Social Development Promoted by Cooperation: A Simple Game Model[1]

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Abstract

This paper presents a simple game theoretic model of development with population growth, based on the idea that the engine of development is cooperation organized by self-interested individuals. The development level of a society as well as population affects the possibility of an organization of cooperation. While the monotone convergence of development holds under the full-cooperation hypothesis, a society develops by repeating growth and decline through the creation of new organizations. Long-run development is determined both by the "fundamentals" of a society and by institutional conditions on organizational costs for cooperation. Dynamic patterns of development are characterized.

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1 Introduction

This paper presents a simple game theoretic model of social and economic development with the idea that the engine of development is cooperative actions of individuals, and investigates a dynamic interrelation between cooperation and development.[2] A society is regarded as an intermixture of conflict and cooperation among individuals where the pursuit of private goals may deviate from cooperation. Many observed failures in developing countries may be caused by non-cooperative actions, termed as moral hazard, rent-seeking and free-riding, at various levels of their societies. The purpose of our analysis is to provide a game-theoretical insight into a basic question of development: "why are some societies well-developed, and others not?" (Olson, 1996).

A society needs some suitable mechanism for attaining cooperation among individuals, which can promote its development. There are many such mechanisms including morals, convention, norm, informal groups, organizations, law, etc. In this paper, we consider the voluntary creation of an organization to enforce cooperative actions on its participants. Our game model is formulated to capture the following dynamic interrelations among individuals, organizations and society. Individuals in a society attempt to create an organization to attain their cooperative actions. Once an organization is successfully created, it increases individuals' welfare and also promotes the development of a society. In turn, the development level of the society affects the group forming behavior of individuals.

To illustrate our problems, we first give two examples in real situations where cooperation plays an important role in development.

Example 1 (voluntary participation in public projects)

The development of a society can be partly described by the level of various kinds of public capital, some of which, such as scientific knowledge and new technology, play a crucial role in economic development as pure public goods. Since costs of R&D for new technology tend to

[2]It is beyond the scope of the present paper to scrutinize the notion of development itself. Rather, it is simply assumed that there exists some appropriate measure of development such as the accumulation of public capital, education, health, and environmental improvements, etc. For a detailed discussion about the concept of development, see Sen (1988).

become huge, it has been the case in some countries that a government has an initiative to organize a public project for promoting basic research of new technology. Private firms are invited to join the project. The participation is costly and voluntary. Since the project is partially financed by the government, scientific and technological knowledges will be open to all firms in the country. Every single firm has an incentive to free-ride on the public project, and the success of the project depends on how many firms will join it.

Example 2 (global environmental problem)

International cooperation is indispensable to the protection of global environment, and some suitable mechanism is needed for collective actions of countries. Driven by increasing concerns on climate changes, the UN Framework Convention on Climate Change (UNFCCC) was signed at the Rio Earth Summit in 1992. The objective of the Convention was to achieve stabilization of greenhouse gas (GHG) concentrations in the atmoshere at a level that would prevent dangerous anthropogen interference with the climate system. Although about 170 countries have ratified the Convention by 1998, a shortcoming of the Convention was the lack of legal obligation. After the UNFCCC, international negotiations were continued for a legally-binding protocol. The Kyoto Protocol was agreed at the third Conference of the Parties (COP-3) to the UNFCCC in 1997. The Kyoto Protocol commited Annex I countries (countries in OECD, former USSR and Eastern Europe) to reducing as a whole GHG emissions by 5.2 per cent below 1990 levels between 2008 and 2012. The reduction commitment to each Annex I country was determined. The reduction commitments are not appied to non-Annex I countries. The prevention of global warming is a controvertial issue in international negotiations, and its success depends on whether or not international cooperation will be successfully attained among Annex I countries and will be expanded to non-Annex I countires in the future. For a detailed explanation of the Kyoto Protocol, see Grubb et al. (1999).

Specifically, our game model is a dynamic version of the n-person prisoners' dilemma with non-overlapping generations. Population growth is incorporated. Every individual's utility depends both on an action profile of all individuals and on the current level of development. Individuals' actions affect their welfare and also determine a new level of development, which is inherited by individulas of the next generation. The more individuals cooperate, the more a society can develop. But, if no cooperation arises, then the society declines. The number of cooperators is endogenously determined by group-forming behavior of self-interested individuals.

A process of organization formation in every generation is formulated as follows. First, all individuals decide independently whether to participate in an organization or not. Secondly, participants negotiate for cooperation. The agreement of cooperation can be reached by the unanimous consent of participants. The participants are burdened with organizational costs of negotiations and enforcements. At the end, all individuals (participants and non-participants) independently select their actions. For simplicity, the enforcement technology of an organization is assumed to be perfect so that the agreement of cooperation is binding. Non-participants are allowed to free-ride on cooperation enforced by the organization. The key factor to the creation of an organization is the organizational surplus, defined by the total cooperative benefits of participants minus the organizational costs and the opportunity costs of the organization. The latter is given by the total payoffs of individuals without the organization, namely, under no cooperation. Given the number of participants, it is proved that the agreement of cooperation is reached if and if the organizational surplus is non-negative. Under the free-participation rule and the presence of an incentive to free-riding, the equilibrium number of participants is determined so that the organizational surplus is zero (or the closest to zero). An organization is successfully created if and only if population is greater than its equilibrium size.

The mechanism of development by cooperation can be explained as follows. Cooperation in one generation promotes development. In turn, the development level affects the possibility of cooperation in future generations through changing the organizational surplus. This surplus may be changed by development in two ways. First, the cooperative benefits increase by development. Secondly, the opportunity costs of an organization also increase by development. For example, the future generations can enjoy high standards of living without the organization in a "developed" society. Depending on which effect is larger, the organizational surplus may increase or decrease through development. By this mechanism, the equilibrium size of an organization changes in the process of development. Furthermore, if population growth is not sufficient, an organization may not be created in equilibrium. The equilibrium condition of an organization makes the development of a society non-monotonic. A society develops through the repetition of growth and decline.

Our game-theoretic model shows two different factors critical to social development. One is the "fundamentals" of a society such as population and production technology, and the other is institutional conditions, represented by organizational costs, for attaining cooperation. Given each number of cooperators, the fundamentals of a society determine the potential level of development that the society can achieve in the long-run. When the potential levels of development are sufficiently high, the development level approaches a certain point in the long-run, repeating growth and decline. The approaching level is determined by the population capacity of a society and organizational costs. When the potential levels of development are low, the development level converges to some intermediate level, not reaching the same long-run level as in the case of the high potential.

This paper is related to two branches of literature, the theory of collective actions and the theory of capital accumulations. Since the seminal work of Olson (1965), the possibility of collective actions has been investigated in many aspects like group size, selective incentives, political entrepreneuerships, and long-term relationships. We incorporate the possibility of a voluntary organization of collective actions into a dynamical model of capital accumulations. On the other hand, the theory of capital accumulations has been one of the primary elements in the literature on growth theory. While neoclassical growth theory explains the long-run growth by exogenous technological progress, recent endogenous growth theory started by the work of Romer (1986) and Lucas (1988) incorporates various kinds of sources of long-run growth into the models, avoiding diminishing returns to capital. Much attention has been focused on human and knowledge capitals, externality, R&D activites, etc. As one of the sources of long-run growth, our approach investigates a broad class of cooperative actions, which play roles of pure public goods in the standard case.

The remainder of this paper is organized as follows. Section 2 describes a dynamic game model of the n-person prisoners' dilemma with non-overlapping generations. Section 3 incorporates the organization of cooperation into the dynamic game model. The equilibrium size of an organization is analyzed. Section 4 characterizes dynamic patterns of development through organizations. Section 5 concludes the paper. All proofs are given in Appendix.

2 A dynamic prisoner's dilemma with non-overlapping generations

Let $N_t = \{1, \cdots, n_t\}$ be the set of players in generation $t(= 1, 2, \cdots)$. Every player i in each generation has two possible actions: $a_i = 1$(cooperation), 0(defection). The model includes a state variable k_t representing a level of development of a society in generation t. We call k_t the development variable in generation t. Examples of development variables are the stock of public capital as public goods and various kinds of environmental indicators. The domain of the development variable k_t is R_+, the set of non-negative real numbers.

