微分積分演習 (6/7)

I (1) $y = e^{-t^2} = e^x \quad (x = -t^2)$

について

$$\frac{dy}{dt} = \frac{dy}{dx} \cdot \frac{dx}{dt} = e^x \cdot (-2t)$$

$$= -2t\, e^{-t^2}$$

(2) $y = \log(1+t^2) = \log x \quad$ (但し $x = 1+t^2$)

について

$$\frac{dy}{dt} = \frac{dy}{dx} \cdot \frac{dx}{dt} = \frac{1}{x} \cdot 2t = \frac{2t}{1+t^2}$$

(3) $y = e^{3t} = e^x \quad (x = 3t)$

$$\frac{dy}{dt} = \frac{dy}{dx} \cdot \frac{dx}{dt} = e^x \cdot 3 = 3e^{3t}$$

(4) $y = e^{-2t} = e^x \quad (x = -2t)$ について

$$\frac{dy}{dt} = \frac{dy}{dx} \cdot \frac{dx}{dt} = e^x(-2) = -2e^{-2t}$$

$$(t\, e^{-2t})' = (t)' e^{-2t} + t(e^{-2t})'$$

$$= 1 \cdot e^{-2t} + t(-2e^{-2t})$$

$$= (1-2t)\, e^{-2t}$$

(5) $(t \log t)' = (t)' \log t + t(\log t)'$

$$= 1 \cdot \log t + t \cdot \frac{1}{t}$$

$$= \log t + 1$$

Ⅱ (1) $y = te^t$ について

$$\frac{dy}{dt} = (t)'e^t + t(e^t)'$$

$$= 1 \cdot e^t + t \cdot e^t = (t+1)e^t$$

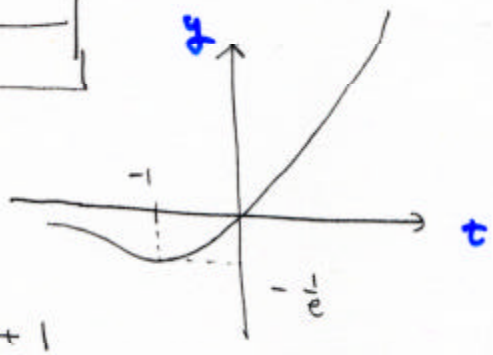
$e^t > 0$ より

$$\frac{dy}{dt} \gtrless 0 \iff t+1 \gtrless 0 \iff t \gtrless -1$$

増減表は

t		-1	
y'	$-$	0	$+$
y	↘	$-\frac{1}{e}$	↗

極小値 minimal value

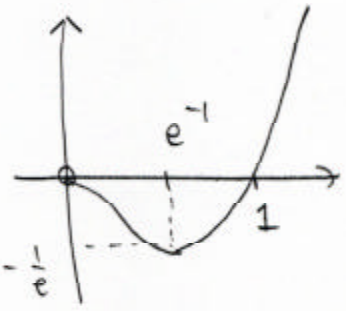


(2) $y = t\log t$ $\quad \frac{dy}{dt} = \log t + 1$

$$\log t + 1 \gtrless 0 \iff \log t \gtrless -1 \iff t \gtrless e^{-1}$$

(よって)

t		e^{-1}	
y'	$-$	0	$+$
y	↘	$-\frac{1}{e}$	↗



$e > 2.7 \rightsquigarrow \frac{1}{e} < 1$

$y = \frac{1}{\sqrt{2\pi}} e^{-\frac{t^2}{2}}$ 标准正规分布.

$\int_{-\infty}^{+\infty} y\,dx = 1$ → 2年, 为统计学. statistics

全概率

$(e^t)' = e^t$

$(\log t)' = \frac{1}{t}$ $\log_e t$ 常用对数.

$\log_e t = \log t$

$a \neq 1,\ a > 0$

$\log_a t = \frac{\log_e t}{\log_e a}$

$(\log_a t)' = \frac{1}{\log a} \times \frac{1}{t}$

$y = e^{ct}$ $c \in \mathbb{R}$ 定数.

