WAGE DIFFERENTIALS AS REPUTATION EQUILIBRIA*

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Abstract: Equilibrium search models obtained endogenous wage differentials among identical firms and workers based on the equal profit assumption. We formulate a more fundamentally dynamic game with identical players where workers can strategically quit and firms cannot commit to future wages, and find that equilibrium wage differentials arise due to different worker expectations on the firms' future behavior. High-wage reputation gives a rent to firms, and workers also prefer to work for firms with high-wage reputation. Implications from the models are consistent with many empirical findings including the firm size-wage effect.

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1. INTRODUCTION

One of the most puzzling facts in labor economics is the persistent wage differentials that cannot be attributed to usual factors such as labor quality (education, skills, age and so on), work conditions and bargaining power (unionization). (See for example, Krueger and Summers 1988, and Thaler 1989.) This is puzzling because wages are supposed to be the prices of the commodity called "labor service" and the evidence seems to contradict the "one good, one price" principle. Efficiency wage theory (for example Shapiro and Stiglitz 1984) formulates that wages are incentive devices. This view still implies that if all firms and workers are identical, the wages should be the same.

Equilibrium search theory such as Burdett and Mortensen (1998) obtained wage differentials by assuming that the firms are indifferent among different wage offers. While the uniqueness of their equilibrium is attractive, two important assumptions in Burdett and Mortensen (1998) are restrictive. One is that the firms commit to the entire future wages. This makes the game essentially static. The other is that different wage offers yield the same profit. Without equal profit, the static equilibrium cannot display wage dispersion.

We think that the game should be more fundamentally dynamic, allowing firms to set wages every period. Coles (1998) modified Burdett and Mortensen (1998) to allow this. His focus was the size effect on wages, so a firm's strategy was a function from its size to a wage offer. Therefore, in the equilibrium same-size firms offer the same wage, but since there are different size firms in the market, the game has an equilibrium wage differential.

We use a simpler dynamic game than Coles (1998) and we allow the same size firms to offer different wages. This highlights the effect of the "reputation" of a firm, which is workers' expectation of the firm's future wages based on its past offers.[1] A firm can freely choose wage levels each period, but it knows that the history of wage offers is used in workers' expectations of its future wages, and this affects their turnover decisions.

In our model, a wage offer is not just a payment for labor service for the relevant period, but also a signal about the future offers. This is a fundamental difference from Coles (1998) and other search models where the wage history was a consequence of changes of other variables such as size and productivity measure. This is because the previous models did not treat the wages as dynamic strategies. In our model, keeping a high wage not only reduces the current period turnover rate but also retains workers' expectation of future high wages. Once a firm loses its reputation, it will become hard to keep and attract workers later. Because of this "reputation mechanism", firms with high-wage reputation want to maintain it, while firms without reputation will just pay the lowest wage allowed. The wage differential arises only due to workers' expectation differences, and requires no heterogeneity of firm or worker characteristics.

The implications of our strategic wage model are also consistent with other important phenomena. Although we do not include productivity differences, firms with high-wage reputations earn a rent, which is consistent with the association of high wages with high profitability, even after controlling for worker characteristics. (See Abowd et al. 1999.) We can also obtain the firm size – wage effect in Brown and Medoff (1989). If workers view large firm size as a signal of future high wages, then a high wage equilibrium is supported, i.e., large firms will maintain the reputation in equilibrium, even without any heterogeneity.

In the following, we give two models. In the first model, a firm hires a sequence of workers over the infinite horizon, and the actions are only the wage level for one period for the firm and the search decision for the workers. All workers have identical productivity and exogenous outside offer distribution. In this simple model we characterize the stationary wage equilibria. There are only two types of stationary equilibria, corresponding to the search - not search actions. The stationary search equilibrium is in fact unique, and there is a range of "high wages" that make no-search equilibria. In the second model we endogenize the outside offers in a full market equilibrium, and show that the same reputation mechanism works.

Note also that our first model contributes to the dynamic game literature by endogenizing the players' quit behavior. Although each period has the same structure, the workers in our game quit strategically, so the dynamic game is not an ordinary repeated game. It has a long-run player (the firm) and players who can choose the duration of stay in the game (workers) strategically, and the long-run player's action affects next period payoffs. This form of dynamic game with endogenous player changes has not been generally analyzed, and we show that multiple equilibria like the Folk Theorem hold.

2. ONE FIRM VS. SEQUENCE OF WORKERS MODEL

We begin with a "partial equilibrium" game where a firm has one worker in each period, but since the workers are free to quit, there can be a sequence of workers in the dynamic game as a whole. We assume that all workers are identical, and they observe all the past actions by the firm. Thus each worker can choose actions dependent on the firm's past actions, so the workers are not "one-shot" players as in Fudenberg et al. (1990). Neither are they "long-run" players because they can strategically quit and therefore cannot be punished forever. Hence this is not a repeated game in the usual sense and we cannot directly apply the usual Folk Theorem. Nevertheless, in this section we show that such a "long-run, short-run" player game has multiple equilibria with a very similar logic to the Folk Theorem. Although the dynamic game has a stationary structure, the firm's action has an effect on the payoff of workers in the next period (if the current employee does not quit), so we cannot separate a stage game each period like in Fujiwara-Greve (1998). Therefore the results there also do not apply directly.

MacLeod and Malcomson (1989) provided a Folk Theorem for a repeated contracting model, but with a more deterministic matching mechanism in the job market, and without on-the-job search. A notable difference between MacLeod and Malcomson (1989) and this paper is that we did not include effort choice. This implies that in our model workers have only quit option to punish the firm's deviation, and that the workers cannot be punished by the firm. With this weaker incentive structure to coordinate on a dynamic strategy combination, we will show that a similar result to theirs holds. Note also that it is easy to extend our model to include effort choice.

An important feature of our model is that the firm can commit for only one period in the future. This is a contrast to the most of the search literature (except Coles 1998) where the firms are assumed to be able to commit to the entire wage schedule of the future. It is more natural to think that the firms can commit only for some short period of time, and we can interpret the duration of a period in the game as the time that a firm can credibly commit.

Once the firm cannot commit to the entire future wage schedule, workers' expectation of the future wage becomes an important part of the strategic equilibrium. We employ the notion of subgame perfect equilibrium where after any history of wage offers, the firm's future wage offers must confirm workers' expectation.

2.1. *Game.* The players are one firm and a sequence of workers it hires in such a way that the firm has one worker each period, $t = 1, 2, \ldots$. The firm and all the workers have the infinite horizon with common discount factor $\delta \in (0, 1)$.

At the beginning of each employment relationship, the firm offers next period wage w, and pays also w as the initial payment. The offer is restricted by $w \geq \underline{w}$ where $\underline{w}$ is the exogenous minimum wage. The firm cannot commit more than one period. After observing the next period wage offer, the current employee can either search for an alternative income source with cost $s/(1-\delta)$ where $s > 0$ or not search with no cost. (The form of the search cost is to capture the idea that it is more costly to search as the worker becomes more patient.) The search is on-the-job search and the employee produces an income v to the firm for this period, regardless of the search decision. If the worker searches, he/she can draw an outside offer which is a constant[2] income stream $x, x, \ldots$ and x follows a known i.i.d. differentiable distribution function F over a compact interval $[\underline{w}, \hat{w}]$ with density f. The outside offers can be interpreted as a stationary equilibrium payoff stream in games with other employers etc. The firm has no incentive to pay more than $\mathrm{Min}\{\hat{w}, v\}$.

If the worker accepts an offer $x, x, \ldots$, he/she quits the firm and starts receiving x from the next period on. In this case the firm incurs turnover cost $Q > 0$ (at the beginning of the next period) to get a replacement and a new employment relationship starts for the firm. If the current worker did not quit, the firm must pay the promised wage at the beginning of the next period and then offers a new wage (valid for the period after, and not necessarily the same as before). If the firm got a new worker, it also offers a new wage w' for the period after and pays w' as the initial payment. The game continues this way.

