Cooperation in Repeated Prisoner's Dilemma with Perturbed Payoffs*

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Abstract

We examine a variant of repeated Prisoner's Dilemma whose stage game payoffs are subject to vary, and show that a payoff perturbation may strictly *reduce* the minimum discount factor to sustain mutual cooperation.

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1 Introduction

Theory of repeated games, a core area of game theory, has made remarkable progress in the past thirty to forty years.[1] Perhaps the most important contribution of the theory is to enable us to understand how cooperation is sustained in non-cooperative way, i.e., without any binding contracts, through long-run interactions among players. There are many elements known to affect sustainability and degree of cooperation, e.g., discount factors, number of players, observability of actions, and so forth. Among those investigated in the literature, we focus on the effect of payoff perturbations on the sustainability of cooperation. Payoff perturbation can be easily caused by (economic, social) environmental changes of the game, as well as players' subjective uncertainty. However, there are few papers that analyzed the effect of the *structure* of payoff perturbation on the sustainability and degree of cooperation.

Payoff perturbation is first introduced into repeated games by Green and Porter (1984). They consider infinitely repeated Cournot games with stochastic demand fluctuations in which firms can observe neither the demand function nor the rival's output. In their model, each firm can infer rival's output only through her own output and a publicly observable signal, a market price. This imperfect (public) monitoring structure makes cooperation difficult because finding and punishing a defector is cumbersome; the players must punish each other when the signal is bad even if no player had actually deviated, which necessarily entails efficiency loss.[2] That is, fluctuated demand functions are bad for cooperation.

When payoff perturbation does not create a monitoring problem, it can still be bad for cooperation, as Rotemberg and Saloner (1986) show. They study infinitely repeated duopoly games (both price-setting and quantity-setting games) with stochastic demand fluctuations, and show that cooperation becomes more difficult when demand is high rather than low.[3] This is because the temptation to deviate is great when the demand is high. Their results also imply that the optimal expected profit under demand fluctuation is weakly lower than the one under the fixed demand function. While Rotemberg and

[1] Pearce (1992) is an excellent survey on repeated games. For the recent developments of repeated games with private monitoring, see Kandori (2002) and Mailath and Samuelson (2006, part 3).

[2] See Abreu, Pearce and Stacchetti (1986) for the complete characterization of optimal collusive payoffs.

[3] This counter cyclical result depends on their i.i.d. assumption of stochastic demand shocks. See Bagwell and Staiger (1997), Haltiwanger and Harrington (1991), and Kandori (1991) for variations of Rotemberg and Saloner (1986) model with correlated demand fluctuations.

Saloner (1986) did not examine the range of discount factors that sustain cooperation, we show in Section 4.1 that introducing payoff perturbations never decreases the lower bound of the discount factor in the class of oligopoly games they consider.[4] That is, payoff perturbations are bad for cooperation.

In this paper, we examine a variant of repeated Prisoner's Dilemma whose stage game payoffs are subject to vary, and analyze how payoff perturbations affect sustainability of cooperation. As a main result, we show that mutual cooperation may become easier to sustain in the perturbed game than in the ordinary repeated Prisoner's Dilemma (without perturbations). Specifically, consider some discount factor under which no cooperation is possible if the game is the ordinary repeated Prisoner's Dilemma. Then, our theorem states that for the same discount factor (i) introducing perturbation never makes it possible for the players to achieve mutual cooperation in *every* realization of the stage game, but (ii) perturbation enables them to sustain cooperation in *some* realization under a certain condition. In this sense, a payoff perturbation may *enhance* cooperation, which is opposed to the shared view in the literature that payoff perturbations make cooperation more difficult to sustain.

The paper is organized as follows. In Section 2, we present two simple examples to show the intuition of our main result. Then, Section 3 introduces the formal model of repeated Prisoner's Dilemma with perturbed payoffs and establishes the main theorem. Applying this result, Section 4 examines two specific classes of perturbed games: linearly perturbed games and public good games. The final section concludes the paper.

2 Examples

In this section, we present two simple examples which give an intuition to understand why payoff perturbations may make cooperation difficult (Example 1) or easy (Example 2) depending on how we perturb the game. Each example considers a modified repeated Prisoner's Dilemma where players randomly play one of the two stage games in each period. Although realized stage game payoffs are different in two examples, their expected values are identical and expressed by the following table:

[4] See also Example 5.5.1 in Mailath and Samuelson (2006), p.176-177.