A payoff for player i in generation t depends upon both the development variable and the actions selected by all players. Following a standard model of the n-person prisoners' dilemma game (for example, Schelling 1978), the payoff function for every player i is represented by

$$f_i(k_t, a_i, h_{-i}),\ k_t \in R_+,\ a_i = 1, 0,\ h_{-i} = 0, 1, \cdots, n_t - 1 \tag{2.1}$$

where h_{-i} is the number of all players except player i selecting cooperative actions. Note that players have no concern about the welfare of future generations. In what follows, we assume that all players have identical payoff functions, and omit player index, $(-)i$, in notations like f_i and h_{-i} whenever no ambiguity arises.

Assumption 2.1. The payoff function (2.1) for every player i satisfies: for all k_t in R_+ and all $h = 0, 1, \cdots, n_t - 1$, (1) $f(k_t, 0, h) > f(k_t, 1, h)$, (2) $f(k_t, 1, n_t - 1) > f(k_t, 0, 0)$, (3) $f(k_t, a_i, h)$ is monotonically increasing in k_t and in h for every $a_i = 1, 0$.

The assumption means that given any level of development k_t, a society in every generation can be described as an n-person prisoners' dilemma game and that development is "good" to all players. Specifically, condition (1) means that action 0 dominates action 1 for every player, i.e., he is better off by choosing defection than cooperation, regardless of what all the other players select. Thus, the action combination $(0, \cdots, 0)$ is a unique Nash equilibrium point of the game. The equilibrium payoff $f(k_t, 0, 0)$ is called the *noncooperative payoff* at development level k_t. On the other hand, condition (2) means that if all players select cooperative actions, each of them is better off than at the Nash equilibrium point. Condition (3) implies that the more other players cooperate, the better every player's life is, regardless of his action. In other words, cooperation by players gives positive externality to others' welfare. Condition (3) also implies that the more a society develops, the higher payoff every player enjoys.

We next introduce dynamical equations of social development and population growth. First, let us formulate the dynamics of development. We assume that players' actions affect not only their payoffs but also a development level in the next generation. The society develops according to the equation

$$k_{t+1} = g(k_t, a_1, \cdots, a_{n_t}), \quad t = 1, 2, \cdots. \tag{2.2}$$

Moreover, assuming that all individuals are identical in their contribution to development, (2.2) can be reduced to a simple form

$$k_{t+1} = g(k_t, s_t), \quad t = 1, 2, \cdots \tag{2.3}$$

where $s_t = 0, 1, \cdots, n_t$ is the number of all players in generation t selecting cooperation. In (2.3), the magnitude of development is determined by the number of cooperators in a society. This formulation describes our main idea that social development is promoted by cooperation among individuals. $g(k_t, s_t)$ is called the *transition function of development*. Remark that population n_t (or the number $n_t - s_t$ of players choosing defection) does not appear as an argument in $g(k_t, s_t)$. To simplify analysis, we assume in (2.3) that noncooperative players never "harm" the development of a society. They only free ride on the development owing to cooperative actions by other players.

Assumption 2.2. (1) $g(k_t, s_t)$ is monotonically increasing in k_t and in s_t, and continuous in k_t, (2) for given $s_t = 0, 1, \cdots, n_t$, the equation $k_{t+1} = g(k_t, s_t)$ has a unique fixed point $k_t^* = k_{t+1}^* \equiv p(s_t)$ such that $k < g(k, s_t)$ and $k > g(k, s_t)$, respectively, if $0 \leq k < p(s_t)$ and $k > p(s_t)$, respectively, and (3) $0 < g(k_t, 0) < k_t$ for all $k_t (\neq 0)$, and $g(0, 0) = 0$.

This assumption implies the following properties of the transition function $g(k, s_t)$ of development. Given the number s_t of cooperators, $g(k, s_t)$ is an increasing function of k and intersects with the 45-degree line uniquely at $k = p(s_t)$. The graph of $g(k, s_t)$ intersects with the 45-degree line from above to below as k increases (see Figure 2.1). As well-known, this property implies that $p(s_t)$ is the globally stable fixed point of dynamical system (2.3). Namely, if the number of cooperators is fixed at s_t over generations, then develpment variables k_t monotonically converge to the fixed point $p(s_t)$, regardless of an initial point. The fixed point $p(s_t)$ is a function of the number s_t of cooperators and is called the *potential function of development.* It describes the potential level of development that the society can achieve in the long run, depending upon the number of cooperators. The potential function $p(s_t)$ of development is regarded as one of the "fundamentals" in a society. From condition (1), the transition function $g(k, s_t)$ shifts upward as s_t increases, that is, the more individuals cooperate, the more a society develops. This implies that the potential function $p(s)$ is an increasing function of the number s of cooperators. Finally, condition (3) implies that if no individuals cooperate over generations, development variables $\{k_t\}$ monotonically decrease and converge to the worst level of zero (see Figure 2.2).

An important aspect of development in our model is that the number of cooperators is not fixed over generations, but that it is endogenously determined by the group forming behavior of individuals. Hence, the transitive function of development may change from generation to generation. This makes the dynamics of development possibly complicated in our model. Dynamics of development will be studied in Section 4.

Figures 2.1 and 2.2 about here

The second dynamical equation describes population growth. Assuming that the magnitude of population growth may depend on the development variable, the population growth has the form of

$$n_{t+1} - n_t = \Delta(k_t, n_t), \quad t = 1, 2, \cdots \tag{2.4}$$

where $\Delta(k_t, n_t)$ represents the change of population from generation t to generation $t+1$.

Assumption 2.3. For any k_t in R_+, there exists a real number[3] $n(k_t)$ such that

(1) $\Delta(k_t, n_t) > 0$ if $n_t < n(k_t)$, $\Delta(k_t, n_t) = 0$ if $n_t = n(k_t)$, and $\Delta(k_t, n_t) < 0$ if $n_t > n(k_t)$,

(2) for every t, $n_{t+1} < n(k_t)$ if and only if $n_t < n(k_t)$,

(3) $\Delta(k_t, n_t)$ is continuous on R_+^2,

(4) $n(k_t)$ is a continuous and monotonically increasing function of k_t, and there exists some N^* such that $n(k_t) \leq N^*$ for all k_t.

This assumption means that population growth in (2.4) has the global stability similar to development in (2.3). Given development level k_t, population n_t monotonically increases (decreases, respectively) and converges to the level $n(k_t)$ when an initial level is below (above, respectively) it. The fixed point $n(k_t)$ of (2.4) shows the population capacity of a society, and it is a function of development level k_t. The $n(k_t)$ is called the *population capacity function*, describing the long-run level of population when development variable k_t is constant over generations. The population capacity function $n(k)$ has an upper bound N^*. The population capacity function $n(k_t)$ as well as the potential function $p(s_t)$ of development constitutes the "fundamentals" of a society in the model.

The following discrete version of the well-known logistic equation (May, 1974)

$$n_{t+1} - n_t = rn_t\left(1 - \frac{n_t}{n(k_t)}\right), \quad 0 < r < 1$$

satisfies Assumption 2.3 over the given range of r. When the initial population is small, popula-

[3]For convenience of analysis, it is assumed in (2.4) that population n_t may take real values. This technical assumption does not affect the results of the model in any crucial way. In Section 4, the number s_t of cooperators is also assumed to be real numbers.

tion grows exponentially with rate r, and beyond a certain level the speed of population growth decreases, and population converges to level $n(k_t)$.

As the first benchmark of analysis, we consider the dynamics of development when the prisoner's dilemma game is played in every generation. The action combination $(0, \cdots, 0)$ is a unique equilibrium point in the game of each generation, regardless of the level of development. Since the outcome of a game in any generation never affects the decision making of players in future generations, we can easily conclude that no cooperation occurs in any generation. The following proposition shows the dynamics of a society under no cooperation.

Proposition 2.1. (no cooperation) For any initial point (k_0, n_0), let $\{(k_t, n_t)\}_{t=1}^{\infty}$ be a sequence of development variables and population generated by (2.3) and (2.4) under no cooperation ($s_t = 0$ for all t). Then, $\{(k_t, n_t)\}_{t=1}^{\infty}$ converges to $(0, n(0))$ as t goes to infinity.

This proposition shows that when players of every generation fail to attain cooperation, the development of a society declines to the worst (zero) level and the population approaches its capacity level $n(0)$. The long-run equilibrium is independent of initial conditions of development and population. This result can be regarded as a dynamic version of the "tragedy of commons" (Hardin, 1968). To avoid this undesirable equilibrium in the long-run, some suitable mechanism for attaining cooperation is needed in the society. This issue will be discussed in the next section.

We conclude this section with an example of our model of development.

