小テスト解答

I (1)

$$(t\sqrt{1+t^2})' = (t)'\sqrt{1+t^2} + t(\sqrt{1+t^2})'$$

$$= \sqrt{1+t^2} + t \cdot \frac{t}{\sqrt{1+t^2}}$$

$$= \frac{(1+t^2)+t^2}{\sqrt{1+t^2}} = \frac{2t^2+1}{\sqrt{1+t^2}}$$

ここで $y=\sqrt{1+t^2}$ の微分を $x=1+t^2$ とおいて

$$\frac{dy}{dx} = \frac{d\sqrt{x}}{dx} \times \frac{d(1+t^2)}{dt}$$

$$= \frac{1}{2}\frac{1}{\sqrt{x}} \times 2t = \frac{t}{\sqrt{1+t^2}}$$

と計算している.

(2) $y = \dfrac{1}{(1+t^2)^2} = \dfrac{1}{x^2}$ （ここで $x=1+t^2$ ）

times

$$\frac{dy}{dt} = \frac{dy}{dx} \times \frac{dx}{dt} = -\frac{2}{x^3} \times 2t = -\frac{4t}{(1+t^2)^3}$$

(3) $y = (1+t+t^2)^4 = x^4$ （ただし $x=1+t+t^2$ ）

$$\frac{dy}{dt} = \frac{dy}{dx}\cdot\frac{dx}{dt} = 4x^3\cdot(1+2t)$$

$$= 4(1+t+t^2)^3(1+2t)$$

(4) $y = \dfrac{1}{(2t+1)^4} = \dfrac{1}{x^4}$ （但し $x=2t+1$ ）

$$\frac{dy}{dt} = \frac{dy}{dx}\cdot\frac{dx}{dt} = -\frac{4}{x^5}\cdot 2 = -\frac{8}{(2t+1)^5}$$

$(fg)' = f'g + fg'$ 　　　　　　　　　　　　　　　　　2

$y = \sqrt{1+t^2} = \sqrt{x}$ 　　 $x = 1+t^2$

$$\frac{dy}{dt} = \frac{dy}{dx} \cdot \frac{dx}{dt}$$

$$= \frac{1}{2}\frac{1}{\sqrt{x}} \cdot 2t = \frac{t}{\sqrt{1+t^2}}$$

$$(\sqrt{x})' = \frac{1}{2} \cdot \frac{1}{\sqrt{x}}$$

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(2) $(x^{-2})' = -2x^{-3}$

$$= -2\frac{1}{x^3}$$

$$\left(\frac{1}{f}\right)' = -\frac{f'}{f^2}$$

$$\frac{dy}{dt} = -\frac{\{(1+t^2)^2\}'}{(1+t^2)^4} = \cdots$$

$$= -\frac{2(1+t^2)\cdot 2t}{(1+t^2)^4}$$

(4) $(x^{-4})' = -4\frac{1}{x^5}$

II $\left(\frac{g}{f}\right)' = \frac{g'f - gf'}{f^2}$

$y = \sqrt{1+t+t^2} = \sqrt{x}$

(ただし $x = 1+t+t^2$

$$\frac{dy}{dt} = \frac{1}{2}\frac{1}{\sqrt{x}} \times (2t+1)$$

II $y = \dfrac{t}{\sqrt{1+t+t^2}}$ に対して

$$\frac{dy}{dt} = \frac{(t)'\sqrt{1+t+t^2} - t\left(\sqrt{1+t+t^2}\right)'}{(1+t+t^2)}$$

$$= \left\{\sqrt{1+t+t^2} - t\cdot\frac{2t+1}{2\sqrt{1+t+t^2}}\right\} \Big/ (1+t+t^2)$$

$$= \frac{2(1+t+t^2) - t(2t+1)}{2\sqrt{1+t+t^2}\times(1+t+t^2)} = \frac{t+2}{2\sqrt{1+t+t^2}\times(1+t+t^2)}$$

ここで $y = \sqrt{1+t+t^2} = \sqrt{x}$ $(x = 1+t+t^2)$

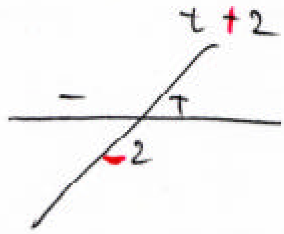
の微分を

$$\frac{dy}{dt} = \frac{1}{2}\frac{1}{\sqrt{x}}\cdot(2t+1) = \frac{2t+1}{2\sqrt{1+t+t^2}}$$