	C	D
C	$3,3$	$-3,6$
D	$6,-3$	$0,0$

Benchmark Prisoner's Dilemma G

If this game is repeated infinitely many times without payoff perturbation, then the usual C-trigger strategy combination becomes a subgame perfect equilibrium if and only if

$$(1-\delta)3+\delta\cdot 3 \geq (1-\delta)6+\delta\cdot 0 \qquad (1)$$
$$\Rightarrow \delta \geq \frac{6-3}{6-0}=\frac{1}{2}(=:\delta_0),$$

which gives us the minimum discount factor to sustain (C,C) at least once in this game. In what follows, we calculate the minimum discount factor under different payoff perturbation, and compare it with the benchmark value of $\delta_0=\frac{1}{2}$.

2.1 Example 1: Perturbation makes cooperation difficult

The following example, motivated by Rotemberg and Saloner (1986), illustrates that a payoff perturbation makes cooperation more difficult to sustain. Suppose that, in each period, the stage game is either one of the two games, G_H and G_L, with the equal probability. One can interpret that H is a "high" demand game and L is a "low" demand game. Both of these are a linear transformation of the benchmark G.

	C	D
C	$4,4$	$-4,8$
D	$8,-4$	$0,0$

G_H

	C	D
C	$2,2$	$-2,4$
D	$4,-2$	$0,0$

G_L

If players sustain (C,C) only in G_H and play (D,D) in G_L, then the minimum discount factor $\underline{\delta}_H$ must satisfy

$$(1-\delta)4+\delta\frac{4+0}{2} \geq (1-\delta)8+\delta\cdot 0$$
$$\Rightarrow \delta \geq \frac{2}{3}=\underline{\delta}_H.$$

Similarly, the minimum discount factor $\underline{\delta}_L$ to sustain (C, C) only in G_L can be calculated by

$$(1-\delta)2 + \delta\frac{2+0}{2} \geq (1-\delta)4 + \delta \cdot 0$$
$$\Rightarrow \delta \geq \frac{2}{3} = \underline{\delta}_L.$$

Hence both $\underline{\delta}_H$ and $\underline{\delta}_L$ are larger than δ_0.

If players aim to achieve (C, C) in every period irrespective of the stage game, then the discount factor must satisfy the following two conditions:

$$(1-\delta)4 + \delta\frac{4+2}{2} \geq (1-\delta)8 + \delta \cdot 0$$
$$\Rightarrow \delta \geq \frac{4}{7} =: \overline{\delta}_H.$$

and

$$(1-\delta)2 + \delta\frac{4+2}{2} \geq (1-\delta)4 + \delta \cdot 0$$
$$\Rightarrow \delta \geq \frac{2}{5} =: \overline{\delta}_L.$$

Therefore, the minimum discount factor $\overline{\delta}$ to sustain (C, C) in every stage game is

$$\overline{\delta} = \max\{\overline{\delta}_H, \overline{\delta}_L\},$$

which is $\frac{4}{7}$ and again larger than δ_0.

2.2 Example 2: Perturbation makes cooperation easy

Let us consider a different type of perturbation, whose stage game payoffs are expressed in the next table:

	C	D
C	$3, 3$	$-3, 6+\epsilon$
D	$6+\epsilon, -3$	$0, 0$

$G_{(+)}$

	C	D
C	$3, 3$	$-3, 6-\epsilon$
D	$6-\epsilon, -3$	$0, 0$

$G_{(-)}$

where ϵ takes a value between 0 and 3. Note that the expectation of these two games coincides with the benchmark Prisoner's Dilemma for any ϵ.[5] Now, it is easy to see that cooperation can be sustained in $G_{(-)}$ for small δ (which is smaller than δ_0) when ϵ is large enough, since the deviation gain is almost negligible then. More specifically, when $\epsilon = 2$, the same calculation as we did in Example 1 gives us:[6]

$$\underline{\delta}_{(+)} = \frac{10}{13}, \underline{\delta}_{(-)} = \frac{2}{5}, \overline{\delta}_{(+)} = \frac{5}{8}, \overline{\delta}_{(-)} = \frac{1}{4}.$$

It is clear that $\underline{\delta}_{(-)}$ is now smaller than δ_0 while $\underline{\delta}_{(+)}$ and $\max\{\overline{\delta}_{(+)}, \overline{\delta}_{(-)}\}$ are larger than δ_0. This means that when a discount factor takes a value between $\underline{\delta}_{(-)}$ and δ_0, no cooperation is possible in the ordinary repeated Prisoner's Dilemma G while it is possible to sustain (C, C) in the one of two stage games in our perturbed game.