Example 2.1. (the accumulation of public capital)

Consider a production economy in which every individual i of generation t is given an initial endowment w_i units of private goods. Let n_t denote population in generation t. The private goods can be consumed and invested to produce public capital as pure public goods. The public capital can have a positive effect on the production of private goods by individuals. Public infrastructure services are examples of such public capital. Let k_t be the stock of public capital which generation t inherits from the past generation. In the beginning of the economy in generation t, all individuals decide independently how much they should contribute to the creation of public capital from their initial endowments. Let $\theta_i (0 \leq \theta_i \leq w_i)$ be individual i's contribution. Then, the new stock of public capital, available for producing private goods in generation t, is given by the equation

$$\tilde{k}_t = (1-\delta)k_t + g(s_t), \quad s_t = \sum_{i=1}^{n_t} \theta_i \tag{2.5}$$

where $\delta (0 < \delta < 1)$ is the depreciation rate of public capital and $g(s)$ is the production function of public capitals, which is an increasing function of total contributions s with $g(0) = 0$. Every individual i's production function of private goods is given in a form of

$$y_t^i = F(\tilde{k}_t, x_t^i) = \tilde{k}_t (l + x_t^i)^{\alpha}, \quad l > 0,\ \alpha > 0 \tag{2.6}$$

where x_t^i is the input of private goods. The constant l can be interpreted to be a predetermined level of labor supply, which is perfectly substitutable for private goods in production. We assume that $x_t^i = \omega_i - \theta_i$, that is, every individual invests the whole initial endowment that remain after he has contributed to the creation of public capital. Individual i's utility $u_i(y_t^i)$ is an increasing function of his consumption y_t^i. Finally, the next generation $t+1$ can inherit the stock $\tilde{k}_t$ of public capital in generation t:

$$k_{t+1} = \tilde{k}_t = (1-\delta)k_t + g(\sum_{i=1}^{n_t} \theta_i). \tag{2.7}$$

We regard the full contribution, "$\theta_i = \omega_i$", as action 1 (cooperation) and zero contribution, "$\theta_i = 0$", as action 0 (defection). We examine under what conditions this economy can be described as the prisoners' dilemma game. Noting that the utility function $u_i(y_t^i)$ is increasing in y_t^i, we can show from (2.5) and (2.6) that "$\theta_i = 0$" is the dominant action of every individual $i = 1, \cdots, n_t$ if

$$g'(\sum_{i=1}^{n_t} \omega_i) < \frac{\alpha(1-\delta)k_t}{l + \omega_i},$$

where $g'(s)$ is the derivative of $g(s)$. The full contribution $(\omega_1, \cdots, \omega_n)$ is Pareto-superior to the

Nash equilibrium $(0, \cdots, 0)$ if

$$\frac{g(\sum_{i=1}^{n_t} \omega_i)}{(1-\delta)k_t} > (1 + \frac{\omega_i}{l})^\alpha - 1.$$

Finally, in the symmetric case that $\omega_i = \omega$ for all i, the potential function of development is given by $p(s) = g(s\omega)/\delta$. Given initial endowment ω, the potential level of development increases either as the number s of cooperators increases, or as the depreciation rate δ of public capital decreases.

3 The organization of cooperation

The prisoners' dilemma game describes an anarchic state in which players are free to choose their actions. The natural outcome in a society is that every player in every generation chooses the dominant action of defection. Then, a society declines to the worst development level in the long run (Proposition 2.1). The society needs some institutional arrangement to escape from the tragedy. In this paper, we assume that players in every generation attempt to create a social institution to enforce their collective action of cooperation. Such a social institution is called an *organization of cooperation*. We consider whether or not an organization of cooperation can be voluntarily created by players in an anarchic situation and if any, how many players participate in the organization.

In an organization, all members negotiate for their collective action of cooperation. An enforcement mechanism has to exist in the organization so that the agreement of cooperation can be effectively implemented. The mechanism has various functions such as monitoring members' actions, punishing members for deviating from cooperation, and distributing organizational costs among members. In general, the construction of an enforcement mechanism itself is a negotiation issue for participants in an organization as we can see in many cases of international treaties. To simplify analysis, we, however, do not present a formal game model to describe such an internal negotiation process for creating an enforcement mechanism. We simply assume that the unanimous agreement of cooperation can be enforced with some organizational costs, and also that the costs are allocated equally to all members.[4] Specifically, the enforcement of cooperation is implemented by punishing any member for deviating from cooperation so that defection is not beneficial. The enforcement of cooperation is not applied to non-members of the organization, and they are allowed to free-ride on cooperative actions enforced by the organization. The organizational cost is described by a function $C(s)$ where s is the number of all participants.

In real situations, punishments may have various forms. For example, if a business firm participating in a public project violates an agreement of the project, it incurs a penalty specified in an explicit contract, or loses subsidy from the government and/or reputations in a market.

We formulate a process of organization formation in each generation as a noncooperative three-stage game.

(1) Participation decision stage:

Given development level k_t, every player of generation $t(= 1, 2, \cdots)$ decides independently whether to participate in an organization or not. Let S be the set of all s participants. If $s = 0, 1$, then no organization is possible.[5]

(2) Organizational negotiation stage:

All participants in S decide independently whether to agree to cooperation or not. The agreement of cooperation is reached if and only if all participants agree. An organization is formed when the agreement of cooperation is reached.

(3) Action decision stage:

When an organization is formed, all players (participants and non-participants) select independently their actions, 1 (cooperation) or 0 (defection). The payoff of every player i is given by

$$\begin{aligned} f(k_t, a_i, h_{-i}) - \tfrac{C(s)}{s}, \qquad & i \in S,\ a_i = 1 \\ f(k_t, a_i, h_{-i}) - \tfrac{C(s)}{s} - p, \quad & i \in S,\ a_i = 0 \end{aligned}$$

[4]We assume here that individuals have transferable utility. This assumption can be relaxed in a more elaborate model.

[5]If $s = 1$, then a single participant has no incentive to taking a cooperative action in a prisoner's dilemma.

$$f(k_t, a_i, h_{-i}), \qquad i \notin S$$

where a_i is player i's action, h_{-i} is the number of all individuals except i selecting 1, and p is the punishment on deviators in S. The organizational cost $C(s)$ is equally allocated to all members. When no organization is formed, all individuals play the original prisoner's dilemma game described in the last section.

We next characterize a subgame perfect equilibrium point for the organization formation game above by the backward induction. Under the assumption of perfect enforceability, the punishment level p is high enough to prevent any participant from defecting. The punishment is not imposed on non-members of the organization even if they select defection. Noting these rules, it is easy to see that the action decision stage has a unique Nash equilibrium point where all members of the organization select cooperation and all non-members select defection.

To analyse the organization forming behavior, we introduce the *organizational surplus* defined by

$$W(k_t, s) \equiv sf(k_t, 1, s-1) - C(s) - sf(k_t, 0, 0). \tag{3.1}$$

The first term is the total payoffs produced by cooperative actions in the organization, the second term is the organizational cost, and the last term is the opportunity cost of the organization given by the sum of all participants' noncooperative payoffs in the prisoner's dilemma. If the organizational surplus is negative, there is no reason for its members to form the organization.

Proposition 3.1. The organizational negotiation stage has a Nash equilibrium point in which an organization is formed, if and only if the organizational surplus $W(k_t, s)$ is non-negative where $s (\geq 2)$ is the number of participants in the organization.

By the rule of our game, an organization is formed through the unanimous agreement of cooperation by participants. This proposition states equivalently that the unanimity is attained in a Nash equilibrium point if and only if all participants can receive their payoffs greater than, or equal to, the noncooperative payoffs of the prisoner's dilemma. We note that there are, however, many other "trivial" Nash equilibrium points leading to the failure of an organization. For example, all situations where two groups each consisting of at least two members make different choices, are such Nash equilibrium points. These equilibrium points are peculiar to the unanimous game (with simultaneous moves). We exclude these trivial equilibrium points from our analysis.

To analyse the participation decision stage, we assume:

Assumption 3.1. The organizational surplus $W(k_t, s)$ is monotonically increasing in the number s of participants.

When the number of participants in the organization is very small, their cooperative actions may not be enough productive, and the organizational surplus may be negative. Then, a small organization may fail. Assumption 3.1 implies that if more players join the organization, it may become more productive, and thus the agreement of an organization is more likely. This assumption holds if the organizational cost $C(s)/s$ per member is a decreasing function of its size s.

Proposition 3.2. Given development level k_t, let $s(k_t)$ be the smallest integer satisfying $W(k_t, s) \geq 0$. (1) When $n_t \geq s(k_t)$, the participation decision stage has a Nash equilibrium point leading to an organization if and only if the number of all participants is exactly equal to $s(k_t)$. (2) When $n_t < s(k_t)$, the participation decision stage has no Nash equilibrium point leading to an organization.