The firm and all the workers maximize the total expected discounted payoff. We assume that the game is common knowledge and has the perfect monitoring of the firm's past actions. In the equilibria we show, the firm does not base the wage level on past worker actions, so it is in fact NOT important whether the firm observes/remembers workers' past actions. The firm controls the next period wage level to adjust the quit rate (and therefore its turnover cost) and the workers have only the search decision. We can include effort choice easily in the framework, but since we are interested in how multiple equilibria arise due to different expectations held by workers, it is better to keep the model simple.

To highlight the role of workers' expectations ("reputations" of the firm) regarding the firm's future wages in the equilibria, we characterize stationary subgame perfect equilibria. There are only two types of such equilibria, one where all the employed workers search every period and ones where the first employee never searches and thus stays with the firm forever. If the firm wants to prevent search, it should do so with the first employee since the future is less valued and all the workers are identical. In the constant-search equilibrium, workers believe that the firm will pay the minimum wage always, so they quit to low outside offers. Because of the high quit rate, the firm in fact pays the minimum wage to minimize the cost. In the no-search equilibria, the first employee believes that the firm will always pay a high wage and therefore does not search. If the firm offers a lower wage, the current and all the future workers believe that it is the minimum-wage firm in the future, so they will search and quit to a low offer. This "loss of high-wage expectation" serves as a punishment and thus the firm prefers to maintain the high wage.

It is possible to construct non-stationary subgame perfect equilibria with different expectation patterns, but we believe that the above two types of equilibria show the importance of the worker expectations enough. Also the stationary equilibria compare well with the steady-state equilibria in the traditional search literature such as Burdett and Mortensen (1998). They showed a unique (Nash) equilibrium assuming the same profit for the firms with different wages, while we show multiple equilibrium wage offers with different payoffs. In particular, there is a rent to the high-wage reputation, which we think is more close to the reality.

2.2. *Worker Behavior under Stationary Expectation.* Since the outside offers are compared in one dimension, the optimal search decision has the reservation level above which the worker accepts any offer and below which the worker rejects all. We compute the optimal reservation level when an employee has a stationary expectation that the firm will always offer the same wage w. In this case the worker's optimization problem is a stationary dynamic programming. Thus an optimal solution is a stationary[3] reservation level. Let the optimal value function for an employee who searches every period using a stationary reservation level $\overline{w}$, when the firm has just paid w, offered w for the next period, and is expected to pay the same stationary future wage $w, w, \ldots$ be $V^S(w^\infty, \overline{w})$. It has the following recursive structure.

$$V^S(w^\infty, \overline{w}) = w - \frac{s}{1-\delta} + \delta \int_{\overline{w}}^{\hat{w}} \frac{x}{1-\delta} f(x)dx + \delta F(\overline{w}) V^S(w^\infty, \overline{w}).$$

The first two terms are the current period wage and the search cost and the third term is the expected total future income when the worker accepts an offer not less than $\overline{w}$, and the last term is the expected continuation value when the realization of the offer was below the reservation level.

Hence

$$V^S(w^\infty, \overline{w}) = [w - \frac{s}{1-\delta} + \delta \int_{\overline{w}}^{\hat{w}} \frac{x}{1-\delta} f(x)dx]/[1 - \delta F(\overline{w})].$$

By differentiation it is straightforward to show that the first order condition is necessary and sufficient to obtain an optimal reservation level $\overline{w}_\delta(w^\infty)$ which is a solution to

$$\frac{\partial V^S}{\partial \overline{w}} = \frac{\delta f(\overline{w})}{1 - \delta F(\overline{w})} \left[- \frac{\overline{w}}{1-\delta} + V^S(w^\infty, \overline{w}) \right] = 0,$$

or

$$[1 - \delta F(\overline{w})]\overline{w} - \delta \int_{\overline{w}}^{\hat{w}} x f(x)dx = (1-\delta)w - s. \tag{1}$$

Denote the solution to (1) by $\overline{w}_\delta(w^\infty)$. The assumption (A1) below will guarantee $\underline{w} < \overline{w}_\delta(\underline{w}^\infty) < \hat{w}$ for large δ's. The left hand side of equation (1) is strictly increasing in $\overline{w}$ and thus the optimal reservation level $\overline{w}_\delta$ is strictly increasing in the stationary wage w and strictly decreasing in the search cost.

Next we show that there is a critical value of a stationary wage offer by the firm below which the workers prefer to search and above which they do not search. Suppose that the current period wage was u and the worker observed the next period offer w and also expects the stationary future wage offer $w, w, \ldots$. If the worker never searches, he/she gets $V^N(u, w^\infty) = u + (\delta w)/(1-\delta)$. To show this is optimal, the worker should not gain by deviating for one period. If he/she deviates and searches this period with appropriate reservation level $\tilde{w}$ and goes back to no-search strategy from next period on, he/she gets

$$D = u - \frac{s}{1-\delta} + \delta \int_{\tilde{w}}^{\hat{w}} \frac{x}{1-\delta} f(x)dx + \delta F(\tilde{w}) \frac{w}{1-\delta}.$$

By differentiation we obtain the optimal reservation level for deviation $\tilde{w} = w$. Substituting this, we get

$$V^N(u, w^\infty) \geq D \iff s \geq \delta \int_{w}^{\hat{w}} (x - w) f(x)dx.$$

This implies (by dividing by $1-\delta$) that when the search cost is not less than the expected future gain from the search, the worker prefers not to search. To make such case possible, we assume that

$$(A1) \qquad \int_{\underline{w}}^{\hat{w}} (x-\underline{w})f(x)dx > s.$$

For each $\delta \in (0,1)$ define w_δ^* implicitly by

$$(2) \qquad s = \delta \int_{w_\delta^*}^{\hat{w}} (x-w_\delta^*)f(x)dx.$$

By assumption (A1), $w_\delta^* > \underline{w}$ and since $s > 0$ and $\int_{\hat{w}}^{\hat{w}}(x-\hat{w})f(x)dx = 0$, we have that $w_\delta^* < \hat{w}$. From the above analysis, we have shown that if the stationary offer was $w \geq w_\delta^*$, the workers do not deviate from no search strategy. Note also that for sufficiently large[4] δ, (A1) implies that

$$\begin{aligned} & s < \delta \int_{\underline{w}}^{\hat{w}} (x-\underline{w})f(x)dx \\ \iff & (1-\delta)\underline{w} - s > [1-\delta F(\underline{w})]\underline{w} - \delta \int_{\underline{w}}^{\hat{w}} xf(x)dx. \end{aligned}$$

that is, the optimal reservation level under the constant minimum wage offer is $\overline{w}_\delta(\underline{w}^\infty) > \underline{w}$.

Suppose that the firm's stationary offer was $w < w_\delta^*$. We will show that in this case the workers want to search every period. Let the current period wage be u (recall that the level of u was irrelevant in the analysis of one period deviation) and suppose that the firm offered w for the next period and is expected to continue offering w. The total expected value of the search with a reservation level $\tilde{w}$ for this period and $\overline{w}_\delta(w^\infty)$ for the future is

$$V^S(u, w^\infty, \overline{w}_\delta(w^\infty)) = u - \frac{s}{1-\delta} + \delta \int_{\tilde{w}}^{\hat{w}} \frac{x}{1-\delta} f(x)dx + \delta F(\tilde{w}) \frac{\overline{w}_\delta(w^\infty)}{1-\delta}.$$

The optimal $\tilde{w}$ is in fact $\tilde{w} = \overline{w}_\delta(w^\infty)$. By substitution, the value for the optimal search is $\frac{\overline{w}_\delta(w^\infty)}{1-\delta} + (u-w)$. If the worker does not search one period and then goes back to the optimal search strategy, the value is $D = u + \delta\frac{\overline{w}_\delta(w^\infty)}{1-\delta}$. Hence the worker does not deviate if $\overline{w}_\delta(w^\infty) \geq w$ which is equivalent to the case where the left hand side of (1) at $\overline{w} = w$ is not more than the right hand side, i.e.,

$$[1-\delta F(w)]w - \delta \int_{w}^{\hat{w}} xf(x)dx \leq (1-\delta)w - s \iff s \leq \delta \int_{w}^{\hat{w}} (x-w)f(x)dx.$$

Since the expected gain from search $\int_w^{\hat{w}}(x-w)f(x)dx$ is strictly decreasing in w, this is equivalent to $w \leq w^*_\delta$. In sum, if the stationary offer was less than w^*_δ, the workers always search with the reservation level $\overline{w}_\delta(w^\infty)$.