This example illustrates that, as players give up cooperation in some realized state ($G_{(+)}$ in our case), it may become easier for them to cooperate in the other realization of stage game ($G_{(-)}$). That is, choosing when to cooperate may effectively relax the players' incentive constraints in a perturbed game so that cooperation becomes easier to achieve, as compared to the original non-perturbed game in which such a strategic choice is not available.[7] To the best of our knowledge, this is the first paper which sheds light on the positive aspect of payoff perturbations in repeated games by showing that they can be good for cooperation.[8]

In the next section, we introduce the formal model and investigate the above issues in detail.

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[5] We would like to thank Ichiro Obara for showing us this intuitive example and for pointing out the connection between our model and the multimarket model introduced by Bernheim and Whinston (1990).

[6] For arbitrary $\epsilon \in (0, 3)$, these cutoffs are given by

$$\underline{\delta}_{(+)} = \frac{6+2\epsilon}{9+2\epsilon}, \underline{\delta}_{(-)} = \frac{6-2\epsilon}{9-2\epsilon}, \overline{\delta}_{(+)} = \frac{3+\epsilon}{6+\epsilon}, \overline{\delta}_{(-)} = \frac{3-\epsilon}{6-\epsilon}.$$

[7] The difference between a perturbed and a non-perturbed game discussed here is similar to the one between a single market and a multi-market contact analyzed in Bernheim and Whinston (1990).

[8] Fujiwara-Greve and Yasuda (2009) introduce outside options into the repeated Prisoner's Dilemma, and show that increasing the volatility of option values may enhance cooperation.

3 Model and Result

3.1 Model

We consider a repeated Prisoner's Dilemma with payoff perturbations denoted by Γ. Suppose two players 1 and 2 engage in an infinitely repeated Prisoner's Dilemma game whose stage game is randomly selected from two different games in each period in the following way. At the beginning of each period, stage game G_1 is chosen with probability p, and game G_2 realizes with the rest of probability, $1-p$. Assume that both players observe the realization of each stage game prior to choosing their actions in that period. Payoffs in stage game G_i, $i = 1, 2$ are given in the next table.

$G_i :=$

	C	D
C	c_i, c_i	ℓ_i, g_i
D	g_i, ℓ_i	d_i, d_i

We impose the regularity assumptions $c_i < g_i$, $\ell_i < d_i$, and $d_i < g_i$ $\forall i$. The first two inequalities guarantee that D becomes a dominant strategy in both stage games. The last inequality combined with the first two implies that d_i is a min-max payoff in G_i, $i = 1, 2$. We do *not* necessarily require each realized stage game to be a Prisoner's Dilemma. For instance, we do not exclude the case in which $c_i < d_i$, i.e., mutual cooperation yields strictly *lower* payoff than mutual deviation does. Instead, we require the average of these two stage game payoffs satisfy the conditions of Prisoner's Dilemma. To be precise, we assume $g > c > d > \ell, 2c > g + \ell > 2d$ where $x(= c, d, g, \ell)$ is the "mean payoff" defined as

$$x := px_1 + (1-p)x_2. \tag{2}$$

Let us denote this non-perturbed counterpart of our perturbed stage games as G_0 and its corresponding infinite repeated game as Γ_0. That is, Γ_0 is an ordinary infinitely repeated Prisoner's Dilemma without payoff perturbations.