The proposition demonstrates that only the smallest feasible organization with $s(k_t)$ participants can be formed in a Nash equilibrium point of the participation decision stage.[6] Any feasible organization with more than $s(k_t)$ participants can not be sustained in equilibrium. The

[6]We consider only pure strategy Nash equilibrium points in this paper. A mixed strategy Nash equilibrium point of a related model is analyzed in Okada (1993).

intuition for this result is the following. If the organization size is strictly greater than $s(k_t)$, the organizational surplus is positive by Assumption 3.1, and any participant has an incentive to deviate from the organization. This is because the remaining smaller organization has still a non-negative surplus and all other participants keep their cooperation. This opting-out process stops at the smallest feasible organization with $s(k_t)$ participants. It is important to see that the equilibrium size $s(k_t)$ of an organization is determined by the development level k_t of a society, and thus the level of cooperation changes in the process of developmehnt. We call $s(k_t)$ the *organization size function* and it reflects the institutional condition for cooperation in a society. For example, if the organizational cost increases, then the organization size function shifts upward. When the population is less than this level, no organization is formed and thus society fails to attain cooperation.

4 The dynamics of social development

We now investigate how a society can develop through the voluntary creation of an organization for cooperation. In the dynamic process of development, every generation plays the organization formation game described in the last section, and the number of participants in the organization determines the magnitude of development. Dynamic patterns of development will be examined by phase diagrams.

For convenience of analysis, we extend the domain of variable s_t, the number of cooperators, from integers to real numbers. To prevent this extension from changing the model crucially, it is assumed that given k_t, the transition function $g(k_t, s_t)$ of development is an increasing step-function of s_t, of which jumping points are at integer values. To be concrete, we assume:

Assumption 4.1. The domain of the transition function $g(k_t, s_t)$ is R^2_+, and for any $k_t, g(k_t, \cdot)$ is constant on semi-closed intervals $(m, m+1]$ for all $m = 0, 1, 2, \cdots$.

Recall that given s_t, the potential function $p(s_t)$ of development is defined by a solution of the equation $k = g(k, s_t)$. This assumption, with Assumption 2.2, implies that the potential function $p(s_t)$ of development is an increasing step-function on R_+ with jumping at integer points. The development level k_t of a society with s_t cooperators increases if and only if k_t is smaller than the potential level $p(s_t)$ of development.

In Section 2, we examined as a benchmark the path of development under no cooperation. For another benchmark, we first consider the development under the full-cooperation hypothesis that all individuals cooperate in all generations. The dynamics of development under this optimistic scenario is given by

$$\begin{aligned} &(1) \quad k_{t+1} = g(k_t, n_t) \\ &(2) \quad n_{t+1} = n_t + \Delta(k_t, n_t). \end{aligned} \tag{4.1}$$

Note that $s_t = n_t$ for all t in this case of full cooperation.

Assumption 4.2. The population capacity function $n = n(k)$ and the potential function $k = p(n)$ of development has a unique intersection (k^+, n^+).

Figure 4.1 is the phase diagram of the dynamic system (4.1) of developmet under full cooperation. In Figure 4.1, the graph of the population capacity function $n = n(k)$ is the locus of (k_t, n_t) at which population n_t is constant. By this reason, we abuse the notation "$\dot{n} = 0$" to indicate the graph of $n = n(k)$. Similarly, the graph of the potential function of development, $k = p(n)$, is indicated as "$\dot{k} = 0$" in Figure 4.1. These notations will be used in the following figures.

The next proposition shows the dynamic pattern of development under full cooperation.

Proposition 4.1. (full cooperation) For any initial point (k_0, n_0) with $k_0 < p(n_0) \leq k^+$ and $n_0 < n(k_0) \leq n^+$, let $\{(k_t, n_t)\}_{t=1}^{\infty}$ be a sequence of development variables and population generated by (4.1). Then, k_t and n_t monotonically increase and converge to (k^+, n^+) as t goes to infinity.

The proposition shows that under the full-cooperation hypothesis, a society monotonically develops and the development level converges to the intersection point k^+ of two loci of $\dot{n} = 0$ and $\dot{k} = 0$ (see Figure 4.1). This pattern of development does not depend on an initial point, as long as it lies in the southwest region of the intersection point (k^+, n^+). The long run development is determined solely by the "fundamentals" of a society, which are described by the population capacity function and the potential function of development. This implies that, under the full-cooperation hypothesis, less developed societies can catch up with developed societies as long as their fundamentals are identical.

<u>Figure 4.1 about here</u>

We next consider the dynamic process of development through the voluntary organization of cooperation. In the continuous version of the model where variables n_t and s_t take real numbers, Proposition 3.2 is modified such that the equilibrium size $s(k_t)$ of an organization is determined by the solution of

$$W(k_t, s) = 0 \tag{4.2}$$

where $W(k_t, s)$ is the organization surplus defined in (3.1). This equation is rewritten as

$$f(k_t, 1, s-1) - f(k_t, 0, 0) = \frac{C(s)}{s},$$

which says that the utility increase of every participant in an organizational is equal to the organization costs per member. If $W(k_t, s) < 0$ for all s, no organization can be formed. To avoid this trivial case, we assume:

Assumption 4.3. For every $k_t \in R_+$, there exists a unique solution $s = s(k_t) \in R_+$ of (4.2). The $s(k_t)$ is called the *organization size function.*

Let F denote the region $\{(k_t, n_t) \in R^2_+ | s(k_t) \leq n_t\}$ of development variables and population. Proposition 3.2 implies that when a society has variables (k_t, n_t) in F, an organization of size

$s(k_t)$ is formed. When not, no cooperation is attained. By this reason, F is called *the feasible region of cooperation.*

The dynamics of development through organized cooperation is given by

$$\begin{aligned}
(1)\ k_{t+1} &= g(k_t, s(k_t)) \quad if\ (k_t, n_t) \in F \\
&= g(k_t, 0) \qquad\qquad otherwise \\
(2)\ n_{t+1} &= n_t + \Delta(k_t, n_t)
\end{aligned} \tag{4.3}$$

where $s(k_t)$ is the number of cooperators in the organization.

Unlike the case of full cooperation, the dynamics of development by the organization crucially depends upon the organization size function $s(k_t)$. Differentiating both sides of (4.2) with respect to development variable k yields

$$\frac{\partial W}{\partial k} + \frac{\partial W}{\partial s}\frac{ds}{dk} = 0 \tag{4.4}$$

Since $\partial W/\partial s > 0$ from Assumption 3.1, we have $ds/dk > 0$ if and only if $\partial W/\partial k < 0$.[7] That is, if the organizational surplus W decreases as development variable k increases, then the organization must become larger to compensate the surplus decrease. In the rest of this section, we analyse only the case that the organization size $s(k)$ is a monotonically increasing function of development variable k. The technique of analysis can be applied to other cases.

Assumption 4.4.

(1) The organization size function $s = s(k)$ is a monotonically increasing and continuous function of development variable k, and has a unique intersection (k^*, n^*) with the population capacity function $n(k)$ satisfying $n^* = s(k^*) = n(k^*)$.

(2) The population capacity function $n = n(k)$ is "flat" over the range $[0, k^*]$ in the sense that

$$[n^*] \leq n(k) \leq n^* \quad for\ all\ k\ with \quad 0 \leq k \leq k^* \tag{4.5}$$

[7] Here the payoff function $f(k, a, h)$ of every player is assumed to be differentiable with respect to k and h for the sake of exposition.

where $[n^*]$ is the largest integer which is smaller than n^*.

For every $i = 1, 2, \cdots$, we define a sub-region $F(i)$ of F by

$$F(i) = \{(k_t, n_t) \in F |\ i - 1 < s(k_t) \leq i\}. \tag{4.6}$$

For every (k_t, n_t) in sub-region $F(i)$, an organization with $s(k_t)$ participants is formed in equilibrium, and the society develops according to the equation $k_{t+1} = g(k_t, s(k_t)) = g(k_t, i)$. Recall that the transition function $g(k_t, s)$ is a step-function being constant on semi-closed intervals $(i-1, i]$ from Assumption 4.1. The feasible region F of cooperation is partitioned into sub-regions $F(i) : F = \bigcup\{F(i)|i = 1, 2, \cdots\}$. When $F(i) \neq \phi$, we define k^i by

$$s(k^i) = i. \tag{4.7}$$

Then, sub-region $F(i)$ is rewritten as

$$F(i) = \{(k_t, n_t) \in F |\ k^{i-1} < k_t \leq k^i\}.$$

At the development level k^i, the number of cooperators changes from i to $i+1$. That is, the transition function of development shifts from $g(k_t, i)$ to $g(k_t, i+1)$. We call $k^i (i = 1, 2, \cdots)$ *turning points of development*. Since the organization size function $s(k)$ is a monotonically increasing function (Assumption 4.4), we have $k^i < k^{i+1}$ for every i.