と計算している。

増減表は

t		-2	
y'	$-$	0	$+$
y	$\searrow$	$-\frac{2}{\sqrt{3}}$	$\nearrow$



$$y' = \frac{t+2}{2(1+t+t^2)\sqrt{1+t+t^2}}$$

$$t^2+t+1 = \left(t+\frac{1}{2}\right)^2 + \frac{3}{4} > 0$$

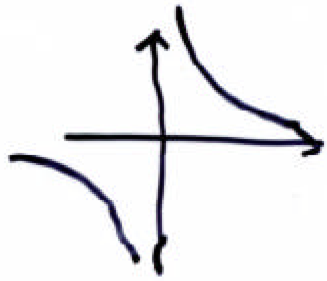
$$y' \gtreqless 0 \iff t+2 \gtreqless 0$$

$t \longrightarrow +\infty \qquad y \longrightarrow ?$

$$y = \frac{t}{\sqrt{1+t+t^2}}$$

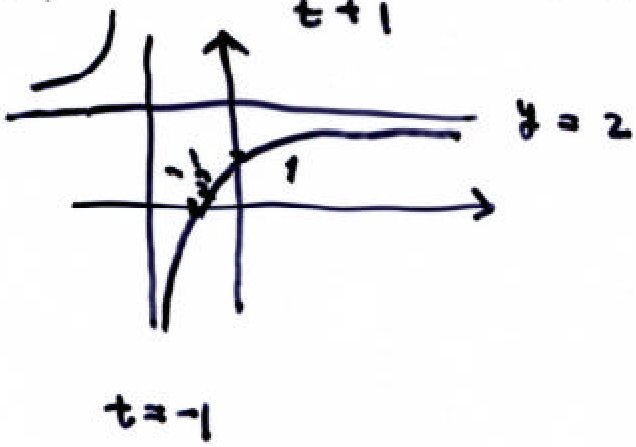
$$\frac{2t+1}{t+1} = \frac{2+\frac{1}{t}}{1+\frac{1}{t}}$$

$$\longrightarrow \frac{2+0}{1+0} = 2$$



$$\lim_{t \to +\infty} \frac{1}{t} = 0$$

$$y = \frac{2t+1}{t+1} = \frac{2(t+1)-1}{t+1} = 2 - \frac{1}{t+1}$$



$y = -\frac{1}{t}$

$3\frac{1}{2}$

$t > 0$ ακα

$$y = \frac{t}{\sqrt{1+t+t^2}} = \frac{1}{\sqrt{\frac{1}{t^2}+\frac{1}{t}+1}} \to \frac{1}{1} = 1$$

$t \to +\infty$ ακα $\frac{1}{t} \to 0$, $\frac{1}{t^2} \to 0$

$$1 + \frac{1}{t} + \frac{1}{t^2} \longrightarrow 1.$$

$$\sqrt{1+\frac{1}{t}+\frac{1}{t^2}} \longrightarrow \sqrt{1} = 1$$

$$\boxed{\begin{array}{l} \alpha > 0 \\ \lim\limits_{t\to\alpha} \sqrt{t} = \sqrt{\alpha} \end{array}}$$

$t < 0$ $\qquad t \longrightarrow -\infty$ $\qquad \frac{1}{t} \to 0$, $\frac{1}{t^2} \to 0$

$$-t = \sqrt{t^2} \rightsquigarrow t = -\sqrt{t^2}$$

$$y = \frac{t}{\sqrt{1+t+t^2}} = \frac{-\sqrt{t^2}}{\sqrt{1+t+t^2}} = -\frac{1}{\sqrt{\underbrace{\frac{1}{t^2}+\frac{1}{t}+1}_{\downarrow\, 1}}}$$

$$\to -\frac{1}{\sqrt{1}} = -1$$

公式　記

$$\lim_{n\to+\infty}\left(1+\frac{1}{n}\right)^n = e$$ とおく。

↳ 収束する。　2.71……

100%の年利。

単利　$1 \longmapsto 1+1$

半年複利　$1 \longmapsto \left(1+\frac{1}{2}\right)^2 = 2.25$

$\frac{1}{3}$年 ― $1 \longmapsto \left(1+\frac{1}{3}\right)^3 = \frac{64}{27} = 2\frac{10}{27} = \cdots$