$G_0 :=$

	C	D
C	c, c	ℓ, g
D	g, ℓ	d, d

A common discount factor $\delta \in (0,1)$ is given, and player i tries to maximize the expected average payoff U_i defined as

$$U_i := (1-\delta)E[\sum_{t=1}^{\infty}\delta^{t-1}u_i^t(s_i^t, s_{-i}^t)], \quad i = 1,2,$$

where u_i^t and s_i^t denote i's stage game payoff and strategy in period t. In what follows, we investigate subgame perfect equilibria of Γ and Γ_0. Let $\underline{\delta}(\Gamma)$ (resp. $\underline{\delta}(\Gamma_0)$) be the minimum discount factor that sustains (C, C) *at least once* in a subgame perfect equilibrium of Γ (resp. Γ_0). Similarly, we define $\overline{\delta}(\Gamma)$ (resp. $\overline{\delta}(\Gamma_0)$) as the minimum discount factor that sustains (C, C) in *every period*. By definition, $\overline{\delta}(\Gamma) \geq \underline{\delta}(\Gamma)$ must hold. Note that in Γ_0 cooperation can be achieved at least once if and only if it is sustained in every period, since Γ_0 is an ordinary repeated Prisoner's Dilemma. So, we denote by δ_0 ($:= \underline{\delta}(\Gamma_0) = \overline{\delta}(\Gamma_0)$) the minimum discount factor that sustains (C, C) at least once (also in every period) in a subgame perfect equilibrium of Γ_0.

3.2 Main Result

We are now ready to show our main result.

Theorem 1 *(i) $\overline{\delta}(\Gamma) \geq \delta_0$ always holds. Moreover, this inequality is strict if and only if $g_1 - c_1 \neq g_2 - c_2$.*

(ii) $\underline{\delta}(\Gamma) < \delta_0$ if and only if

$$\min\{\frac{g_1 - c_1}{g_1 - \{(1-p)c_1 + pd_1\}}, \frac{g_2 - c_2}{g_2 - \{pc_2 + (1-p)d_2\}}\} < \frac{g-c}{g-d}.$$

Proof. (i) Let us first consider Γ_0. Since Γ_0 is an ordinary repeated Prisoner's Dilemma, (C, C) is sustained (at least once) in an equilibrium if and only if

$$\begin{aligned} & c \geqq (1-\delta)g + \delta d \\ \Longleftrightarrow \quad & \delta \geqq \frac{g-c}{g-d} =: \delta_0. \end{aligned}$$

In Γ, (C, C) is sustained in both G_1 and G_2 if and only if

$$(1-\delta)c_i + \delta c \geqq (1-\delta)g_i + \delta d \;\; \text{for } i = 1,2. \tag{3}$$

Multiplying both sides by p and $(1-p)$ respectively, we obtain

$$p[(1-\delta)c_1 + \delta c] \geqq p[(1-\delta)g_1 + \delta d], \text{ and}$$
$$(1-p)[(1-\delta)c_2 + \delta c] \geqq (1-p)[(1-\delta)g_2 + \delta d].$$

Adding each side gives us

$$c \geqq (1-\delta)g + \delta d,$$

which proves the first part of (i). For the second part, note that (3) is written as

$$\delta \geq \frac{g_i - c_i}{g_i - c_i + c - d} \quad \text{for } i = 1, 2. \tag{4}$$

It is clear that the right hand side takes different value across different stage game if and only if $g_1 - c_1 \neq g_2 - c_2$. This means that for given δ, if (3) is satisfied for $i = 1, 2$ and $g_1 - c_1 \neq g_2 - c_2$, then the condition holds with strict inequality in at least one of the stage games, which eventually implies

$$c > (1-\delta)g + \delta d \Leftrightarrow \delta > \delta_0.$$

(ii) Alternatively, (C, C) can be played in only one type of the two possible stage games. Namely, an equilibrium play that (C, C) in G_1 and (D, D) in G_2 is sustained if and only if

$$\begin{aligned}
&(1-\delta)c_1 + \delta\{pc_1 + (1-p)d_2\} \geqq (1-\delta)g_1 + \delta d \\
&\Leftrightarrow \delta p(c_1 - d_1) \geqq (1-\delta)(g_1 - c_1) \\
&\Leftrightarrow \delta \geqq \frac{g_1 - c_1}{g_1 - \{(1-p)c_1 + pd_1\}} := \underline{\delta}_1(\Gamma),
\end{aligned}$$

where we use the property that $d = pd_1 + (1-p)d_2$ from the first to the second line. Similarly, (C, C) in G_2 and (D, D) in G_1 is sustained if and only if

$$\begin{aligned}
&(1-\delta)c_2 + \delta\{(1-p)c_2 + pd_1\} \geqq (1-\delta)g_2 + \delta d \\
&\Leftrightarrow \delta(1-p)(c_2 - d_2) \geqq (1-\delta)(g_2 - c_2) \\
&\Leftrightarrow \delta \geqq \frac{g_2 - c_2}{g_2 - \{pc_2 + (1-p)d_2\}} := \underline{\delta}_2(\Gamma).
\end{aligned}$$