When variables (k_t, n_t) in generation t lie in sub-region $F(i)$, the development of a society is governed by the dynamical system $k_{t+1} = g(k_t, i)$. If the current development level k_t is lower than the potential level $p(i)$ under cooperation by i players, the development level increases, and otherwise it decreases. This property of transitive dynamics in development variables suggests that dynamic pattern of development is critically affected by the locations of two sequences $\{k^i\}$ and $\{p(i)\}$.

The first theorem considers the case that for every number $i(= 1, 2, \cdots)$ of cooperators, the potential $p(i)$ of development is larger than the turning point k^i of development. Geometrically, this means that the curve of the potential function of development is located right of that of the organization size function (see Figure 4.2). In other words, the former lies outside the feasible region of cooperation.

Theorem 4.1. (high potential case) For any initial point (k_0, n_0) in the feasible region F of cooperation, let $\{(k_t, n_t)\}_{t=1}^{\infty}$ be a sequence of development variables and population generated by (4.3). If

$$k^i < p(i) \quad \text{for every } i = 1, \cdots, [n^*],^{8} \tag{4.8}$$

where n^* is the population level at which the two curves of the population capacity function and the organization size function intersect, then there exists some sufficiently large t such that $[n^*] \leq n_t$ and $k^{[n^*]} \leq k_t$.

The condition in the theorem means that the potential of development is so high that the development level increases whenever an organization is successfully formed. The theorem shows that in such a high potential case, the long-run development of a society can exceed the largest turning point $k^{[n^*]}$ below the intersection level k^* of the population capacity function and the organization size function. If we employ a counting rule for the number of players finer than the integer (for example, if we can say meaningfully that 10.322 players cooperate), then the largest switching point $k^{[n^*]}$ can become closer to k^*. Therefore, by employing a sufficiently fine counting rule, if necessary, we can say from the theorem that the variables (k_t, n_t) of development and population can approach the intersection point (k^*, n^*) of the population capacity curve and the organization size curve in the long run (see Figure 4.2).

<u>Figure 4.2 about here</u>

The dynamic pattern of development in Theorem 4.1 is as follows. When the variable (k_t, n_t) is in sub-region $F(i)$ of cooperation with $k^{i-1} < k_t \leq k^i$, the change of k_t is given by

[8] We put $k^i = 0$ when $F(i) = \phi$.

$k_{t+1} = g(k_t, i)$. (4.8) implies that development variable k_t increases since it is less than the potential level $p(i)$. If the variable (k_t, n_t) remains in the sub-region $F(i)$ during the process of development, development variable k_t converges to the potential level $p(i)$. Since $k^i < p(i)$ holds, in some future generation t development variable k_t will go beyond the turning point k^i of development, which yields $k^i < k_t < p(i)$. Once this happens, the transition function shifts upward from the i-th level to some higher level. Thereafter, the same mechanism of development starts again under a new transition function. This process continues as long as an organization is successfully formed. In the process of development, the size of an organizations is expanded. In Figure 4.2, numbers of sub-regions indicate the sizes of organizations. Since the feasible region F of cooperation is bounded by the curve of the organization size function, the variable (k_t, n_t) may move outside of the feasible region in some future generation. If this happens, no organization is formed, and development variable k_t starts to decrease with the failure of cooperation. Population, however, keeps growing when its initial level is low. Owing to the increased population, the variables (k_t, n_t) can move back towards the feasible region F and can re-enter it after sufficiently many generations. Again, the process of development starts. In this way, the society develops with the creation of new organizations, repeating growth and decline, and approaches the point (k^*, n^*).

Finally, Assumption 4.4.(2) guarantees that the lower part of the feasible region F of cooperation, bounded by the locus $\dot{n} = 0$, is not too "narrow". If this lower part is too narrow, the variable (k_t, n_t) may enter the upper part of the feasible region when the development level decreases in the argument above. In this case, population does not keep increasing in the process of development. This population change may add some complex patterns of development, for example, cyclic patters, and the theorem may not hold.

The next theorem characterizes the dynamic pattern of development when the potential of development is low so that its level is not larger than the turning point of development for some number of cooperators. Geometrically, two curves of the potential function of development and of the population capacity function intersect. In other words, the potential curve of development enters the feasible region of cooperation (see Figure 4.3).

Theorem 4.2. (low potential case) If there exists some integer $i = 2, 3, \cdots, [n^*] - 1$ such that

$$k^j < p(j) \quad \text{for all} \quad j = 1, \cdots, i-1 \tag{4.9}$$
$$p(i) \le k^i.$$

then, for any initial point $(k_0, n_0) \in F$ with $k_0 \le p(i)$, the sequence $\{(k_t, n_t)\}_{t=1}^{\infty}$ of development and population generated by (4.3) converges to $(p(i), n(p(i)))$ as t goes to infinity.

Theorem 4.2 shows that when the potential of development is low, the development level converges to some intermediate level, not reaching the same long-run level k^* as in the high potential case of Theorem 4.1. This result may be regarded as a "poverty trap" in our model. (4.9) is different from (4.8) in that for some integer i the potential level $p(i)$ of development lies in the interval $(k^{i-1}, k^i]$. In this case, once variables (k_t, n_t) enter sub-region $F(i)$ of cooperation corresponding to this interval, it remains in this sub-region in all future generations and development levels $\{k_t\}$ converge to the potential level $p(i)$ as time goes to infinity. Population $\{n_t\}$ increases and converges to its capacity $n(p(i))$ at the potential level $p(i)$ of development. The dynamic path of development before entering subregion $F(i)$ is similar to that of Theorem 4.1. When the initial population n_0 is smaller than the level $i-1$, the development process may repeat "up-and-down" movements until population exceeds it. As Figure 4.3 shows, a society with the low potential of development can not pass through the sub-region $F(i)$ of cooperation with its own development engine ($i = 6$ in Figure 4.3). The sub-region $F(i)$ may be regarded as a kind of "hurdle" that the society should overcome to achieve higher levels of development. If the society can "jump" this hurdle, for example, by some exogenous help, then the self-development process may start in the next sub-region $F(i+1)$ if condition (4.8) of Theorem 4.1 holds for all $j = i+1, \cdots, [n^*]$.

<u>Figure 4.3 about here</u>

5 Conclusion

We have presented a game theoretic model of social development with the idea that the engine of development is cooperative actions of individuals. Incorporating the possibility of voluntary organizations for cooperation into an n-person dynamic prisoners' dilemma game, we have characterized patterns of long-run development. The main theorems show that two different factors, the "fundamentals" of a society imposing upper limits on population and development levels, and institutional conditions on the organization costs determine the long-run level of development. The group-forming behavior with population growth may yield the repetition of growth and decline in the process of development.

The model in this paper provides us with a game-theoretic insight into the problem of divergences in development. It can be easily understood that the fundamentals of societies such as population capacity and production technology affect the long-run level of development. We discuss here how differences in organizational costs for cooperation and in initial conditions of development and population affect the long-run development of societies with identical fundamentals. If the organizational cost becomes higher, the organization size curve shifts upward and the feasible region of cooperation becomes smaller in the phase diagram of development.

First, we consider the high potential case (Theorem 4.1). When societies have the same organizational costs, their long-run development levels are the same, regardless of their initial conditions. In this case, less developed societies can catch up with developed societies in the long-run. When societies have different organizational costs, societies with lower organizational costs can attain higher levels of development in the long-run than other societies. Institutional differences yields divergence in development.

More complicated is the low potential case (Theorem 4.2). Even when societies have identical organizational costs, their long-run development levels may differ very much, depending on whether their initial levels of development are below or above the "hurdle" of development. When the initial level is below the hurdle, a "poverty trap" arises and its development level converges to an intermediate level. When the initial level exceeds the hurdle, a society can develop by its own force if no further hurdles lie ahead.