Finally we show a useful remark.

Remark 1. *For sufficiently large δ, $\overline{w}_\delta(\underline{w}^\infty) < w^*_\delta$. (Hence $\overline{w}_\delta(\underline{w}^\infty) < \hat{w}$.)*

Proof. When the stationary offer is $\underline{w}$, equation (1) is equivalent to

$$s + (1-\delta)(\overline{w}_\delta(\underline{w}^\infty) - \underline{w}) = \delta \int_{\overline{w}_\delta(\underline{w}^\infty)}^{\hat{w}} (x - \overline{w}_\delta(\underline{w}^\infty))f(x)dx.$$

We have already shown that for sufficiently large δ, the reservation level $\overline{w}_\delta(\underline{w}^\infty)$ is strictly bigger than $\underline{w}$. Thus the left hand side is bigger than $s = \delta\int_{w^*_\delta}^{\hat{w}}(x - w^*_\delta)f(x)dx$ so that $\overline{w}_\delta(\underline{w}^\infty) < w^*_\delta$. Q.E.D.

2.3. *Unique Stationary Search Equilibrium.* We show that among "naive" stationary expectations such that if a worker observed an offer w from the firm, he/she expects the same offer forever after, the only expectation that can be confirmed by the firm (i.e., the offer that the firm would actually like to stick to) is $\underline{w}, \underline{w}, \ldots$.

We again show that the firm cannot gain by deviating one period. If the firm takes a stationary wage strategy $w, w, \ldots$, where $w < w^*_\delta$, the workers always search with the reservation level $\overline{w}_\delta(w^\infty)$. The value of this strategy for the firm has the following recursive structure.

$$W^S(w^\infty) = v - w + \delta F(\overline{w}_\delta(w^\infty))W^S(w^\infty) + \delta[1 - F(\overline{w}_\delta(w^\infty))][W^S(w^\infty) - Q],$$

where the first two terms make this period net payoff and the third term is the continuation value when the worker did not quit and the last term is the continuation value when the firm lost a worker. In short,

$$W^S(w^\infty) = \frac{1}{1-\delta}[v - w - \delta[1 - F(\overline{w}_\delta(w^\infty))]Q].$$

When the workers have naive stationary expectations, the firm can alter the stationary offer and the worker expectation at any time. So the firm should choose w that maximizes $W^S(w^\infty)$. By differentiation,

$$\frac{\partial W^S}{\partial w} = \frac{1}{1-\delta}[-1 + \delta f(\overline{w}_\delta(w^\infty))Q\frac{\partial \overline{w}_\delta}{\partial w}] = -\frac{1}{1-\delta} + \frac{\delta f(\overline{w}_\delta(w^\infty))Q}{1-\delta F(\overline{w}_\delta(w^\infty))}.$$

Hence for sufficiently large δ, the derivative is negative and the unique optimal stationary offer is $\underline{w}, \underline{w}, \ldots$. Since $\underline{w} < w^*_\delta$, this makes a consistent belief of workers.

Proposition 1. *For sufficiently large δ, the following strategy combination is a subgame perfect equilibrium.*

Firm: Offer $\underline{w}$ after any history of past offers.

Workers: When the current offer is w, search with reservation level $\overline{w}_\delta(\underline{w}^\infty) + (1-\delta)(w - \underline{w})$.

Note that this equilibrium is the unique stationary search equilibrium in the sense that there is no other stationary wage offer that induces constant search and from which the firm does not deviate.

Proof. In a subgame where a worker received u, observed the next period offer w, and expects that the firm will offer $\underline{w}$ forever after, the value of search with reservation level $\tilde{w}$ is

$$\begin{aligned} V^S =& u - \frac{s}{1-\delta} + \delta \int_{\tilde{w}}^{\hat{w}} \frac{x}{1-\delta} f(x)dx \\ &+ \delta F(\tilde{w})[w - \frac{s}{1-\delta} + \delta \int_{\overline{w}_\delta(\underline{w}^\infty)}^{\hat{w}} \frac{x}{1-\delta} f(x)dx + \delta F(\overline{w}_\delta(\underline{w}^\infty))\frac{\overline{w}_\delta(\underline{w}^\infty)}{1-\delta}], \\ =& u - \frac{s}{1-\delta} + \delta \int_{\tilde{w}}^{\hat{w}} \frac{x}{1-\delta} f(x)dx + \delta F(\tilde{w})[\frac{\overline{w}_\delta(\underline{w}^\infty)}{1-\delta} + (w - \underline{w})]. \end{aligned}$$

By differentiation, the optimal reservation level for this period is $\overline{w}_\delta(\underline{w}^\infty) + (1-\delta)(w - \underline{w})$. One period deviation to no-search gives

$$D = u + \delta[\frac{\overline{w}_\delta(\underline{w}^\infty)}{1-\delta} + (w - \underline{w})].$$

For sufficiently large δ, the reservation level $\tilde{w} = \overline{w}_\delta(\underline{w}^\infty) + (1-\delta)(w - \underline{w})$ is close to $\overline{w}_\delta(\underline{w}^\infty)$ which is less than w^*_δ as shown in Remark 1. Since $\int_w^{\hat{w}}(x-w)f(x)dx$ is strictly decreasing in w,

$$s = \delta \int_{w^*_\delta}^{\hat{w}} (x - w^*_\delta) f(x)dx < \delta \int_{\tilde{w}}^{\hat{w}} (x - \tilde{w}) f(x)dx,$$

which is equivalent to $V^S - D > 0$. Thus, when the workers are sufficiently patient and when they expect that the firm is the minimum wage firm, they always search.

Next we show that the firm does not deviate to other wage level for one period. Suppose that it has paid u and is supposed to offer $\underline{w}$ forever after. One period deviation to other offer y this period (and then offering $\underline{w}$ forever) gives a different reservation level $\tilde{w}(y) = \overline{w}_\delta(\underline{w}^\infty) + (1-\delta)(y - \underline{w})$ so that the value is

$$\begin{aligned} D(y) &= v - u + \delta F(\tilde{w}(y))[W^S(\underline{w}^\infty) - (y - \underline{w})] \\ &\quad + \delta[1 - F(\tilde{w}(y))][W^S(\underline{w}^\infty) - (y - \underline{w}) - Q] \\ &= v - u + \delta[W^S(\underline{w}^\infty) - (y - \underline{w})] - \delta[1 - F(\tilde{w}(y))]Q. \end{aligned}$$

By differentiation, the optimal deviation is in fact $y = \underline{w}$ for sufficiently large δ. Thus neither the firm nor workers want to deviate from the above strategy in any subgame. Q.E.D.

We interpret this equilibrium as a stable situation where the firm is considered to be the minimum wage payer and because of this expectation it cannot attract the employee to stay with high probability, even if it raises wage for one period. Therefore the firm would rather pay only the minimum wage.

2.4. *Stationary No-Search Equilibria.* We now show that with different expectation adjustment than the "naive" stationary ones considered in 2.3, the game has more equilibria, and in particular, any stationary wage not less than w^*_δ can be an equilibrium offer.

Proposition 2. *For sufficiently large δ, any $w^* \in [w^*_\delta, \mathrm{Min}\{v, \hat{w}\}]$ and any $Q \geq \frac{w^* - \underline{w}}{\delta[1 - F(\overline{w}_\delta(\underline{w}^\infty))]}$, the following strategy combination is a subgame perfect equilibrium.*

Firm: Start the game by offering w^*. If all the past offers were at least w^*, then offer w^*. Otherwise offer $\underline{w}$.