Thus, $\underline{\delta}(\Gamma) = \min\{\underline{\delta}_1(\Gamma), \underline{\delta}_2(\Gamma)\}$, which establishes the theorem. ■

The above theorem shows that a perturbation makes cooperation easier only if players can *selectively* cooperate in a perturbed repeated game. To be more precise, the first part claims that when (C, C) can be sustained in *both* G_1 and G_2 in an equilibrium under some discount factor δ, then (C, C) is also sustained in Γ_0 under the same δ. In other words, whenever players can *fully* cooperate in a perturbed game, cooperation can also be sustained in the corresponding non-perturbed game. Furthermore, achieving the full cooperation in Γ is strictly more difficult than in Γ_0 when the deviation gains are strictly different across stage games.

The second part provides a necessary and sufficient condition for the selective cooperation being *strictly* easier in a perturbed game than in a non-perturbed game. Note that the condition tends to be easily satisfied when the deviation gain are volatile across stage games in Γ_0. In the next section, we apply these results and examine two specific classes of perturbed games.

4 Applications

In this section, applying Theorem 1, we investigate two specific perturbed games, 1) linearly perturbed (affine transformed) games, and 2) public good games with perturbed valuations.

4.1 Linearly Perturbed Games

Assume that payoffs in one stage game are linear (affine) transformations of the payoffs in another stage game. That is, for $x_i = c_i, d_i, g_i, l_i$, $i = 1, 2$,

$$x_2 = \alpha x_1 + \beta \text{ for } \alpha > 0. \tag{5}$$

Then, the mean payoff $x(= c, d, g, l)$ can be written as

$$\begin{aligned} x &= px_1 + (1-p)x_2 \\ &= \{p + (1-p)\alpha\}x_1 + (1-p)\beta. \end{aligned} \tag{6}$$

When $\beta = 0$ payoffs vary proportionally across stage games. Many models fall into this proportionally perturbed games. For instance, in repeated duopoly (oligopoly, in general) games with linear demand shifts investigated by Rotemberg and Saloner (1986) and others, profits under high demand are proportionally increased from those under low demand. Repeated games with fluctuated discount factors examined by Dal Bó (2007) have a similar structure as well. In his model, changing discount factors proportionally increases or decreases the continuation payoffs starting from the next period compared to the current period payoffs.

The next proposition shows that, under these linearly perturbed games, a payoff perturbation never makes cooperation easier.

Proposition 1 *In any linearly perturbed games, $\underline{\delta}(\Gamma) \geq \delta_0$. Moreover, this inequality is strict if and only if $\alpha \neq 1$.*

To show Proposition 1, let us first establish the following lemma.

Lemma 1 *In any linearly perturbed games, $c > d \Leftrightarrow c_1 > d_1 \Leftrightarrow c_2 > d_2$.*

Proof. By (5),

$$c_2 - d_2 = \alpha(c_1 - d_1).$$

Since $\alpha > 0$, $c_1 > d_1$ if and only if $c_2 > d_2$. By (6),

$$c - d = \{p + (1 - p)\alpha\}(c_1 - d_1).$$

Since $p + (1 - p)\alpha > 0$, $c > d$ if and only if $c_1 - d_1$, which concludes the proof. ■

Now we show Proposition 1.

