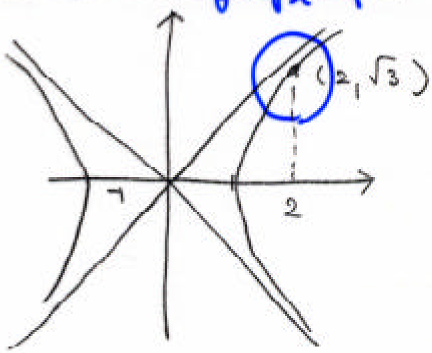
(II) $g(x,y) = x^2 - y^2 - 1 = 0$ の $(2,\sqrt{3})$ を通る
接線の傾き

$y = \sqrt{x^2-1}$



$(2,\sqrt{3})$ の近くで

$$y = \varphi(x)$$

と解けているとすると

$$x^2 - \varphi(x)^2 - 1 \equiv 0$$

が成立する。両辺を x で微分すると

$$2x - 2\varphi(x)\cdot\varphi'(x) \equiv 0$$

を得る。$x = 2$ を代入すると

$$2\times 2 - 2\times\sqrt{3}\,\varphi'(2) = 0$$

から

$$\boxed{\varphi'(2) = \frac{2}{\sqrt{3}}}$$

を得る。

$$\frac{dy}{dx} = \frac{1}{2}\frac{2x}{\sqrt{x^2-1}} = \frac{x}{\sqrt{x^2-1}}$$

$$\boxed{\begin{aligned}\frac{du^2}{dx} &= \frac{du^2}{du}\cdot\frac{du}{dx} \\ &= 2u\cdot\varphi'(x)\end{aligned}}$$

(III) $f(x,y) = x^3 + 2x^2y + 5y^4$ に対して

$\dfrac{\partial f}{\partial x}$, $\dfrac{\partial f}{\partial y}$

を求める.

定数.

$$f_x = 3x^2 + \overset{2y\cdot 2x}{4xy} + 0 = \boxed{3x^2 + 4xy} = x(3x+4y)$$

$$f_y = 0 + 2x^2\cdot 1 + 20y^3$$

$$= 2x^2 + 20y^3.$$

停留点 $f_x = f_y = 0$ $\Leftarrow$ 極大・極小.

$f_x = 0 \Leftrightarrow x = 0$ or $3x + 4y = 0$ $x = 0$

(i) $x = 0$ $0 + 20y^3 = 0 \rightsquigarrow y = 0$ $(0,0)$

(ii) $3x + 4y = 0 \rightsquigarrow x = -\frac{4}{3}y$

$2\times\frac{16}{9}y^2 + 20y^3 = 0$ → $y = 0$ or $y = -\frac{8}{45}$

$\frac{32}{9} + 20y = 0$

$x^2 + xy - y^2 = 1$ 上の点 (a, b) で

$\Updownarrow$

$y = \varphi(x)$ と書けていたとする.

$$x^2 + x\varphi(x) - \varphi(x)^2 - 1 \equiv 0$$

$\boxed{\varphi'(x) \text{ を } x \text{ と } \varphi(x) \text{ で表す.}}$

$$y^2 - xy + 1 - x^2 = 0$$

$$y = \frac{x \pm \sqrt{x^2 - 4(1 - x^2)}}{2}$$

$$= \frac{x \pm \sqrt{5x^2 - 4}}{2}$$

$$2x + 1 \cdot \varphi(x) + x\varphi'(x) - 2\varphi(x)\varphi'(x) = 0$$

$$2x + \varphi(x) + \varphi'(x)(x - 2\varphi(x)) = 0$$

$$\varphi'(x) = \frac{2x + \varphi(x)}{2\varphi(x) - x}$$

$$= \frac{2x + y}{2y - x}$$

예 8.12. $z = f(x, y) = x^3 - xy - y^2$

$$\frac{\partial z}{\partial x} = z_x = 3x^2 - y \cdot 1 - 0$$

$$\begin{cases} z_x = 3x^2 - y = 0 \\ z_y = 0 - x \cdot 1 - 2y = 0 \end{cases}$$

$$\begin{cases} y = 3x^2 \\ x + 2y = 0 \quad \dashrightarrow \quad x = -2y \end{cases}$$

