The character varieties of Montesinos knots of Kinoshita-Terasaka type

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1. Motivations

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2. Montesinos knots of Kinoshita-Terasaka type

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3. Main result and a sketch of its proof

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4. Some considerations in characteristic p>2

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The computational complexity involved requires that theoretical ways to study and describe them are explored.

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Standard components for a hyperbolic (Montesinos) knot:

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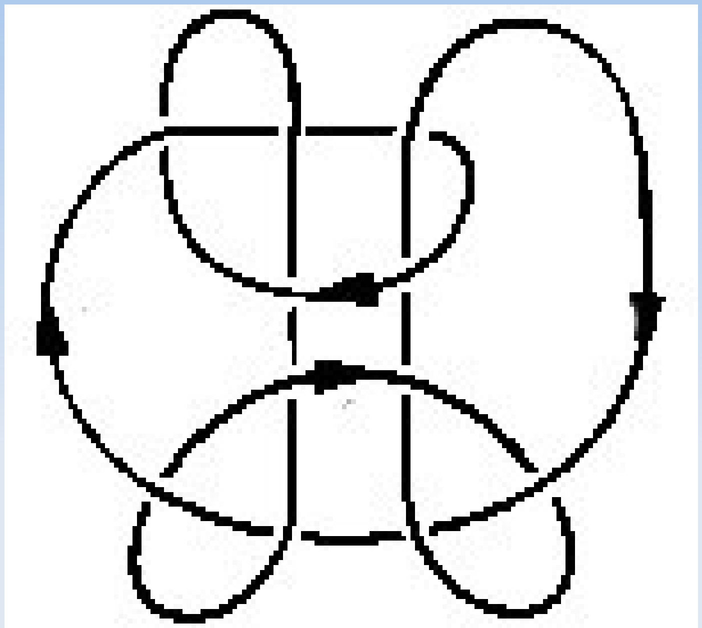
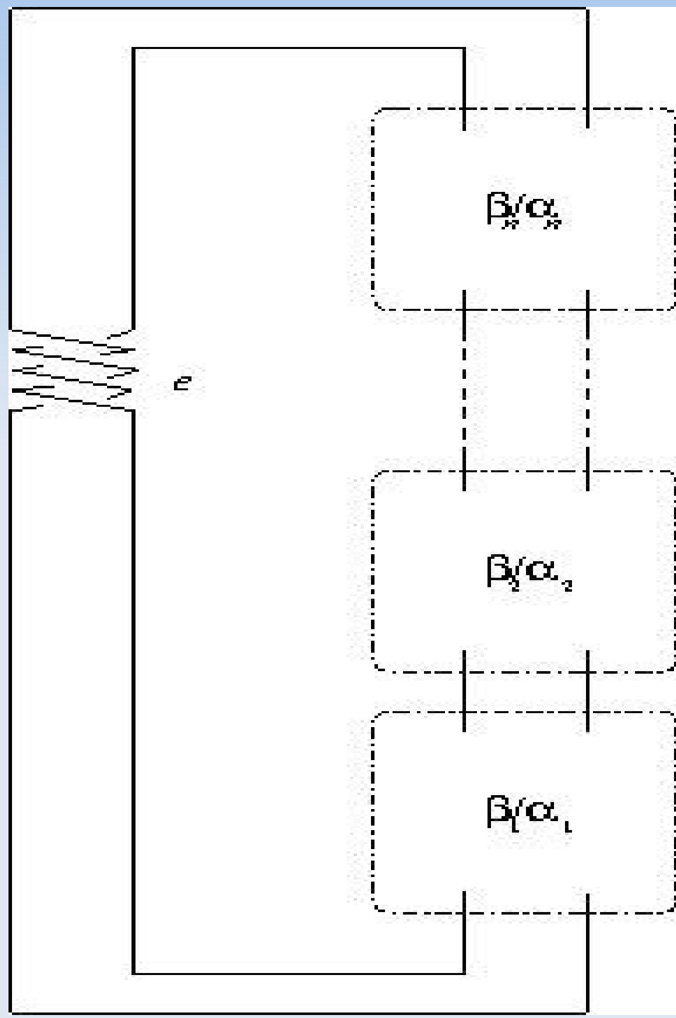
2. For certain pretzel knots: they are due to “symmetry” (Mattman's *r-components*)