$= e^x$ $x = ct$

$\frac{dy}{dt} = \frac{dy}{dx} \cdot \frac{dx}{dt} = e^x \cdot c = ce^{ct}$

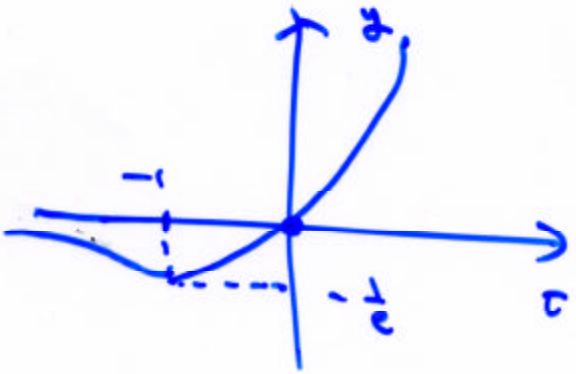
$\boxed{(e^{ct})' = ce^{ct}}$

(4) $(fg)' = f'g + fg'$

(4)

$y = te^t \rightsquigarrow 0$

$y \gtrless 0 \iff t \gtrless 0$



① $\lim_{t \to -\infty} te^t = 0$

(基本的な極限)

$\lim_{t \to +\infty} \frac{t}{e^t} = 0$

→ 証明は省略

$s = -t$ とおく

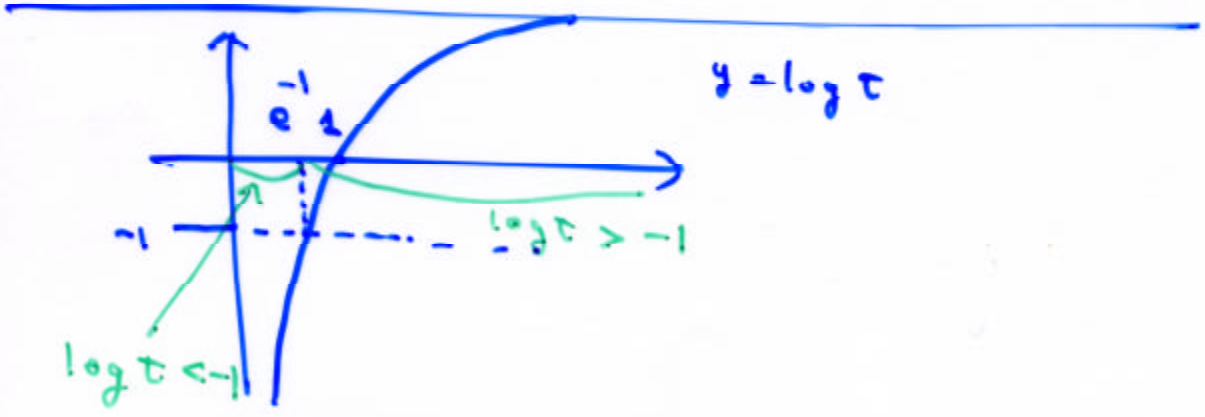
$t \to -\infty$ のとき $s \to +\infty$

$te^t = -se^{-s} = -\frac{s}{e^s}$

$\longrightarrow (-1) \times 0 = 0$

② $t \to +\infty$ のとき $e^t \to +\infty$

$te^t \to +\infty$



補足 $\log t \longrightarrow +\infty \quad (t \to +\infty)$

$\log t \longrightarrow -\infty \quad (t \to -\infty)$

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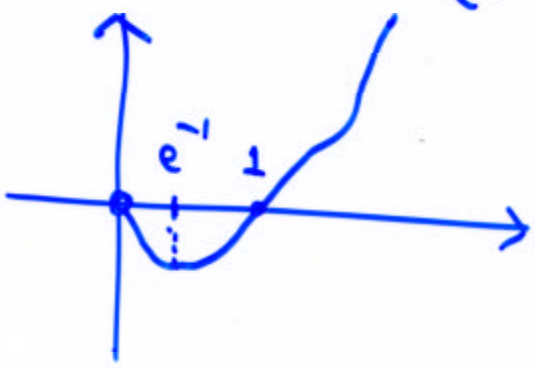
1) $t \longrightarrow +\infty$ のとき $t\log t \longrightarrow +\infty$