In conclusion, the analysis of the paper shows that, depending on several factors such as the potential of development, population capacity, organization costs and initial points, divergent patterns in development appear in a simple game-theoretic model. The population growth plays an important role in our model. If population does not grow, the development of a society remains to stop once the variables of development level and population lie outside the feasible region of cooperation in the phase diagram. This is caused by the lack of population because there is no more possibility of a benefitial organization with positive surplus. However, when population grows, a new organization will be formed after some periods of decline in future generations with larger populations. The process of development can continue as long as the population reach its capacity.

Finally, for analytical convenience, our model has several restrictions. The model is limited to the case that individuals have binary choices to cooperate or not, and that the outcome of cooperation serves as pure public goods. Although the prisoners' dilemma is a standard model for social cooperation, there are many other models suitable for the analysis of coopeation and development. Other obvious restrictions are non-overlapping generations, symmetric individuals, and the presence of a single organization. The internal structure of an organization is highly simplified in our model. We have assumed that the same mechanism for cooperation is employed for all generations in a society. It is an interesting problem to consider the evolution of social mechanisms for cooperation in the process of development.[9] We think that game-theoretic models of cooperation and development are worth of further investigation.

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[9] Related to this issue, Okada, Sakakibara and Suga (1997) consider the transformation of two political systems, centralized (monarchy) and decentralized (democracy), of the state in the accumulation process of social capitals.

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Appendix.

Proof of Proposition 2.1: Given any initial point (k_0, n_0), let $\{k_t, n_t)\}_{t=1}^{\infty}$ be a sequence of development variables and population generated by the system

$$k_{t+1} = g(k_t, 0) \quad \text{and} \quad n_{t+1} - n_t = \Delta(k_t, n_t),\ t = 1, 2, \cdots. \tag{A.1}$$

It follows from Assumption 2.2.(3) that $k_{t+1} < k_t$ whenever $k_t \neq 0$. Since $\{k_t\}$ is a monotonically decreasing sequence bounded below, it has the limit point $k^*(\geq 0)$. Consider two cases:

(i) $n(k_t) \leq n_t$ for some t, and (ii) otherwise. In case (i), Assumptions 2.3.(1) and (2) imply that $n(k_t) \leq n_{t+1} \leq n_t$. Since $k_{t+1} < k_t$, Assumption 2.3.(4) implies that $n(k_{t+1}) \leq n(k_t)$. Therefore, $n(k_{t+1}) \leq n_{t+1} \leq n_t$. By repeating the same argument, it can be shown that $\{n_t\}$ decreases monotonically and is bounded from below. In case (ii), since $n_t < n(k_t)$ for all t, it follows from Assumptions 2.3.(1) and (2) that $\{n_t\}$ increases monotonically and is bounded from above. In either case, $\{n_t\}$ converges to some value m^* as t goes to infinity. By taking the limit of t in (A1), we obtain $k^* = g(k^*, 0)$ and $0 = \Delta(k^*, m^*)$. From Assumptions 2.2.(3) and 2.3.(1), we have $k^* = 0$ and $m^* = n(0)$. Q.E.D.

Proof of Proposition 3.1: If the unanimous agreement is reached in the organizational negotiation stage, every member receives the payoff

$$f(k_t, 1, s-1) - \frac{C(s)}{s} = \frac{W(k_t, s)}{s} + f(k_t, 0, 0)$$

where s is the number of all participants in the organization. If any one member does not agree to cooperation, negotiations break down and the deviating member will receive the noncooperative payoff $f(k_t, 0, 0)$ in the prisoners' dilemma. Therefore, the organization can be supported by a Nash equilibrium point if the organizational surplus $W(k_t, s)$ is non-negative. Q.E.D.

Proof of Proposition 3.2: (1) Let s be the number of participants in an organization. Suppose that $s = s(k_t)$. Any non-member does not have an incentive to joining the organization. If one member deviates from the organization, then the remaining organization with $s(k_t) - 1$ members has a negative surplus and thus it is no longer formed by Proposition 3.1. Then, the deviating member results in receiving the noncooperative payoff in the prisoner's dilemma, which is not better than joining the organization. Therefore, the organization can be supported by a Nash equilibrium point. Suppose that $s > s(k_t)$. In this case, every member is better-off by deviating unilaterally from the organization because the organization without himself still has a non-negative surplus by Assumption 3.1 and thus it is formed from Proposition 3.1. An organization can not be supported in equilibrium in this case. (2) When $n_t < s(k_t)$, the organization surplus

$W(k_t, s)$ is negative for all $s \leq n_t$ by Assumption 3.1. Then, it follows from Proposition 3.1 that no organization is formed. Therefore, there is no Nash equilibrium point leading to an organization. Q.E.D.

Proof of Proposition 4.1: By (2.3) and Assumption 2.2, the initial condition $k_0 < p(n_0) \leq k^+$ implies that $k_0 < k_1 < p(n_0) \leq k^+$. Similarly, by (2.4) and Assumption 2.3, $n_0 < n(k_0) \leq n^+$ implies that $n_0 < n_1 < n(k_0) \leq n^+$. Then, since $n(k)$ and $p(n)$ are monotonically increasing, we have $n(k_0) \leq n(k_1) \leq n^+$ and $p(n_0) \leq p(n_1) \leq k^+$. Combining the inequalities above yield $n_0 < n_1 < n(k_1) \leq n^+$ and $k_0 < k_1 < p(n_1) \leq k^+$. By repeating the same arguments as above, we can prove that $n_{t-1} < n_t < n(k_t) \leq n^+$ and $k_{t-1} < k_t < p(n_t) \leq k^+$ for all t. Since $\{k_t\}_{t=1}^{\infty}$ and $\{n_t\}_{t=1}^{\infty}$ are monotonically increasing sequences bounded from above, there exist some limit points $\bar{n}$ of n_t and $\bar{k}$ of k_t such that $\bar{k} \leq k^+$ and $\bar{n} \leq n^+$. By taking the limit of t in both sides of equations (4.1), we have $\Delta(\bar{n}, \bar{k}) = 0$ and $\bar{k} = g(\bar{k}, \bar{n})$, which means $\bar{n} = n(\bar{k})$ and $\bar{k} = p(\bar{n})$. Note that $g(k_t, \cdot)$ is a left-continuous step function. Since (k^+, n^+) is a unique intersection of $n = n(k)$ and $k = p(n)$, we have $\bar{n} = n^+$ and $\bar{k} = k^+$. Q.E.D.

Proof of Theorem 4.1. Define the following three sub-regions of F:

$$
\begin{aligned}
F_1 &= \{(k, n) \in F |\ n \leq [n^*]\}, \\
F_2 &= \{(k, n) \in F |\ [n^*] < n \leq n(k) \text{ and } k < k^{[n^*]}\}, \\
F_3 &= \{(k, n) \in F |\ n(k) < n \text{ and } k < k^{[n^*]}\}.
\end{aligned}
$$

Case (1). $(k_0, n_0) \in F_2$:

We first claim that if $(k_t, n_t) \in F_2$, then $n_t \leq n_{t+1}$ and $k_t < k_{t+1}$. Since $[n^*] < n_t \leq n(k_t)$, the property of population growth yields $n_t \leq n_{t+1} \leq n(k_t)$. $k_t < k^{[n^*]}$ implies $k^{i-1} < k_t \leq k^i$ for some $i = 2, \cdots, [n^*]$. Then, the development equation is given by $k_{t+1} = g(k_t, i)$. With Assumption 4.2 and (4.8), this yields $k_t < k_{t+1}$, which proves the claim. Since $n(k)$ is an increasing function of k, we have $n_{t+1} \leq n(k_t) \leq n(k_{t+1})$. By way of contradiction, assume that

$k_t < k^{[n^*]}$ for all t. Then, starting with $(k_0, n_0) \in F_2$, the result above implies $(k_t, n_t) \in F_2$ for every $t = 1, 2, \cdots$. Since the sequence $\{k_t\}_{t=1}^{\infty}$ monotonically increases, there exists some $i = 2, \cdots, [n^*]$ such that $k^{i-1} < k_t \leq k^i$ for almost all t. By Assumption 4.1, we can prove that sequence $\{k_t\}_{t=1}^{\infty}$ converges to $p(i)$. Since $k^i < p(i)$ by (4.8), it must be true that $k^i < k_t < p(i)$ for almost all t. A contradiction.

Case (2). $(k_0, n_0) \in F_3$:

When $(k_t, n_t) \in F_3$, the property of population growth yields $n(k_t) < n_{t+1} < n_t$. Then, Assumption 4.4.(2) implies that one and only one of the three cases holds: (i) $k^{[n^*]} \leq k_{t+1}$, (ii) $(k_{t+1}, n_{t+1}) \in F_2$, and (iii) $(k_{t+1}, n_{t+1}) \in F_3$. If the second case happens, we can apply the argument in case (1). Therefore, it is sufficient to consider the case that $(k_t, n_t) \in F_3$ for all t. By (4.8), we can prove $k_t < k_{t+1}$ for all t. Then, by using the same argument as in case (1), a contradiction arises.