Workers: If all the past offers were at least w^* including the current one, do not search. Otherwise search with reservation level $\overline{w}_\delta(\underline{w}^\infty) + (1-\delta)(w - \underline{w})$, where w is the current offer.

In these equilibria, it is important whether the firm maintained the "high wage" w^* throughout the past. In that case the worker considers it as a high-wage firm and does not search. Once the firm signals otherwise, all the current and the future workers think it has become a low-wage firm that permits turnover. Among low wages that permit turnover, the minimum wage was the only incentive compatible stationary offer, so the workers should expect that in a stationary equilibrium, the firm has become the minimum-wage firm.

Proof. We only need to show that as long as the firm has maintained at least w^* (including the first period), the firm or a worker does not deviate.

Suppose that the firm has paid u to the employee and offered $w \geq w^*$ for the next period. If the worker does not search, the value is $V^N = u + \delta w + (\delta^2 w^*)/(1-\delta)$ under the above strategy. If he/she searches this period with a reservation level $\tilde{w}$, the value is

$$D = u - \frac{s}{1-\delta} + \delta \int_{\tilde{w}}^{\hat{w}} \frac{x}{1-\delta} f(x)dx + \delta F(\tilde{w})[w + \frac{\delta w^*}{1-\delta}].$$

The optimal reservation level is $\tilde{w} = (1-\delta)w + \delta w^* \geq w_\delta^*$. The definition of w_δ^* (2) implies that

$$s = \delta \int_{w_\delta^*}^{\hat{w}} (x - w_\delta^*) f(x)dx \geq \delta \int_{\tilde{w}}^{\hat{w}} (x - \tilde{w}) f(x)dx.$$

This is equivalent to $V^N \geq D$, hence a worker does not deviate.

If the firm offers $w > w^*$ after a history where it has maintained at least w^*, the current employee's behavior does not change but it has to pay more than w^*. Hence the firm does not deviate to a higher wage. If the firm deviates to a lower wage $w < w^*$, because of the worker expectation and thanks to Proposition 1, we know that the optimal deviation is to offer $\underline{w}$ forever after. So (in any subgame where the firm maintained at least w^*) the firm compares $W^N(w^{*\infty}) = (v - w^*)/(1-\delta)$ with $W^S(\underline{w}^\infty) = \{v - \underline{w} - \delta[1 - F(\overline{w}_\delta(\underline{w}^\infty))]Q\}/(1-\delta)$. The high wage strategy is better if

$$W^N(w^{*\infty}) \geq W^S(\underline{w}^\infty) \iff Q \geq \frac{w^* - \underline{w}}{\delta[1 - F(\overline{w}_\delta(\underline{w}^\infty))]}.$$

Q.E.D.

When δ and Q is sufficiently large as required in the two propositions, both search and no-search equilibria exist. As the turnover cost Q increases, more and more no-search equilibria are supported, as shown in Figure 1. All the no-search equilibria are efficient for the firm-worker pair, since search is costly and all the workers are identical.

=== Insert Figure 1 about here.===

When Q is large such that $Q \geq [\text{Min}\{v, \hat{w}\} - \underline{w}]/[\delta[1 - F(\overline{w}_\delta(\underline{w}^\infty))]]$, the firm may pay up to $\text{Min}\{v, \hat{w}\}$, which can be the "competitive price" v. But when $v > \hat{w}$, all equilibrium offers show a monopsony.

2.5. *Discussion.* We want to emphasize two things. First, the two types of stationary equilibria are derived only based on the difference in how expectation changes after each action, and among no-search equilibria, the only difference is what is considered to be a "high wage". We did not need asymmetric information or asymmetric characteristics of the players, such as labor quality v, firm's turnover cost Q and so on, to derive different equilibrium wages.

Second, the high-wage reputation gives a "rent" to the firm when the turnover cost is not too low. As we showed at the end of the proof of Proposition 2, the firm's equilibrium payoff in a high-wage equilibrium $W^N(w^{*\infty})$ is larger than the one in the minimum-wage equilibrium $W^S(\underline{w}^\infty)$. It is well established in empirical literature that high wages are associated with high profitability, (see for example Abowd et al. 1999) so this is a realistic implication, compared to the usual *assumption* of equal profit over wage dispersion in macro literature.[5] As Remark 1 shows, the equilibrium payoff for workers in a high-wage equilibrium is not less than $w^*_\delta/(1-\delta)$, which is bigger than the one in the minimum-wage equilibrium $\overline{w}_\delta(\underline{w}^\infty)/(1-\delta)$. So workers also prefer any high-wage no-search equilibrium. This is also a natural conclusion.

Finally, we note that if we interpret the firm as a job, jobs with high-wage reputation will not be left while jobs with low-wage reputation can be left often. Thus firms with high-wage jobs will have more employees, which is consistent with another famous empirical finding that larger firms have higher wages after controlling for labor and employer characteristics. (See for example Brown and Medoff 1989.)

3. MARKET MODEL

It is also desirable to have multiple firms in the model, to make a full market equilibrium and endogenize the outside offer distribution. In this section we make a similar game with a continuum of firms and workers where some firms can be vacant and some workers can be unemployed due to random matching in the job market. The main statement of this section is that there is a stationary (Nash) equilibrium where identical firms pay different wages and identical workers can end up in different wage firms. The equilibrium payoffs exhibit rents to high-wage firms and workers as before. We do not try to construct a subgame perfect equilibrium. The main difficulty is that a subgame perfect equilibrium has to form a Nash equilibrium after any positive measure of firms deviated, and since the stationary proportion of different state (vacant or filled) firms depends on the proportion of high-wage firms (see Section 3.2), it is too much to assume the proportion of firms jumps to another stationary proportion in such subgames. Once we drop stationarity, the analysis becomes very complicated and we think that is also too much to pursue here. Nevertheless, our Nash equilibrium has a flavor of perfection, that is, after any unitary deviation, each player's continuation strategy is sequentially rational.

We discuss the differences between our model and results and similar search models at the end of the section.

3.1. *Game.* The players are a continuum $[0,1]$ of firms and a continuum $[0,1]$ of workers. All firms and workers are identical respectively. We construct a game where a firm can commit to only one-period wage level and if it is vacant, can enter the job market to search for a worker (but it cannot enter the job market if it has an employee), while workers can enter the job market at any time even while employed. Each firm can hire up to one worker. (Alternatively, a firm can be viewed as a job, as before.) Workers make decisions only regarding job changes, and there is no effort choice, to keep the model simple as in the previous section.

In each period, a firm is either vacant or filled. A worker is either employed or unemployed. A firm with a worker offers end-period wage level $w \geq \underline{w} > 0$ to be paid if the employee does not quit during the period. After observing the current employer's wage, an employee can decide whether to go to the job market with cost $s > 0$ or not to search with no cost. If the current employee went to the job market and received and accepted another offer, the firm becomes vacant for this period (and the payoff is normalized to be zero for this period) and starts next period as a vacant firm. If the employee stayed (regardless of the search decision), he/she produces a value $v > \underline{w}$ to the firm for this period and the firm must pay the promised wage w. At the end of the period, each worker (employed or unemployed) has probability $p < 1$ to survive for the next period. With probability $1 - p$, a worker dies and a new person enters the worker population immediately as an unemployed worker. Thus even if the firm retained the current employee, if he/she dies, the firm becomes vacant at the beginning of the next period.

If a firm starts as a vacant firm, it can either stay vacant with no cost or search with cost $S > 0$ and post a wage offer $w' \geq \underline{w}$. After vacant firms and all workers made search decisions, a random matching process makes one-to-one matches of the searching firms and workers. The matching probability is endogenous and determined by the relative measure of searching firms and workers. If measure m of firms and measure $n > m$ (say) of workers are in the job market, then all the searching firms get a match with a worker, but $n - m$ of workers go unmatched, and other cases are similar.

