=0}^{n} {}_nC_k\, \theta^k$$

$$= 1 + n\theta + \frac{n(n-1)}{2}\theta^2 + \frac{n(n-1)(n-2)}{3!}\cdot\theta^3 + \cdots$$

$$+ \frac{n(n-1)\cdots(n-k+1)}{k!}\theta^k + \cdots$$

$$\cdots + \theta^n$$

$${}_nC_k = \binom{n}{k} = \frac{n!}{(n-k)!\,k!}$$

$$= \frac{n(n-1)\cdots(n-k+1)}{k!}$$

$$a_n = \left(1+\frac{1}{n}\right)^n \qquad \theta = \frac{1}{n}$$

$$= 1 + n \times \frac{1}{n} + \frac{n(n-1)}{2!}\left(\frac{1}{n}\right)^2 + \frac{n(n-1)(n-2)}{3!}\left(\frac{1}{n}\right)^3$$

$$+ \cdots + \frac{n(n-1)\cdots(n-k+1)}{k!}\left(\frac{1}{n}\right)^k + \cdots$$

$$+ \left(\frac{1}{n}\right)^n$$

$$= 1 + 1 + \underbrace{\frac{1}{2!}\left(1-\frac{1}{n}\right)}_{(1)} + \frac{1}{3!}\left(1-\frac{1}{n}\right)\left(1-\frac{2}{n}\right) + \cdots$$

$$\cdots + \frac{1}{k!}\left(1-\frac{1}{n}\right)\left(1-\frac{2}{n}\right)\cdots\left(1-\frac{k-1}{n}\right) + \cdots$$

$$\frac{1}{n} > \frac{1}{n+1}$$

$$+ \frac{1}{n!}\left(1-\frac{1}{n}\right)\left(1-\frac{2}{n}\right)\cdots\left(1-\frac{n-1}{n}\right)$$

$$a_{n+1} = 1 + 1 + \overbrace{\frac{1}{2!}\left(1-\frac{1}{n+1}\right)}^{大} + \frac{1}{3!}\left(1-\frac{1}{n+1}\right)\left(1-\frac{2}{n+1}\right) +$$

$$\cdots + \frac{1}{k!}\left(1-\frac{1}{n+1}\right)\cdots\left(1-\frac{k-1}{n+1}\right) + \cdots$$

$$+ \cancel{(n+1)}\ \frac{1}{n!}\left(1-\frac{1}{n+1}\right)\cdots\left(1-\frac{n-1}{n+1}\right)$$

$$+ \boxed{\frac{1}{(n+1)!}\left(1-\frac{1}{n+1}\right)\cdots\left(1-\frac{n}{n+1}\right)}$$

増える →

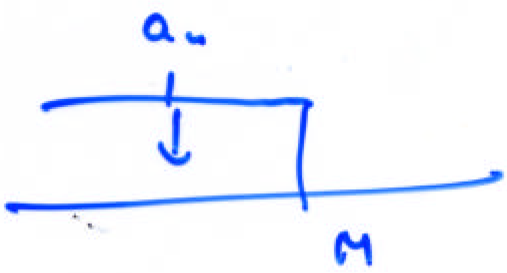
$$\rightsquigarrow \quad a_{n+1} > a_n$$

(II) $\{a_n\}$ は上に有界.

2ps i.e.

ある M に対して

$a_n \leq M$ (全ての n)



ido est

$$a_n \leq 1 + 1 + \frac{1}{2!} + \frac{1}{3!} + \cdots + \frac{1}{n!}$$

帰納法から $k! \geq 2^{k-1}$

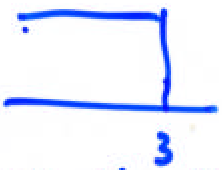
$$1 \cdot 2 \cdot 3 \cdot \cdots (k-1) \cdot k$$
$$\geq 1 \cdot 2 \cdot 2 \cdot \cdots \cdot 2 \cdot 2 = 2^{k-1}$$

$$a_n \leq 1 + 1 + \frac{1}{2} + \frac{1}{2^2} + \frac{1}{2^3} + \cdots + \frac{1}{2^{n-1}}$$
$$= 1 + \frac{1 - \left(\frac{1}{2}\right)^n}{1 - \frac{1}{2}} = 1 + 2\left(1 - \left(\frac{1}{2}\right)^n\right)$$
$$\leq 1 + 2 = 3.$$

まとめ

(I) $\{a_n\}$ は単調増加

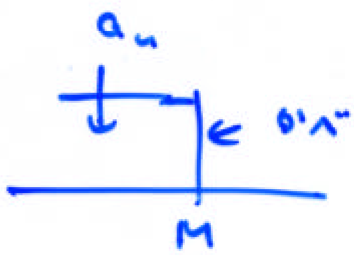
(II) $\{a_n\}$ は上に有界



bounded from above

定理 → $\{a_n\}$ は収束する.