Case (3). $(k_0, n_0) \in F_1$:

We first note that $k_t \leq p(N^*)$ for all t when the initial point (k_0, n_0) is in F_1. Recall that N^* is an upper bound of the population capacity function $n(k)$ (Assumption 2.3.(4)). Since $\Delta(n_t, k_t)$ is continuous and positive on the compact set $E = \{(k, n) | 0 \leq n \leq [n^*] \text{ } and \text{ } 0 \leq k \leq p(N^*)\}$, there exists some $\varepsilon > 0$ such that $\Delta(k_t, n_t) \geq \varepsilon$ for all $(k_t, n_t) \in E$. Therefore, in the sequence $\{(k_t, n_t)\}_{t=1}^{\infty}$, we have $n_t + \varepsilon \leq n_{t+1}$ as long as $n_t \leq [n^*]$. This means that, starting at $(k_0, n_0) \in F_1$, n_t goes above $[n^*]$ after many finite steps, independent of the movement of k_t. Then, the following two cases are possible for some large t: (i) $(n_t, k_t) \in F_2$, and (ii) $k^{[n^*]} \leq k_t$ and $[n^*] \leq n_t$. If the first case happens, we can go to case (1). Q.E.D.

Proof of Theorem 4.2: (4.9) implies that $k^{i-1} < p(i-1) < p(i) \leq k^i$. Let $i^- = s(p(i))$. Then, $i - 1 < i^- \leq i$. Similarly as in the proof of Theorem 4.1, define the following three sub-regions of F.

$$
\begin{aligned}
F_1 &= \{(n, k) \in F | \ n \leq i^- \text{ and } k \leq k^{i-1}\}, \\
F_2 &= \{(n, k) \in F | \ i^- < n \leq n(k) \text{ and } k \leq k^{i-1}\},
\end{aligned}
$$

$$F_3 \quad = \quad \{(n,k) \in F |\ n(k) < n \text{ and } k \leq k^{i-1}\}.$$

First notice that (4.9) and $k_0 \leq p(i)$ implies that $k_t < p(i)$ for all t.

Case (1). $(n_0, k_0) \in F_2$:

By the same argument as case (1) in the proof of Theorem 4.1, we can prove that if $(k_t, n_t) \in F_2$, then $n_t \leq n_{t+1} \leq n(k_{t+1})$ and $k_t < k_{t+1}$. If $k_t \leq k^{i-1}$ for all t, then a contradiction arises as in case (1) in the proof of Theorem 4.1. Thus, we can assume without loss of generality that $k^{i-1} < k_t$ for alomost all t. In this case, the society develops according to development equation $k_{t+1} = g(k_t, i)$ for almost all generations t. Then, $\{k_t\}_{t=1}^{\infty}$ converges to $p(i)$ as t goes to infinity. Since $\{n_t\}_{t=1}^{\infty}$ is a monotonically increasing sequence bounded from above, $\{n_t\}_{t=1}^{\infty}$ converges to some point v as t goes to infinity. By taking the limit of t in both sides of population growth equation (2.3), we have $\Delta(v, p(i)) = 0$. This implies that $v = n(p(i))$.

Case (2). $(n_0, k_0) \in F_3$:

By Assumption 4.4.(2), it can be shown that $(n_t, k_t) \in F_3$ implies either (i) $(n_{t+1}, k_{t+1}) \in F$ with $k^{i-1} < k_{t+1} < p(i)$ or (ii) $(n_{t+1}, k_{t+1}) \in F_2 \cup F_3$. If (i) happens, we can prove that $\{k_t\}$ converges to $p(i)$ as t goes to infinity. If $(n_{t+1}, k_{t+1}) \in F_2$, then case (1) can be applied. By these arguments, without loss of generality, we can assume $(n_t, k_t) \in F_3$ for every t. The same arguments as in case (1) implies that $\{k_t\}_{t=1}^{\infty}$ converges to $p(i)$ as t goes to infinity. Since $\{n_t\}_{t=1}^{\infty}$ is a monotonically decreasing sequence bounded from below, we can prove the theorem similarly as in case (1).

Case (3). $(n_0, k_0) \in F_1$:

By employing the same argument as in case (3) in the proof of Theorem 4.1, it can be shown that we have either (i) $(n_{t+1}, k_{t+1}) \in F$ with $k^{i-1} < k_{t+1} < p(i)$ or (ii) $(n_{t+1}, k_{t+1}) \in F_2$, after sufficiently many finite steps. Then, the same arguments as in case (2) can be applied. Q.E.D.

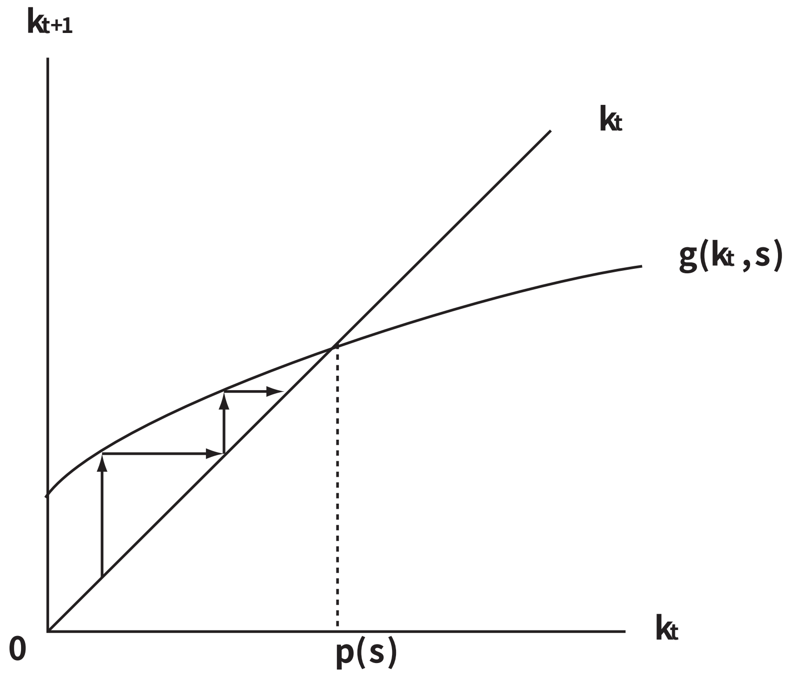

Figure 2.1 Development when the number of cooperators is s.

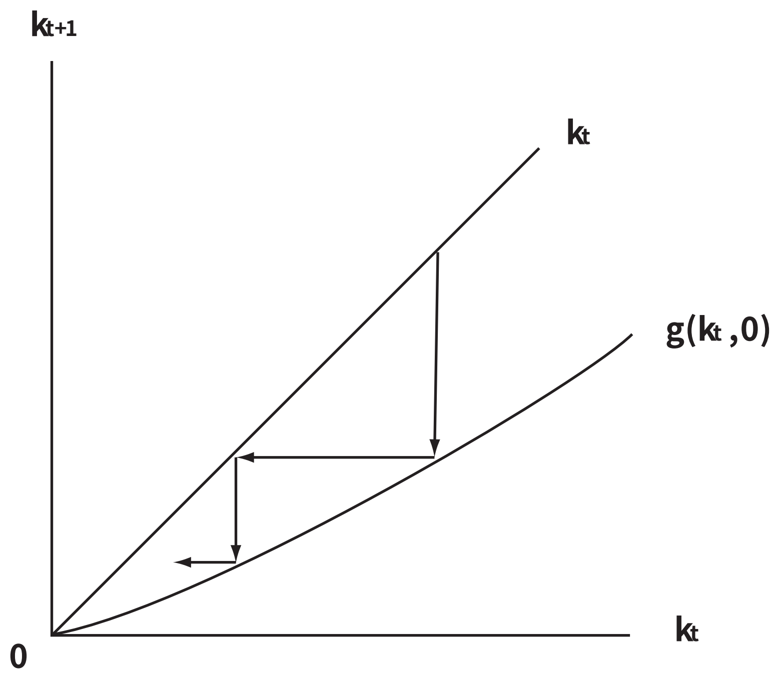

Figure 2.2 Development under no cooperation.