$y = 12y^2 \xrightarrow[0]{\otimes} 1 = 12y$

또는 $y = 0$

$y = 0$ or $\frac{1}{12}$

$\downarrow \qquad \downarrow$

$x = 0 \qquad x = -\frac{1}{6}$

$z = \underbrace{e^{-x^2 - y^2}}_{= e^{-x^2} \cdot e^{-y^2}} (2x^2 + y^2)$

$(e^{-t^2})' = e^{-t^2} \cdot (-2t)$

$z_x = e^{-y^2} \cdot e^{-x^2} (-2x)(2x^2 + y^2)$

$\quad + e^{-x^2 - y^2} \cdot 4x$

or

$= \boxed{e^{-x^2 - y^2}} \times 2x(-2x^2 - y^2 + 2)$

$z_y = e^{-x^2} \cdot e^{-y^2} (-2y)(2x^2 + y^2)$

$\quad + e^{-x^2 - y^2} \cdot 2y$

or

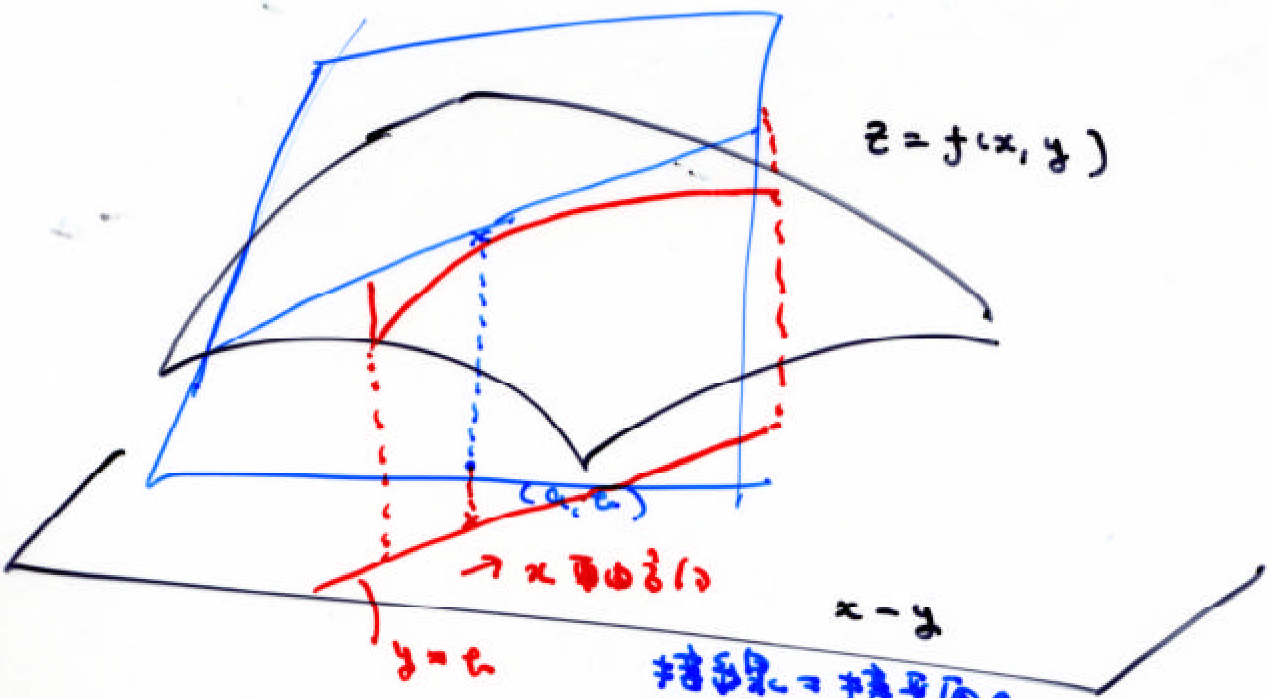
$= \boxed{e^{-x^2 - y^2}} \times 2y(-2x^2 - y^2 + 1)$