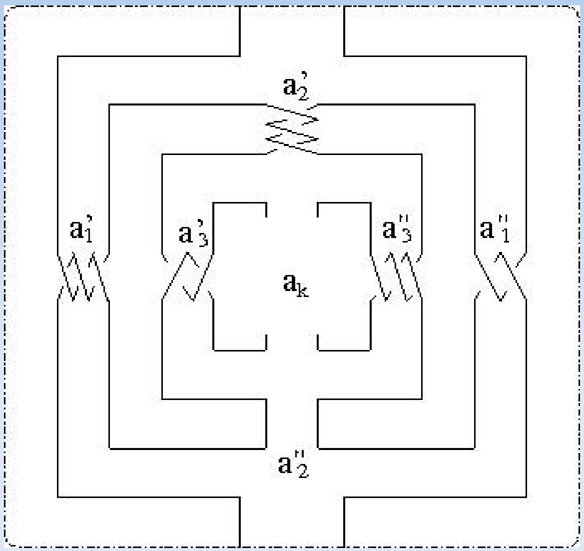
2. Montesinos knots of Kinoshita-Terasaka type: Riley's example



Riley studied this Montesinos knot and found that it admits several parabolic representations into PSL(2,p)

2. Montesinos knots of Kinoshita-Terasaka type: arbitrary Montesinos links





A Montesinos link (left) and a rational tangle (right)

Orientations of the two vertical arcs on the far left can be the same or opposite.

2. Montesinos knots of Kinoshita-Terasaka type: definition and properties

Definition:

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These Montesinos knots are "closely related" to composite knots whose summands are 2-bridge knots.

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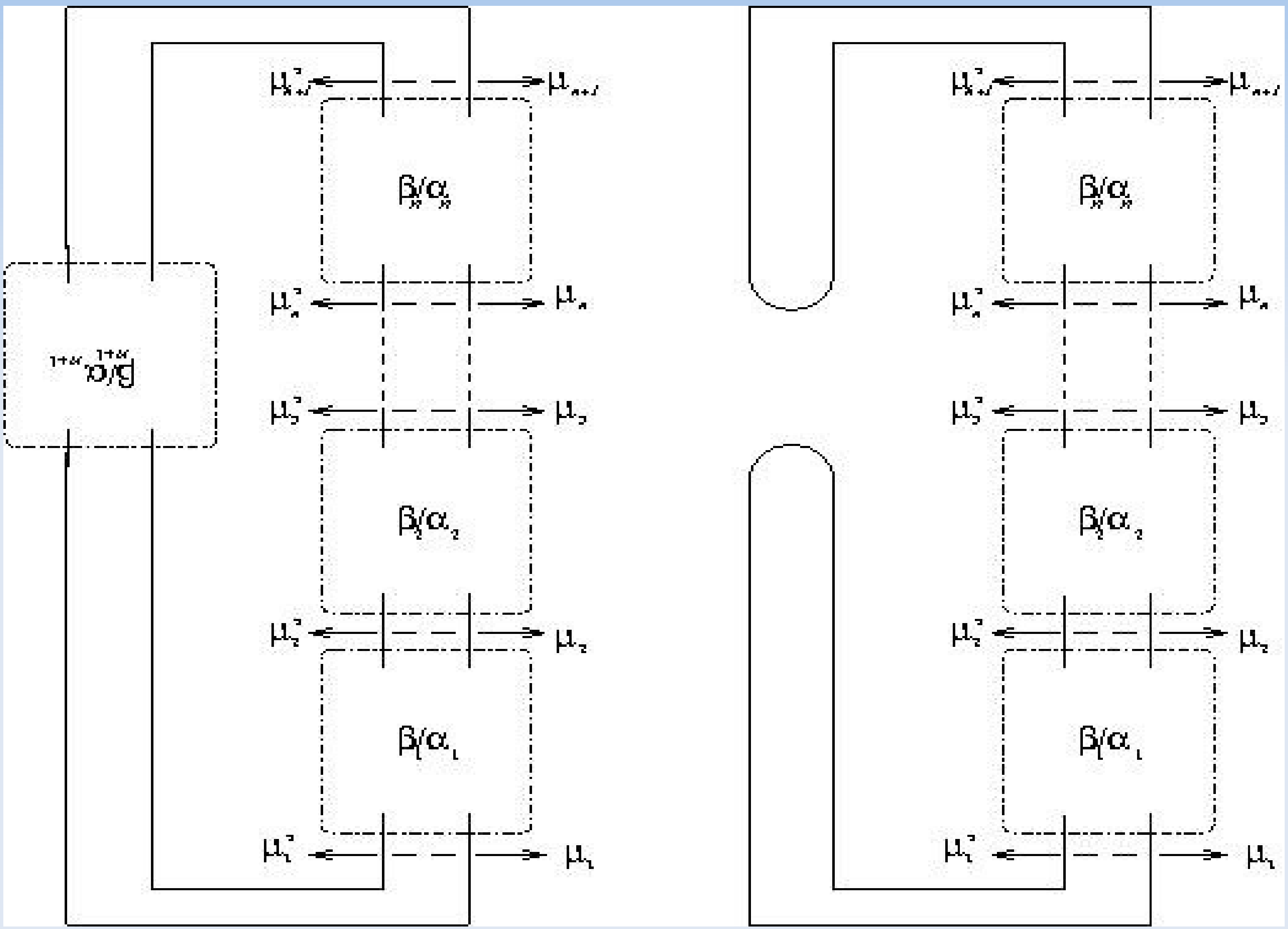