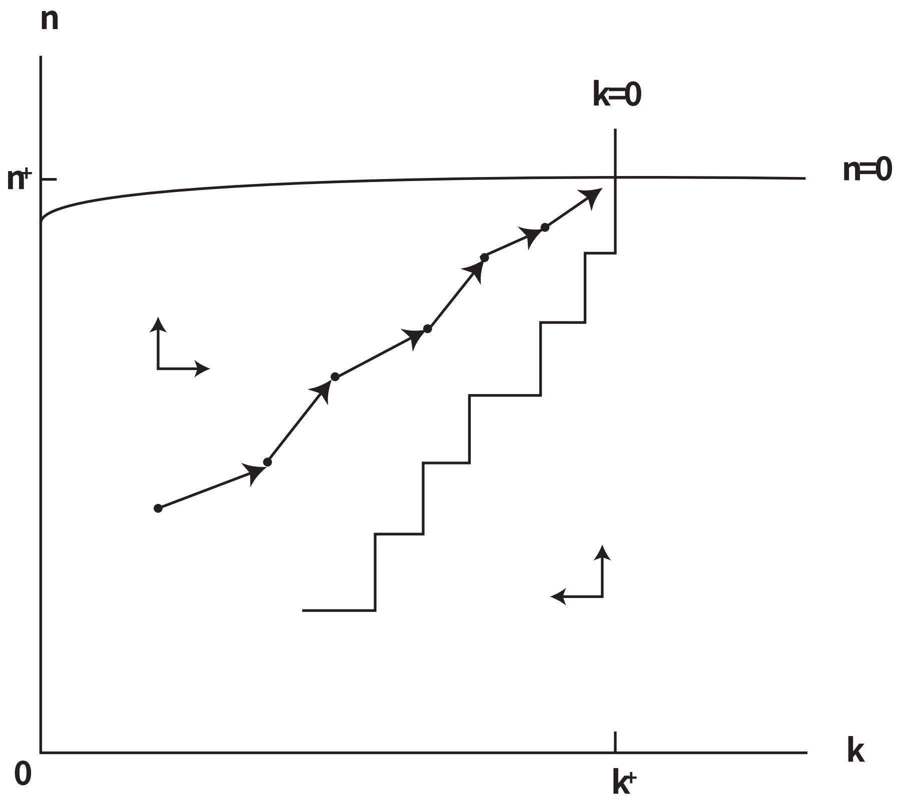

Figure 4.1. Development under full cooperation.

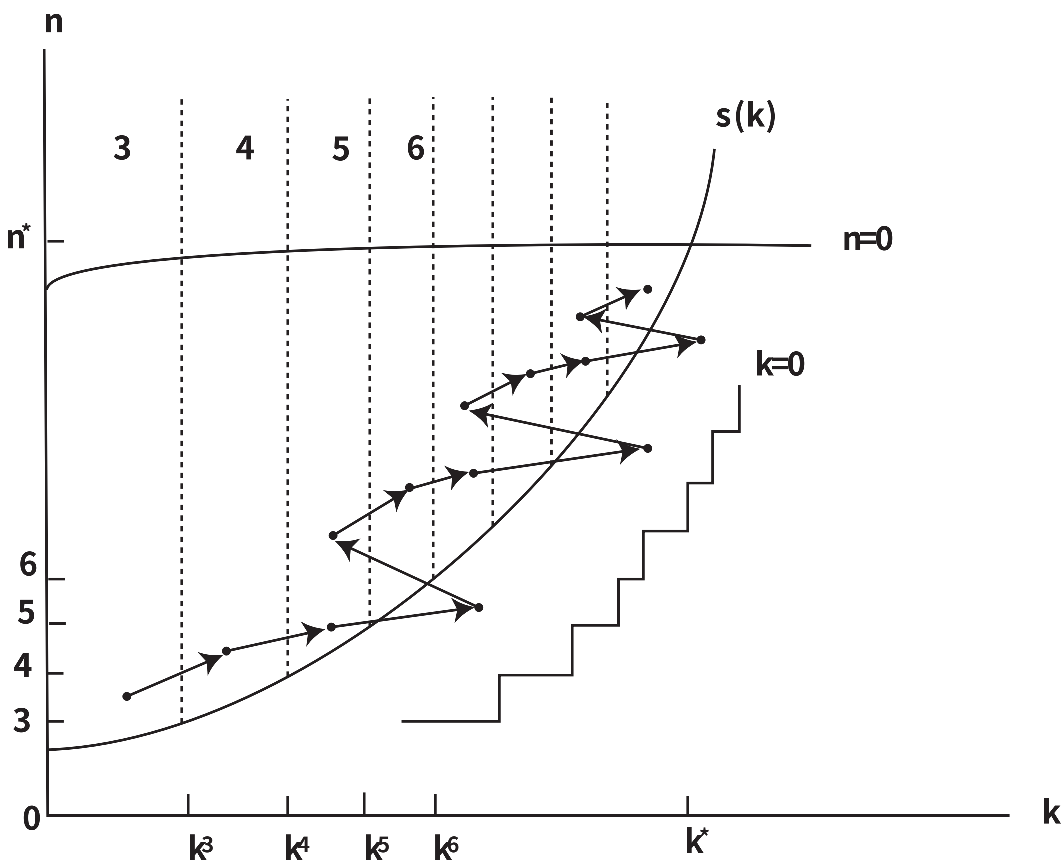

Figure 4.2. Development under organized cooperation when the potential of development is high.

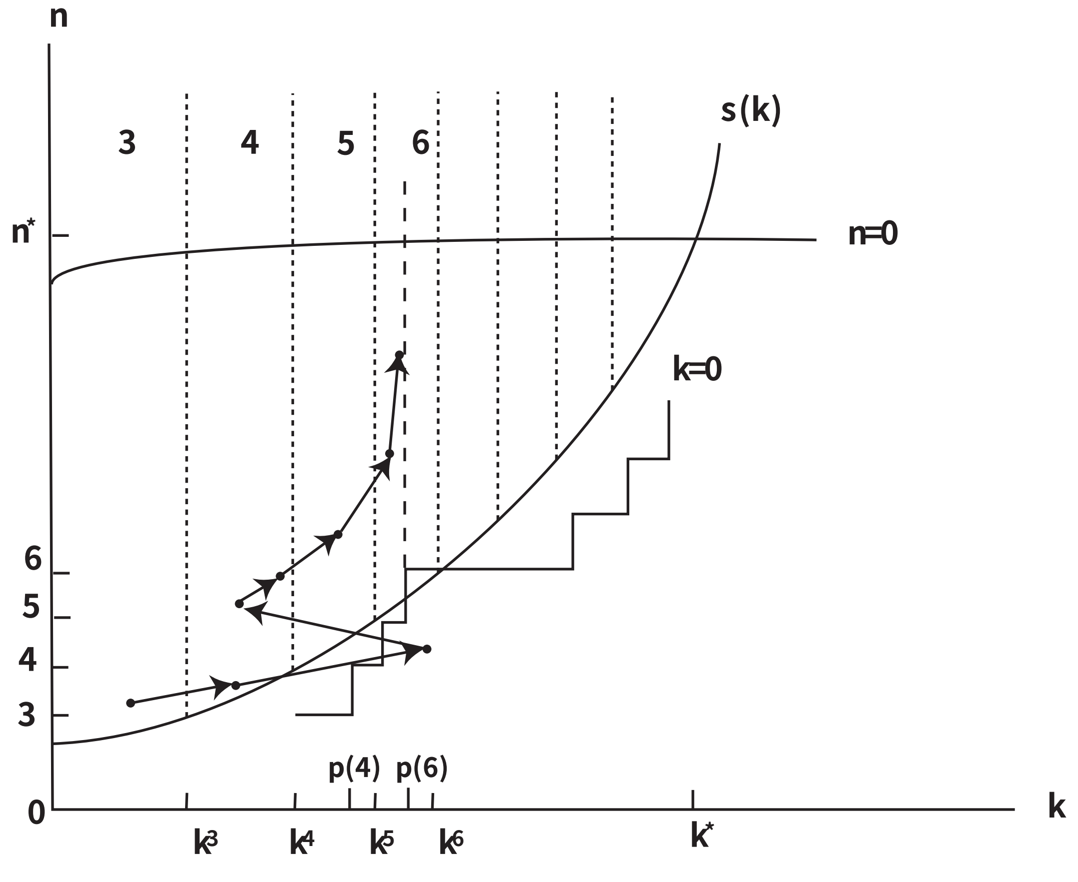

Figure 4.3. Development under organized cooperation when the potential of development is low.