$\frac{1}{n}$年　$1 \longmapsto \left(1+\frac{1}{n}\right)^n = a_n$ とおく。

$$a_1 < a_2 < \cdots\cdots < a_n < a_{n+1} < \cdots < \underset{\substack{\shortparallel \\ 2.71\cdots}}{e}$$

- 100α%の年利

$\frac{1}{n}$年複利　$\left(1+\frac{\alpha}{n}\right)^n \longrightarrow e^\alpha \quad (n\to+\infty)$

スポットレート

数学の得意な人「金融工学」

東工大，一橋，東大……

(5

$$\boxed{\lim_{h\to 0} \frac{e^h - 1}{h} = 1}$$

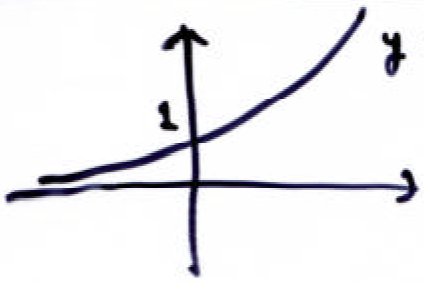
大事

$$\lim_{n\to+\infty} \left(1+\frac{1}{n}\right)^n = e$$

$$\frac{e^{a+h} - e^a}{h} = e^a\left(\frac{e^h - 1}{h}\right) \to e^a \cdot 1 = e^a$$

$(e^t)' = ?$ $\qquad e^{a+h} = e^a \cdot e^h$

$$\boxed{(e^t)' = e^t}$$



$y = e^t$ $\qquad (e^t)' = e^t > 0$

$y = e^t$ 単調増加

$$(-\infty, +\infty) \longrightarrow (0, +\infty)$$

$$t \longmapsto s = e^t \qquad \underline{s > 0.}$$

$$t = \log_e s \longleftarrow s$$

$s = \log_e t \longrightarrow$ $t = e^s$

$$\frac{ds}{dt} = \frac{1}{\frac{dt}{ds}} =$$

$$t = \log_e s \longleftrightarrow \boxed{s = e^t} \qquad \frac{ds}{dt} = e^t$$

$$\frac{dt}{ds} = \frac{1}{\frac{ds}{dt}} = \frac{1}{e^t} = \frac{1}{s}$$

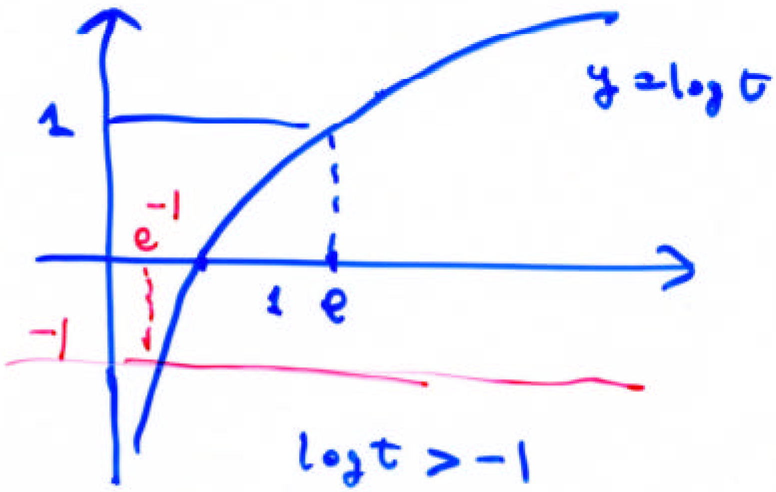
$$\boxed{(\log_e s)' = \frac{1}{s}}$$

$$(\log_e x)' = \frac{1}{x}$$

$$y = e^{-t^2} = e^s \qquad \frac{dy}{dt} = \frac{dy}{ds} \cdot \frac{ds}{dt}$$

$$s = -t^2$$

$$e^{-1} = \frac{1}{e} \qquad \log \frac{1}{e} = -1$$



$$\log t > -1$$

$$\iff \log t + 1 > 0$$

小テスト

I 微分せよ.

(1) e^{-t^2}

(2) $\log_e(1+t^2)$

(3) e^{3t}

(4) $t\,e^{-2t}$

(5) $t\log t$ $(t>0)$

II 増減表

(1) $t\,e^{t}$ 真数条件.

(2) $t\log t$ $t>0$

対数⟷真数 は正