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Composite knots have lots of representations obtained by *bending*.

3. Main result: statement

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2. At least one irreducible component of dimension (at least) n-2 of irreducible characters on which the trace of the meridian is not constant.

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3. There are non standard irreducible components of dimension d, for each d=1,...,n-2 (uses surjections between knot groups) and again their number can be arbitrarily large.

4. There might be other non standard components.

Sketch of proof (parabolic component):

1. Construct irreducible parabolic representations for the associated composite knot $K'=B_1\#...\#B_n$.

Choose $\rho_i : \pi_1(B_i) \to SL(2,\mathbf{C})$ parabolic irrep (e.g. holonomy)

Build $\rho=\rho_1 * a_1\rho_2 a_1^{-1}*...*a_{n-1}\rho_n a_{n-1}^{-1}$ a parabolic irrep for K' (here a_i is any element belonging to the centraliser of $\rho_i(\mu_i)$ in $PSL(2,\mathbf{C})$)

This gives an (n-1) parameter family of parabolic irreps for K' up to conjugacy.

Sketch of proof (parabolic component):

2. Show that the family of representations just constructed meets the variety of representations of Γ, thus giving representations of the Montesinos knot K.

It is sufficient to choose a_{n-1} so that the fixed point on $\mathbf{CP}^1$ of $\rho_1(\mu_1)$ and $a_{n-1}\rho_n a_{n-1}^{-1}(\mu_{n+1})$ are the same: two parabolic elements commute if and only if they have the same fixed point on $\mathbf{CP}^1$.

This way one loses one degree of freedom at the most.

Sketch of proof (second component):

Same strategy as before but

1. In principle one more degree of freedom (the trace of the meridian is non constant);

2. Two non parabolic elements commute if and only if they have the same axis (i.e. they fix the same two points of CP^1): one needs to conjugate twice to make sure that this is indeed the case (the argument uses cross-ratios).

4. Characteristic p: more motivations

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For almost every prime p the character variety in characteristic p “looks exactly like” the one in characteristic 0.

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Why bother?

One can use the character variety to detect essential surfaces (Culler-Shalen) but not all of them can be detected in characteristic 0 (Schanuel-Zhang).

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Hope: (extremely hard)

Can one detect them by looking at character varieties in characteristic p, where p is a ramified prime?

Proposition (P-Porti):

Consider $\Gamma_p = \Gamma/\langle \mu^p \rangle$. For almost every prime $p>2$ the character variety $X(\Gamma_p)$ of Γ_p ramifies at p, in the sense that its dimension is at least n-2 while its expected dimension is at most n-3.

4. Characteristic p: a toy example

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Remark:

The extra ideal points should correspond to Conway spheres.

Proof:

1. $X(\Gamma_p) = X(\Gamma) \cap (\cup_k$ {trace of meridian $= 2\cos(k\pi/p)$})

So, for almost all prime characteristics >2, dim $X(\Gamma_p) \leq n-3$.

Proof:

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So, for almost all prime characteristics >2, dim $X(\Gamma_p) \leq n-3$.

2. In characteristic p>2 we have $X(\Gamma_p)$=Xpar(Γ) and we know that in characteristic 0 dim Xpar(Γ) $\geq n-2$.