Proof of Proposition 1. By (5),

$$\frac{g_2 - c_2}{g_2 - d_2} = \frac{\alpha g_1 + \beta - (\alpha c_1 + \beta)}{\alpha g_1 + \beta - (\alpha c_1 + \beta)} = \frac{g_1 - c_1}{g_1 - d_1}.$$

By (6),

$$\delta_0 =: \frac{g-c}{g-d} = \frac{\{p+(1-p)\alpha\}(g_1-c_1)}{\{p+(1-p)\alpha\}(g_1-d_1)}$$
$$= \frac{g_1-c_1}{g_1-d_1}(= \frac{g_2-c_2}{g_2-d_2}).$$

Now, by Theorem 1-(ii), it is enough to show that $\underline{\delta_i}(\Gamma) > \frac{g_i-c_i}{g_i-d_i}$, for $i=1,2$.

$$\underline{\delta_1}(\Gamma) =: \frac{g_1-c_1}{g_1-\{(1-p)c_1+pd_1\}}$$
$$> \frac{g_1-c_1}{g_1-d_1} \text{ (since } c_1 > d_1 \text{ by Lemma 1).}$$

By the same argument, we can show $\underline{\delta_2}(\Gamma) > \frac{g_2-c_2}{g_2-d_2}$, which concludes the proof of the first part. For the second part, we know from Theorem 1-(i) that $\overline{\delta}(\Gamma) > \delta_0$ if and only if

$$(g_1-c_1)-(g_2-c_2) \neq 0 \Leftrightarrow (1-\alpha)(g_1-c_1) \neq 0.$$

Since $(g_1-c_1) > 0$ by assumption, the condition holds if and only if $\alpha \neq 1$. ■

Proposition 1 implies that, as long as we consider linearly perturbed games, the shared view in the literature is indeed correct since the payoff perturbation never makes cooperation easier to sustain. However, this claim does not necessarily hold in general non-linearly perturbed games including the next application.

4.2 Public Good Games

Assume that each player obtains a public good when at least one player chooses to contribute (action C). A public good's valuation takes either v_1 or v_2, with probability p and $1-p$ respectively. The total cost of producing a public good in each period is $2c$, which is equally shared by players who choose C. Therefore, each stage game G_i, $i=1,2$, can be expressed by the following payoff matrix.

$G_i :=$

	C	D
C	v_i-c, v_i-c	v_i-2c, v_i
D	v_i, v_i-2c	$0, 0$

We do not necessarily require that each stage game must be Prisoner's Dilemma, but assume that the non-perturbed counterpart is Prisoner's Dilemma. That is, $v - c > 0 > v - 2c$ where $v = pv_1 + (1-p)v_2$. The next proposition shows a necessary and sufficient condition such that a payoff perturbation makes cooperation (public good provision) easier.

Proposition 2 *In public good games (with perturbed valuations),* $\underline{\delta}(\Gamma) < \delta_0$ *if and only if* $c > \min\{v_1, v_2\}$.

Proof. By Theorem 1-(ii), $\underline{\delta}(\Gamma) < \delta_0$ if and only if

$$\min\{\frac{c}{pv_1 + (1-p)c}, \frac{c}{(1-p)v_2 - pc}\} < \frac{c}{pv_1 + (1-p)v_2}.$$

Then, $\frac{c}{pv_1+(1-p)c} < \frac{c}{pv_1+(1-p)v_2}$ if and only if $c > v_2$. Similarly, $\frac{c}{(1-p)v_2-pc} < \frac{c}{pv_1+(1-p)v_2}$ if and only if $c > v_1$, which concludes the proof. ■

An interpretation of this model is that players have small doubt about the efficiency of cooperation (producing a public good); one of the stage games, say G_1 has the property that $v_1 < c$. The theorem shows that such small doubt makes it easier to cooperate in G_2, when they are assured that cooperation is efficient, while they can happily defect in G_1. This is a new insight in repeated games with some uncertainty.

5 Conclusion

In this paper, we show how payoff perturbations affect sustainability of cooperation in the repeated Prisoner's Dilemma. Specifically, we provide the necessary and sufficient condition such that a payoff perturbation strictly *reduces* the minimum discount factor to sustain mutual cooperation. Although we assume a binary and i.i.d. distribution on stage games, one can straightforwardly extend our analyses to the cases in which there are more than two possible stage games or there is some (exogenous) correlation among stage games. In these general models, calculating the exact condition inevitably becomes more complicated than in our simple model. However, we believe that the basic insight remains to be unchanged. Namely, a perturbation makes cooperation easier only if players can *selectively* cooperate in some of the possible stage games.

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