If a vacant firm got a match, the matched worker can either take the posted offer w' or not. If the worker accepted the offer, then the firm becomes filled and receives the income v from the production and must pay the promised wage w' for that period. Otherwise it receives zero payoff for this period. At the end of the period, the employee (if any) has probability $1 - p$ of dying as above. We assume that unemployed workers who did not find a firm get payoff zero for that period.

Although all workers produce the same income v to any firm, we introduce one asymmetry. If a worker is unemployed, getting employed does not cost anything (except that he/she has to search with cost s), but employed workers must incur relocation cost $r > 0$ in addition to the search cost, upon moving to another firm. This is justified by human capital theory that if a worker changes employers, some firm-specific human capital is lost. From firms' point of view, a firm with the same wage offer as the current employer cannot attract an employed worker.

In this game we did not use a direct turnover cost parameter, but the search cost and the possibility of not getting a replacement in the job market serves as an indirect turnover cost for the firms. All firms maximize the total discounted (with factor $\gamma \in (0, 1)$) expected payoff over the infinite horizon, while all workers maximize the total expected payoff with random death probability $1 - p$, which is like discounting.

In the rest of this section we will construct a stationary Nash equilibrium where $q \in (0, 1)$ of the firms offer a "high wage" $w^* > \underline{w}$ and $1 - q$ of the firms offer the minimum wage $\underline{w}$ every period. The labels "high-wage firm" and "low-wage firm" are initially attached to some q and $1 - q$ firms respectively,[6] but afterwards if a high-wage firm deviates to a wage less than w^*, it will be considered as a low-wage firm by all the workers. This is the reputation effect.

We will construct an equilibrium with the following property. Unemployed workers and employees at the low-wage firms always search, while employees at the high-wage firms do not. All the vacant firms search in the job market. Unemployed workers accept any offer, but employees at the low-wage firms accept only offers from the high-wage firms. First, we show that such stationary movement of players is possible.

3.2. *Stationary Proportions.* At the beginning of each period, the states for workers are **U**nemployed, employed at a **L**ow wage firm, or employed at a **H**igh wage firm. Let α_U, α_L, and α_H be the stationary proportions of these three states, then $\alpha_U + \alpha_L + \alpha_H = 1$. Since workers at high-wage firms do not search, the measure of high-wage firms with employee must equal to α_H. Note also that vacant high-wage firms will search and attract a worker for sure, so the measure of vacant high-wage firms $q - \alpha_H$ must equal to the measure of those who suffered the death of the employees. Thus,

$$q - \alpha_H = (1 - p)q. \tag{3}$$

This implies that $\alpha_H = pq$.

The remaining $1-q$ firms split into vacant low-wage firms and filled ones. Let β be the measure of filled low-wage firms. The measure of firms searching for workers is $(1-p)q+1-q-\beta = 1-pq-\beta$. On the other hand, the measure of workers who are not currently employed by a high-wage firm (that is, the measure of searching workers) is $1-pq$. Thus all searching firms will get a match, while the probability for a searching worker to find any offer is $(1 - pq - \beta)/(1 - pq)$ and the probability to find an offer from a high-wage firm is $(1 - p)q/(1 - pq)$.

In a steady state we have

$$\alpha_U = (1 - p) + p\alpha_U \big(1 - \frac{1 - pq - \beta}{1 - pq}\big), \tag{4}$$

which means that the steady-state proportion of the unemployed workers equals the measure of the newly born workers plus the measure of surviving p workers of the previously unemployed ones who failed to find a firm in the job market.

Rearranging (4) and using the fact that $\alpha_L = \beta = 1 - pq - \alpha_U$, we obtain

$$p\beta^2 - (1 - pq)(1 + p)\beta + p(1 - q)(1 - pq) = 0. \tag{5}$$

The left hand side of (5) is positive when $\beta = 0$ and $-(1-q)(1-p) < 0$ when $\beta = 1-q$. Thus (5) has a solution $\beta \in (0, 1-q)$. From now on use β as this steady state proportion of filled low-wage firms. Note that β only depends on the survival rate p and the proportion of high-wage firms q.

To summarize, at the beginning of each period, there are pq firms that pay high wage and have employee, $(1-p)q$ firms that pay high wage but had lost employee, β firms that pay low wage and have employee, and $1-q-\beta$ firms that pay low wage and are vacant. On the other hand, there are pq workers who are employed by some high-wage firms so do not search for this period, β workers who are employed by some low-wage firms, and $1-pq-\beta$ workers who are unemployed. Even though there are the same measure of firms and workers, because of the on-the-job search, involunatry unemployment results.

3.3. *Workers.* We construct conditions that must hold to generate a stationary equilibrium with two-tier wage distribution in the economy $\{\underline{w}, w^*\}$ every period. For the reason stated at the beginning we only check deviations by a single player and subgames that has at most one player deviation. The subgames with a unitary deviation are not necessary for a Nash equilibrium, but it gives sequential rationality for those cases.

The strategies in the relevant subgames are as follows. A firm with "high wage" reputation offers $w^* > \underline{w}$ as long as it has offered at least w^* in the past, and once it offered a lower wage (off-equilibrium path) it becomes a "low wage" firm and offers $\underline{w}$ forever after. The firms with "low wage" reputation offer the minimum wage after any history. On the equilibrium path, no "high wage" firm will deviate so there are always the same q firms with high-wage reputation.

Given these strategies by firms, it will be shown that the following reactions are best responses for the workers.

(U) Unemployed workers always search and take any offer.

(L) An employee at a low-wage firm with the current employer's offer being $w \geq \underline{w}$ (which includes the possibility that the firm deviated) searches if and only if $w < R$, and once searched, takes offers not less than $w + r$ if the offer was from a low-wage firm, and takes an offer from a high-wage firm if it is not less than $w + r + pV^S(\underline{w}) - \frac{pw^*}{1-p}$. ($R$ and $V^S(\underline{w})$ will be defined below.) Under the equilibrium wage distribution $\{\underline{w}, w^*\}$, this implies that this state workers always search and accept offers only from high-wage firms.

(H) An employee at a high-wage firm with the current offer $w \geq w^*$. He/she expects that the employers will offer w^* forever after, so does not search. But in the subgames where he/she searched (off-equilibrium path), he/she does not accept any offer.[7] Once the current employer offered less than w^*, then he/she behaves as in (L).

First, we show the equilibrium value of the worker strategies described above, given that no firm deviates.

An employee at a high-wage firm offering w^* forever receives (under (H)) $w^*/(1-p)$. An employee at a low-wage firm offering $\underline{w}$ forever after can become a high-wage firm's employee with probability $(1-p)q/(1-pq)$, and otherwise stay, so the value is (under (L))

$$\begin{aligned} V^S(\underline{w}) &= -s + \frac{(1-p)q}{1-pq}[w^* - r + \frac{pw^*}{1-p}] + (1 - \frac{(1-p)q}{1-pq})[\underline{w} + pV^S(\underline{w})], \\ &= \frac{1-pq}{1-p}[-s + \frac{(1-p)q}{1-pq}(\frac{w^*}{1-p} - r) + \frac{1-q}{1-pq}\underline{w}] \end{aligned}$$

Remark 2. *When $w^* > \underline{w}$, the equilibrium value for an employee at a high-wage firm is larger than the one for an employee at a low-wage firm, i.e., $w^*/(1-p) > V^S(\underline{w})$.*

Proof. By computation,

$$\frac{w^*}{1-p} - V^S(\underline{w}) = [(1-q)(w^* - \underline{w}) + q(1-p)r + (1-pq)s]/(1-p) > 0.$$

Q.E.D.