定理 $\{a_n\}$: 単調増加, 上に有界



$\Rightarrow$ $\{a_n\}$ は収束する.

$a_n = \left(1+\frac{1}{n}\right)^n$ は収束する.　　$e = 2.71\cdots$

$$\left(1+\frac{1}{n}\right)^n \longrightarrow e \quad (n \to +\infty)$$

Napier の数.

$$\cdots\cdots \longrightarrow$$

$$\lim_{h \to 0} \frac{e^h - 1}{h} = 1$$

α $$\boxed{\alpha \in \mathbb{R} \quad \left(1+\frac{\alpha}{n}\right)^n \longrightarrow e^\alpha}$$

100α % の年利率で瞬間複利

$100 \times \underline{\underline{e^\alpha}}$ %

瞬間スポットレート.

$(e^x)' = ?$ $\qquad e^{a+h} = e^a \cdot e^h$

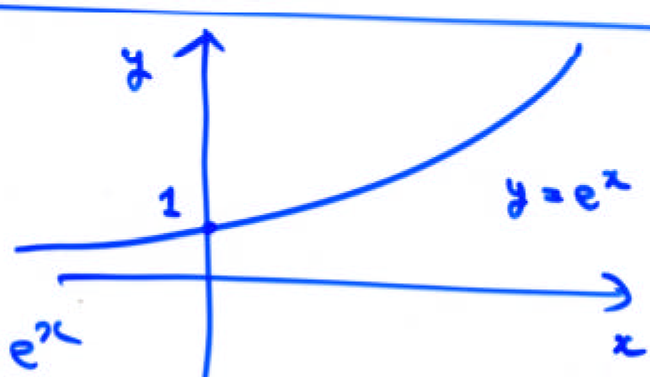
$$\frac{e^{a+h} - e^a}{h} = e^a \boxed{\frac{e^h - 1}{h}}$$

$$\longrightarrow e^a \times 1 = e^a$$

$$\boxed{(e^x)' = e^x}$$

$y = e^x$

$x = \log_e y$



$y = e^x \qquad y' = e^x$

$$(-\infty, +\infty) \longleftrightarrow (0, +\infty)$$

$$x \longmapsto y = e^x$$

$$x = \log_e y \longleftrightarrow y > 0$$

$$\log_e y = \log y = \ln y$$ 自然対数

$$\frac{dx}{dy} = \frac{1}{\frac{dy}{dx}} = \frac{1}{e^x} = \frac{1}{y}$$

$$\boxed{(\log t)' = \frac{1}{t}}$$

$t > 0 \qquad t^{\alpha} = (e^{\log t})^{\alpha} = e^{\alpha \log t}$

$\alpha \in \mathbb{R} \qquad \boxed{t = e^{\log t}}$

$t^{\sqrt{2}}$

$$y = t^{\alpha} = e^{\alpha \log t} = e^{u}$$

但し $u = \alpha \log t \qquad \boxed{(\log t)' = \frac{1}{t}}$

$$\frac{dy}{dt} = \frac{dy}{du} \cdot \frac{du}{dt} = e^{u} \cdot \frac{\alpha}{t}$$

$$= t^{\alpha} \cdot \frac{\alpha}{t} = \alpha t^{\alpha - 1}$$

$$\boxed{(t^{\alpha})' = \alpha t^{\alpha - 1}}$$

今からの $\lambda = 2$ —

微分の応用, 凹 凸

Taylor 展開

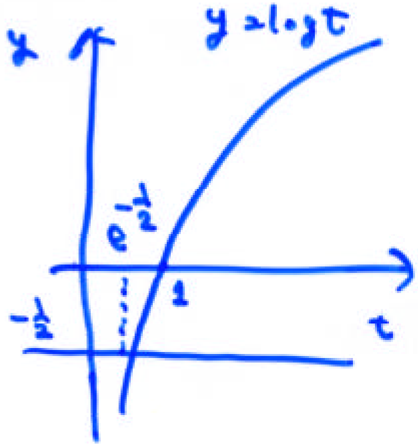
$$e^{t} = 1 + t + \frac{1}{2!} t^{2} + \cdots$$

偏微分 (時間があると積分)

小テスト

I 次の関数を微分せよ

(1) $\dfrac{\log t}{t}$



(2) e^{-3t}

(3) $t^{\frac{1}{3}}(t-1)$

(4) $t^2 e^{-t}$

II. $y = t^2 \log t$ の増減表.

$(fg)' = f'g + fg'$