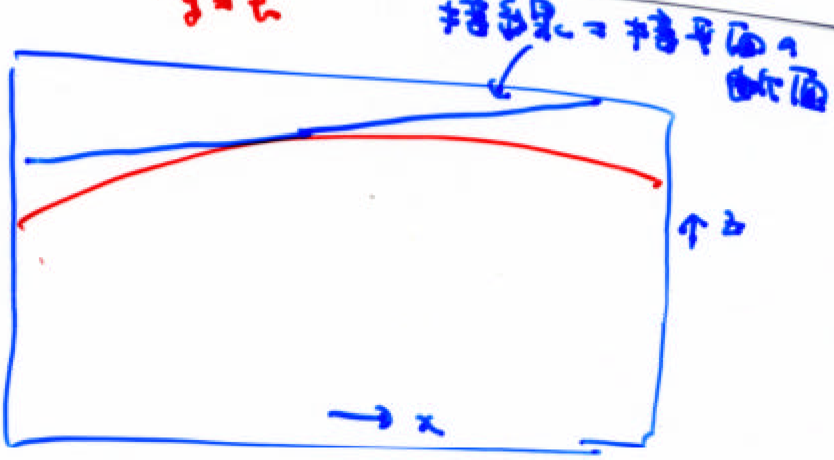
$z_x = 0 \iff x = 0$ 또는 $2x^2 + y^2 = 2$

$z_y = 0 \iff y = 0$ 또는 $2x^2 + y^2 = 1$

(1) $x = y = 0$ (2) $x = 0$, $2x^2 + y^2 = 1 \rightsquigarrow y = \pm 1$

(3) $2x^2 + y^2 = 2$, $y = 0 \rightsquigarrow x = \pm 1$





接平面

$$A(x-a)+B(y-b)+C(z-f(a,b))=0$$

$\boxed{C \neq 0}$

何故か

$C = 0 \rightsquigarrow$ 法線

$$\begin{pmatrix} A \\ B \\ 0 \end{pmatrix} \parallel x\text{-}y \text{平面}$$

$\rightsquigarrow$ 平面 $\perp$ x-y 平面

$$z = -\frac{A}{C}(x-a) - \frac{B}{C}(y-b) + f(a,b)$$

$$\boxed{z = \alpha(x-a) + \beta(y-b) + f(a,b)}$$

接平面の方程式

→ $y=b$ の断面

$$z = \alpha(x-a) + f(a,b)$$

$$z = F(x) = f(x,b)$$

$$\frac{\partial f}{\partial x}(a,b) = F'(a) = \alpha \qquad \boxed{\alpha = \frac{\partial f}{\partial x}(a,b)}$$

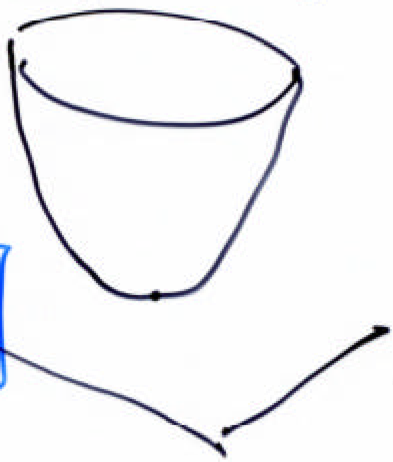
同様に $x=a$ の断面で考えて

$$\leadsto \boxed{\beta = \frac{\partial f}{\partial y}(a,b)}$$

接平面

$$z = \frac{\partial f}{\partial x}(a,b)(x-a) + \frac{\partial f}{\partial y}(a,b)(y-b) + f(a,b)$$

例



$z = x^2 + y^2$ at $(1,1,2)$

$$\left.\begin{array}{ll} z_x = 2x & z_x = 2 \\ z_y = 2y & z_y = 2 \end{array}\right) \text{at } (1,1)$$

$$\boxed{z = 2(x-1) + 2(y-1) + 2}$$

小テスト

I　$z = xy(x^2+y^2+1)$ の停留点を求める

II　$x^4 + y^4 - 4xy = 0$ が定める

$$y = \varphi(x)$$

に対して $\varphi'(x)$ を $x, \varphi(x)$ で表す.

III　$z = e^{2x-3y}$ の $(0,0,1)$ における接平面を求める.

$= e^{2x} \cdot e^{-3y}$

$(e^{ct})' = c\,e^{ct}$