An unemployed worker's strategy (U) gives the value as follows.

$$\begin{aligned} V^U &= -s + \frac{(1-p)q}{1-pq}\frac{w^*}{1-p} + \frac{1-q-\beta}{1-pq}[\underline{w} + pV^S(\underline{w})] + \frac{\beta}{1-pq}[0 + pV^U], \\ &= \frac{1}{1-pq-p\beta}\Big[-s\frac{(1-pq)(1-p(q+\beta))}{1-p} \\ &\quad + \frac{q(1-p(q+\beta))}{1-p}w^* - pq(1-q-\beta)r + \frac{(1-pq)(1-q-\beta)}{1-p}\underline{w}\Big]. \end{aligned}$$

Note that an unemployed worker does not incur relocation cost.

Remark 3. *Assume that the relocation cost is not too high so that*

$$\beta\underline{w} - q(1-p)r \geq 0. \tag{A2}$$

Then, the equilibrium value for an employee at a low-wage firm is not less than the one for an unemployed, i.e., $V^S(\underline{w}) \geq V^U$.

Proof. By computation,

$$V^S(\underline{w}) - V^U = [\beta\underline{w} - q(1-p)r]/[1 - p(\beta + q)].$$

Q.E.D.

Next, we construct parameter conditions to warrant no one-period deviation is beneficial for each state of workers. Take a worker employed by a high-wage firm at the beginning of a period who observed an offer $w \geq w^*$ from the employer. The worker expects that the employer will offer w^* forever after. For backward solving, consider that the worker has sunk the search cost and received another offer w' in the job market (which is off-equilibrium path). If the offer is from another high-wage firm and $w' \geq w^*$, taking this offer gives $w' - r + pw^*/(1-p)$ in total, while rejecting gives $w + pw^*/(1-p)$. Thus moving to another high-wage firm is better if and only if $w' \geq w + r$. Hence a high-wage firm's employee does not take another high-wage firm's offer (even if he/she searched) if no firm has deviated, because $w = w^*$ and $w' = w^*$ in that case. If the worker met a low-wage firm in the job market with offer $w' \geq \underline{w}$ (or previously high-wage firm which offered $w' < w^*$), he/she expects that this firm will only offer $\underline{w}$ forever after, so taking this offer gives $w' - r + pV^S(\underline{w})$. Therefore moving is better if and only if $w' \geq r + w + pw^*/(1-p) - pV^S(\underline{w})$. Note again that the worker does not take a low-wage firm's offer in the job market if it is $\underline{w}$. Given this, it is not worth searching with positive cost. Thus:

REMARK 4. *If the firms follow the stationary strategies, it is optimal for an employee at a high-wage firm not to search.*

Take an employee at a low-wage firm (a firm with the past or current offer below w^*) who observed $w \geq \underline{w}$ for this period. The worker expects that the employer will offer $\underline{w}$ forever after. Once the search cost is sunk, and the worker received an offer from a high-wage firm $w' \geq w^*$, he/she compares the value of accepting this offer $w' - r + pw^*/(1-p)$ and the value of rejection $w + pV^S(\underline{w})$. Thus he/she should take offer w' if and only if $w' \geq w + r - p[w^*/(1-p) - V^S(\underline{w})]$. If he/she received an offer from another low-wage firm, the future wages are expected to be the same, so he/she should move if and only if the offer covers the relocation cost $w' \geq w + r$, which does not occur if the low-wage firm offers only $\underline{w}$. The following condition (A3) guarantees that if the current employer offers $\underline{w}$ (on the equilibrium path) and the job market offer by a high-wage firm was w^*, the worker changes the firms as in (L).

REMARK 5. *Assume that*

$$w^* - \underline{w} \geq (1-p)r + (1-pq)s/q. \tag{A3}$$

If the firms follow the stationary strategies, it is optimal for an employee at a low-wage firm to follow strategy (L), where

$$R := -\frac{(1-pq)^2}{q(1-p)}s + (1-pq)[\frac{w^*}{1-p} - r] - \frac{(1-q)p}{1-p}\underline{w}.$$

RROOF. By computation,

$$w^* \geq \underline{w} + r - p[w^*/(1-p) - V^S(\underline{w})] \iff w^* - \underline{w} \geq (1-p)r - ps,$$

which is satisfied under (A3). So the equilibrium offer w^* by a high-wage firm will be accepted in the job market. Now go back to the search decision, when the current employer's offer was $w \geq \underline{w}$. If the employee searches and accepts only a high-wage firm's offer in the job market, the value is

$$V^S(w) = -s + \frac{(1-p)q}{1-pq}[w^* - r + \frac{pw^*}{1-p}] + \frac{1-q}{1-pq}[w + pV^S(\underline{w})].$$

Note that we are allowing the firm's deviation w. If the worker does not search for one period, the value is $D = w + pV^S(\underline{w})$. So search for this period is better if and only if

$$V^S(w) \geq D \iff w \leq -\frac{(1-pq)^2}{q(1-p)}s + (1-pq)[\frac{w^*}{1-p} - r] - \frac{(1-q)p}{1-p}\underline{w} =: R.$$

(A3) is equivalent to $R \geq \underline{w}$, so on the equilibrium path when the low-wage firm's offer is $\underline{w}$, it is optimal to search. Q.E.D.

Finally, consider that an unemployed worker has sunk the search cost. If he/she gets an offer from a high-wage firm $w' \geq w^*$, taking this offer gives $w' + pw^*/(1-p)$ given the stationary expectation. If he/she receives an offer w' from a low-wage firm, the expected value is $w' + pV^S(\underline{w})$. Rejection gives $0 + pV^U$. (A2) implies that $\underline{w} > 0$ as well as $V^S(\underline{w}) \geq V^U$, so any offer from a low-wage firm is better than rejection. By Remark 2, an offer from a high-wage firm (which is at least $w^*/(1-p)$) is better than $\underline{w} + pV^S(\underline{w})$, so it is also better than rejection. Thus an unemployed worker accepts any offer. Going back to the search decision, the value of searching and accepting any offer gives (in equilibrium) V^U, while one period deviation to not searching gives $0 + pV^U$. (A2) and (A3) imply that $V^U \geq qr/\beta > 0$, hence an unemployed worker always searches.

Remark 6. *Assume (A2)-(A3). If the firms follow the stationary strategies, it is optimal for an unemployed worker to follow strategy (U).*

3.4. *Firms.* The equilibrium values for a firm with high-wage (resp. low-wage) reputation are computed as follows. Let the value for a high-wage firm with an employee (resp. without an employee) be $W_H(w^*)$ (resp. $W'_H(w^*)$). For a firm with high-wage reputation, it loses a worker only to death and when it searches it can attract a worker for sure. Thus these values satisfy

$$W_H(w^*) = v - w^* + \gamma[pW_H(w^*) + (1-p)W'_H(w^*)],$$

$$W'_H(w^*) = -S + v - w^* + \gamma[pW_H(w^*) + (1-p)W'_H(w^*)].$$

Hence $W'_H(w^*) = W_H(w^*) - S$ so that

$$W_H(w^*) = [v - w^* - \gamma(1-p)S]/(1-\gamma).$$

Let the value for a low-wage firm with an employee (resp. without an employee) be $W_L(\underline{w})$ (resp. $W'_L(\underline{w})$). A low-wage firm with an employee may lose him/her to a high-wage firm (with probability $(1-p)q/(1-pq)$), or to death after the production. A vacant low-wage firm can only attract an unemployed worker (with meeting probability $(1-pq-\beta)/(1-pq)$).

$$\begin{aligned} W_L(\underline{w}) &= \frac{(1-p)q}{1-pq}[0 + \gamma W'_L(\underline{w})] + \frac{1-q}{1-pq}[v - \underline{w} + \gamma p W_L(\underline{w}) + \gamma(1-p)W'_L(\underline{w})], \\ W'_L(\underline{w}) &= -S + \frac{1-pq-\beta}{1-pq}[v - \underline{w} + \gamma p W_L(\underline{w}) + \gamma(1-p)W'_L(\underline{w})] + \frac{\beta}{1-pq}[0 + \gamma W'_L(\underline{w})]. \end{aligned}$$