2) $t \to +0$ $\quad t\log t \longrightarrow ?$

$-\log t = s$ とおく. $\quad t = e^{-s}$

$t \to +\infty$ のとき $s \to +\infty$

$$t\log t \longrightarrow e^{-s} \times (-s) = -\frac{s}{e^s} \longrightarrow 0$$



$y = t\log t \quad (t > 0)$

$y \geqq 0 \iff \log t \geqq 0 \iff t \geqq 1$

$y' = \log t + 1$

$y'' = \frac{1}{t} > 0 \rightsquigarrow$ 下に凸

1次近似式　approximate

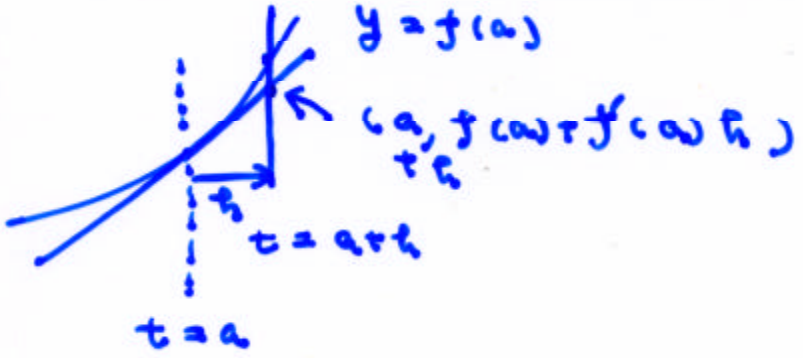
$$\frac{f(a+h)-f(a)}{h} \longrightarrow f'(a) \quad (h \to 0)$$

$|h|$ が a と比べて十分小さいとき

$$\frac{f(a+h)-f(a)}{h} \fallingdotseq f'(a)$$

$$f(a+h)-f(a) \fallingdotseq f'(a)\cdot h$$

$$\boxed{f(a+h) = f(a) + f'(a)\cdot h}$$



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1) $y = f(x) = \sqrt{1+x}$

$$f'(x) = \frac{1}{2}\cdot\frac{1}{\sqrt{1+x}}\cdot 1 = \frac{1}{2}\cdot\frac{1}{\sqrt{1+x}}$$

$a = 0$　$\sqrt{1+h} \fallingdotseq 1 + \frac{1}{2}h$

$f(0) = 1$, $f'(0) = \frac{1}{2}$

$\sqrt{1.01} = (1.01)^{\frac{1}{2}} \fallingdotseq 1 + 0.005 = 1.005$

$(1.1)^{\frac{1}{2}} \fallingdotseq 1 + 0.05 = 1.05$

$= 1.0488$

成長率

A　A(1+r)

$$\frac{A(1+r)^4 - A}{A} = (1+r)^4 - 1$$

$$\fallingdotseq 1 + 4r - 1 = 4r$$

$f(t) = (1+t)^4$　　$f'(0) = 4$

$f'(t) = 4(1+t)^3 \cdot 1$　　$f(0) = 1$

$= 4(1+t)^3$

$(1+r)^4 \fallingdotseq 1 + 4r$

$f(a+h) \fallingdotseq f(a) + h f'(a)$

$f(a+h) \fallingdotseq f(a) + h f'(a) + \frac{1}{2} h^2 f''(a)$

$\vdots$ Taylor 展開

C言語　情報処理

次の関数の増減表を求めよ.
(できたらグラフも)

I $y = \frac{\log t}{t}$

II $y = t^2 e^{-t}$

III $y = t e^{-t^2}$

$\frac{t^a}{e^t} \to 0$ $(t \to +\infty)$

$t^2 = s$

$t > 0$

$t e^{-t^2} = s^{\frac{1}{2}} e^{-s}$

$= \frac{s^{\frac{1}{2}}}{e^s} \to 0$

$t \to +\infty$ $s \to +\infty$

$a = \frac{1}{2}$