Let $x := \beta\gamma + q[1 - \gamma(1-p)]$ and $a := 1 - p(q + \gamma - \gamma q) > 0$. Since $\beta < 1 - q$, we have that $0 < x < 1$. By solving the system of simultaneous equations above,

$$\begin{aligned} W_L(\underline{w}) &= \frac{1}{(1-\gamma)(1-px)}[-\gamma(1-p)S + (1-x)(v - \underline{w})], \\ W'_L(\underline{w}) &= \frac{1}{(1-\gamma)(1-px)}[-aS + (1-pq-\beta)(v - \underline{w})]. \end{aligned}$$

Let us consider possible one-period deviations by a low-wage firm with an employee. If it deviates to a higher offer, there is a critical value R above which it can prevent the current employee from searching for this period. So, among $w < R$, the optimal offer is $\underline{w}$ and among $w \geq R$, the optimal offer is R. We only need to find a condition that the low-wage firm does not prefer offering R. The value of this deviation is

$$D = v - R + \gamma[pW_L(\underline{w}) + (1-p)W'_L(\underline{w})].$$

Thus offering $\underline{w}$ is better if and only if $W_L(\underline{w}) - D \geq 0$ i.e., the mark-up R is large enough

$$(A4) \qquad R - \underline{w} \geq \frac{(1-p)q}{1-px}[p\gamma S + (v - \underline{w})].$$

Note that (A3) was equivalent to $R \geq \underline{w}$, so (A4) implies (A3).

When a low-wage firm is vacant, it can either post an offer $\underline{w}$ to attract only unemployed workers, or post $\underline{w} + r$ to attract a worker for sure. (Other offers are not optimal.) Posting $\underline{w}$ is better if the mark-up r is not too small;

$$\frac{1-pq-\beta}{1-pq}[v - \underline{w} + \gamma p W_L(\underline{w}) + \gamma(1-p)W_L'(\underline{w})] + \frac{\beta}{1-pq}[0 + \gamma W_L'(\underline{w})]$$

$$\geq v - \underline{w} - r + \gamma p W_L(\underline{w}) + \gamma(1-p)W_L'(\underline{w})$$

$$(A5) \qquad \iff \frac{(1-px)r}{\beta} \geq v - \underline{w} + \gamma p S.$$

Remark 7. *Assume (A4)-(A5). Then a low-wage firm with or without an employee does not deviate from the stationary offer* $\underline{w}$.

Consider a high-wage firm (with past offers only w^* or more) with an employee. If it offers not less than w^*, it can maintain the reputation (among such offers w^* is optimal) and thus the employee does not search. If it offers less than w^*, by the above analysis the optimal offer is $\underline{w}$ under (A4). Hence maintaining the reputation is better if $W_H(w^*) \geq W_L(\underline{w})$.

A vacant high-wage firm (with past offers not less than w^*) can either offer w^* to attract any worker or to offer $\underline{w}$ to attract only unemployed ones. (Once it offers less than w^*, (A5) implies that $\underline{w}$ is optimal.) Offering w^* is better if $W_H'(w^*) \geq W_L'(\underline{w})$.

By computation,

$$W_L(\underline{w}) - W_L'(\underline{w}) \geq S \iff \beta \geq q(1-p).$$

So, assume[8] that $\beta \geq q(1-p)$. Then $W_H(w^*) \geq W_L(\underline{w})$ implies that $W_H'(w^*) \geq W_L'(\underline{w})$ as well. $W_H(w^*) \geq W_L(\underline{w})$ is equivalent to

$$(A6) \qquad v - w^* \geq \frac{1}{1-px}[(1-x)(v - \underline{w}) - px\gamma(1-p)S].$$

Remark 8. *Assume $\beta \geq q(1-p)$ and (A4) - (A6). Then a high-wage firm does not deviate from the stationary offer w^*.*

Combining Remarks 4-8, we have obtained a Nash equilibrium. Notice that there is plenty of freedom in the parameters $(p, q, s, r, v, w^*, \underline{w}, S)$ for the inequalities $\beta \geq q(1-p)$, (A2), (A4), (A5), (A6) to be simultaneously satisfied. In fact, there is a range of w^* that can be an equilibrium offer given all other parameter values. An example is shown in Figure 2.

=== Insert Figure 2. ===

3.5. *Discussion.* To summarize, we have obtained a equilibrium wage dispersion among identical firms and workers with endogenous matching probability and with firms that cannot commit to future wage levels. Our model was as simple as possible to demonstrate the sheer reputation mechanism was effective enough to generate wage differentials. Theoretically, this is a stronger result than models with heterogeneity.

Extending the model to include effort choice or worker quality heterogeneity is straightforward and preserves the results. Once workers can choose productivity (effort), high-wage firms earn even more rent because the incentive is stronger, and it is easier to sustain multiple equilibrium wages. High-wage firms attract workers better, so they can also attract better-quality workers if such heterogeneity is included in the model.

As mentioned in the introduction, Burdett and Mortensen (1998) and Coles (1998) are very closely related to the above model. Burdett and Mortensen (1998) showed a unique equilibrium wage differential among identical firms and workers, but their result relied on the firms' commitment to the future wages and the equal profit across different wage offers.

Coles (1998) modified their model to allow firms to set wages every period and found that their equilibrium wage dispersion was robust under this modification. The main focus in his paper was to link firm size (the number of employees) and equilibrium wage offers, and he showed that larger firms offer higher wages and make greater profit. In his equilibrium the same-size firms offer the same wage but since there are different-size firms in the economy, wage differentials arise. The equilibrium was obtained by a reputation mechanism similar to ours, but what workers react to was a deviation from size-wage function, not the history of wage offers. Because the history was not important, the game was not fundamentally dynamic.

Our focus was the role of worker expectation based on the history of wages offered by each firm. From a game-theoretic insight, once the future actions can be used to punish a current action, it is not difficult to obtain diverse equilibrium behavior based on different expectations. (This is the idea behind the Folk Theorem.) We think that the history-dependent punishment mechanism constructed here is realistic and the resulting diverse wages and payoffs match observed wage and profit distributions.

Another important difference is that in Burdett and Mortensen (1998) or Coles (1998), the probability of contacting a firm is exogenous, while in our model the matching probability in the job market is endogenous. If endogenizing wage offers are important for equilibrium wage offers, as van den Berg and Ridder (1998) argued, endogenizing the matching probability is also an attractive feature of the model. The endogenous matching probability exhibits non-zero probability of involuntary unemployment even though there are the same measure of firms and workers.

With the above simple model we provided only two-tier wage distributions, but we conjecture that more diverse wage offers can be generated by some additional structure such as workers' search intensity. For "medium" wages to be offered, firms need to accept "medium" rate of turnover, which should be possible by allowing search behaviors that are more flexible than the above all-or-nothing decision.

4. CONCLUDING REMARKS

To conclude the paper, we discuss testable implications from the model as well as possible extensions. Both games provide implications for job change behavior that are consistent with empirical findings. Equation (1) in the single-firm model implies that for a given outside offer distribution, the quit probability decreases as the current employer's wage or the worker's search cost increases. In the market model, the definition of the wage offer R for a low-wage firm to prevent search shows that it is easier to prevent search (small R) when workers' search or relocation costs are large. For example, Brewer (1996) showed a negative relationship between quit rate and the current wage. Like other search models, the workers quit only for higher wages so the longer tenure implies higher wages. This is also well-documented. From Proposition 2 and (A6), firms with high turnover or search costs want to reduce quit rates even at the cost of high wages. Campbell (1993) obtained such an empirical result.

Section 2 shows the important effect of the outside offer *distribution* (not just the mean) on the reservation wages. The more firms offering high wages in the outside offer distribution, the more picky a worker becomes. Fujiwara-Greve and Greve (1998) tested this prediction and found support for it.

An increase in the minimum wage does not eliminate the rent to the high-wage firms as long as the assumptions are satisfied. Thus, wage differentials persist. However, (A4) may be violated if the minimum wage is so large that the mark-up R is only slightly above the minimum wage. Such a high minimum wage could eliminate the rent to high-wage reputation and therefore the wage differentials.

This paper has proposed a fundamentally dynamic and strategic wage setting model, and can be extended in several directions. First, the multiplicity of the equilibria invites further study and refinement. We have already clarified that existing Folk Theorems do not apply when endogenous quits are involved, but we have shown that a similar result can be established. This may not be a good news because the number of equilibria can be large and prediction is harder, especially when some heterogeneity is added to the model. Renegotiation-proofness or other concepts can be used to reduce the set of equilibria. However, to our knowledge, general dynamic games with endogenous player changes and renegotiation have not been studied yet.

Second, new interpretations of our results are possible. One extension is to the discrimination literature. We have shown that identical workers can end up in different-wage firms. This can be interpreted that jobs for women (say) are expected to be low-wage jobs and because of this expectation women who have the same productivity as men quit often, and that makes the firms to pay low wages. Such a rational but vicious cycle is an equilibrium, so inequality will persist unless expectations change. This idea should be examined more carefully.

Finally, we hope that the reputation mechanism is tested in the near future. We believe that the strategic aspect of wages is more and more accepted in labor economics, and it should be possible to test whether the wages (or wage changes) are due to strategic considerations. Bulow and Summers (1986) noted that the firms have great interest in wage surveys which suggests that they are concerned with the relative standings in the wage distribution. It should be possible to find out whether the relative rankings in the wage distribution (which is quite stable) and the history of wages affect the firm and worker behavior. Fujiwara-Greve and Greve (1998) showed that the distribution of outside offers affected worker behavior, which is a first step toward testing the whole reputation mechanism.

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Footnotes

1. Our game has complete information and the "reputation" is an interpretation of a trigger strategy. So this paper is not in the line of incomplete information reputation literature.

2. The constant income stream is for simplicity. If the outside offers are comparable in one dimension, for example if all the offers are long-term wage schedule with the same growth rate, then only the initial wage matters and essentially the same analysis carries through.

3. See Blackwell (1965).

4. The precise statement is of course "there exists $\underline{\delta} < 1$ such that for any $\delta \geq \underline{\delta}$, ...", but for ease of reading we will write this way.

5. In the traditional search models, the wages give only one shot effect on labor supply etc., (see for example Stiglitz, 1985, and Burdett and Mortensen, 1998), and therefore unless different wages give the same profit, the firms have no reason to choose different wages. In our model, a wage offer has a long-term effect in future worker behavior.

6. One justification of this assumption is that the economy has already converged to a steady state and we analyze the game after the convergence. It is an interesting research area to develop a model how a reputation (or a name, such as a "high-wage firm") is built. See Mailath and Samuelson (1998).

7. This is optimal only in the subgames where no firm deviated. In subgames where both the worker and a firm matched with him/her deviated, this may not be optimal.

8. This is not the only way to obtain the equilibrium. If $\beta < q(1-p)$, we can impose another assumption to warrant $W_H'(w^*) \geq W_L'(\underline{w})$.

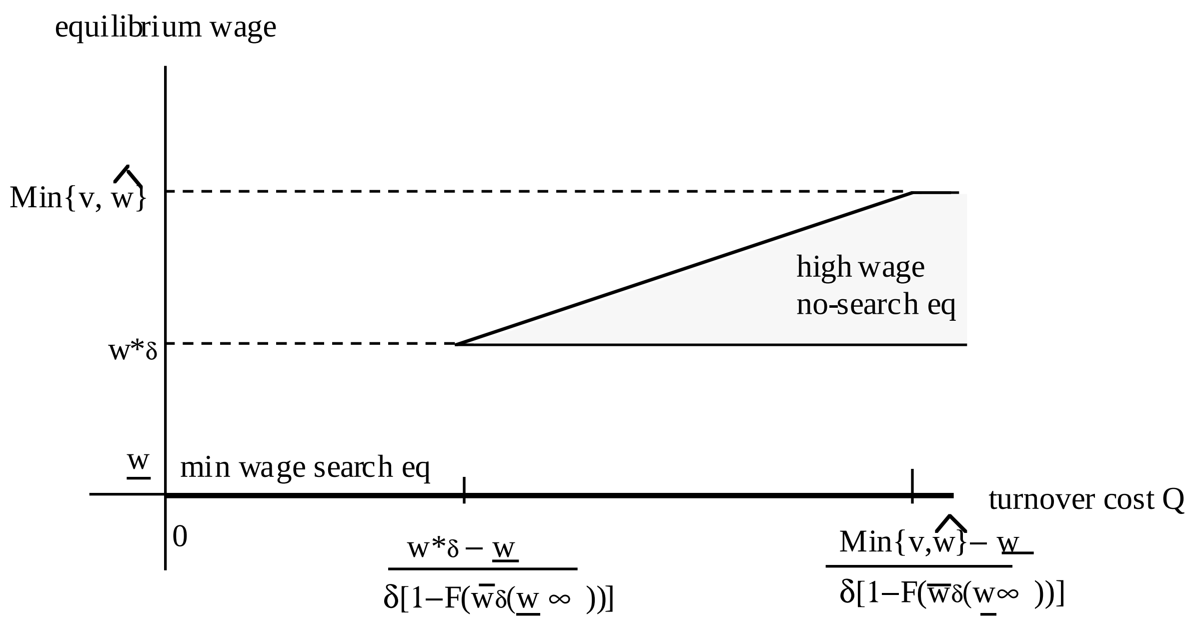

FIGURE 1

STATIONARY EQUILIBRIUM WAGES FOR EACH (SUFFICIENTLY LARGE) δ

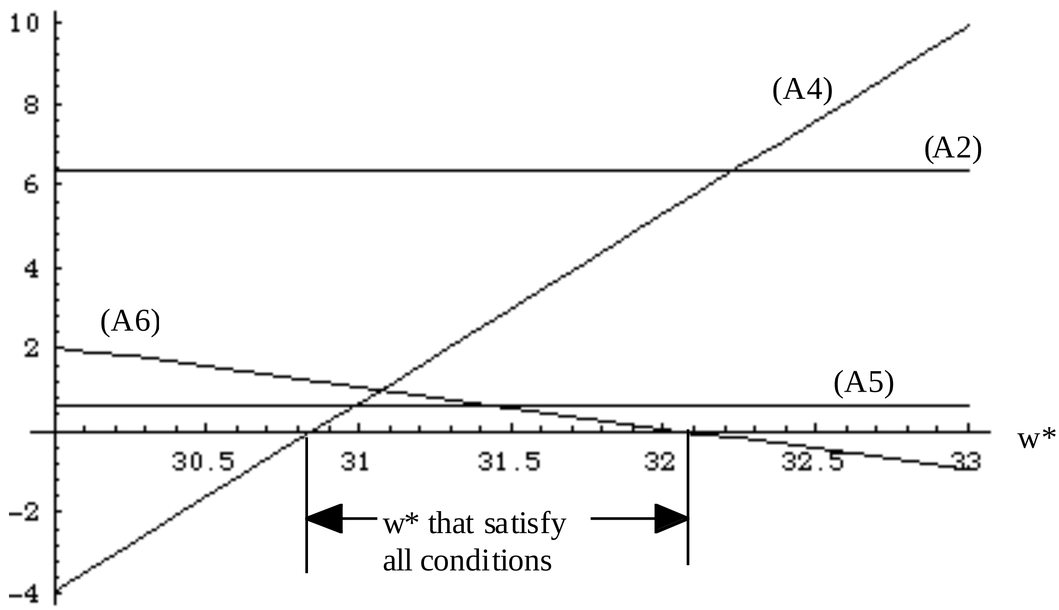

FIGURE 2

EQUILIBRIUM W*: Parameter values
v = 42, w =28, r = 14, s = 1, S = 2, p = 0.9
q = 0.6, $\gamma = 0.9$