Accounting for persistence and volatility of good-level real exchange rates: the role of sticky information

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Motivation: Law-of-One-Price (LOP) deviation

- Like PPP, the speed of adjustment toward a long-run LOP level is measured by estimating $\alpha_j$ of

$$q^j_t = \alpha_j q^j_{t-1} + \nu^j_t.$$ 

$q^j_t$: (log) real exchange rate for good $j$

- Kehoe and Midrigan (2007, KM) proved that the Calvo sticky price model implies

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where $\lambda_j$: the probability of no price change (Calvo parameter, degree of price stickiness)
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KM’s findings on persistence and volatility

- Using recent micro studies, $\lambda_j$ is observable. KM find the following two puzzles:

1. If the model is correct, $\alpha_j = \lambda_j$. However,

   $\hat{\alpha}_j \gg \lambda_j$ (Persistence puzzle)

2. If the model is correct, it will fully explain volatility. However,

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Contributions of this paper

1. Confirm KM’s findings with highly disaggregated panel data
   - Crucini and Shintani’s (2007) data
   - Highly disaggregated data with 165 goods. (KM: 66 goods)
   - Panel data of good-level RER between cities in US and Canada. (KM: time series)

2. Propose a model to solve the two puzzles.
   - Integrate sticky information with the standard Calvo sticky price model
   - Add sticky information by Mankiw and Reis (2002)
   - We call the model ‘dual stickiness’ model.
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Intuition: Why can dual stickiness model solve the puzzles?

1. Persistence Puzzle
   - Even if price adjustment is very fast, good-price adjustment can be slow due to information stickiness ($\omega_j \uparrow$).
   - Persistence $\uparrow$.

2. Volatility Puzzle
   - Even if price adjustment is very fast, good-prices can be almost unaltered due to information stickiness ($\omega_j \uparrow$).
   - RER keeps track of volatile nominal exchange rate.
   - Volatility $\uparrow$. 
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Overview of the model: Two-country general equilibrium model

- **Households**
  - $U(c_t, n_t) = \log c_t - \chi n_t$ with cash-in-advance constraint.

- **Firms producing good $j$**
  - sell goods in monopolistically competitive domestic and foreign local markets.
  - set price for good $j$ in each local market (local currency pricing)
  - face two constraints:
    1. cannot change price with prob. $\lambda_j$.
    2. cannot update info. with prob. $\omega_j$.

- **Governments**
  - control money growth rates. AR(1)
Sketch of dual stickiness model (Calvo model with info. delay)

- Dual stickiness model has two nominal rigidities. (price & information)
- Under sticky prices, the domestic optimal price is

\[
\hat{P}_{H,t}^j = (1 - \beta \lambda_j) \sum_{h=0}^{\infty} (\beta \lambda_j)^h \mathbb{E}_t(\hat{W}_{t+h})
\]

\[
\hat{P}_{F,t}^j = (1 - \beta \lambda_j) \sum_{h=0}^{\infty} (\beta \lambda_j)^h \mathbb{E}_t(\hat{S}_{t+h} + \hat{W}^*_t)
\]

\(\hat{W}_t(\hat{W}^*_t)\): nominal wages in the home (foreign) country. \(S_t\): nominal exchange rate.
Sketch of dual stickiness model (2)

- Sticky information: a fraction of firms cannot have the newest information
  - Prob. $\omega_j$: use info. set they last updated $\Rightarrow E_{t-k} \hat{P}^j_{H,t}$,
    $E_{t-k} \hat{P}^j_{F,t}$
  - Prob. $1 - \omega_j$: use the newest info. set $\Rightarrow \hat{P}^j_{H,t}, \hat{P}^j_{F,t}$
  - The index for newly set prices $\hat{X}^j_t$ collects these prices.
- Due to Calvo assumption,
  \[
  \hat{P}^j_t = \lambda_j \hat{P}^j_{t-1} + (1 - \lambda_j) \hat{X}^j_t
  \]
- Good-level RER is given by $q^j_t = \hat{S}_t + \hat{P}^j_{t} - \hat{P}^j_t$. 
Overview of Data: *Worldwide Cost of Living Survey*

- Prices from 13 US $\times$ 4 CAN cities:
- Total of 52 cross-border city pairs
Persistence Puzzle

1. KM’s benchmark case \((\omega_j = 0)\)
   - Calvo model without info. delay

2. Our dual stickiness model \((\omega_j \geq 0)\)
   - Calvo model with info. delay
Good-level Real Exchange Rates

- Persistence
- Calvo model

Calvo model’s predictions (No information delay)

- We estimate the model with the annual data.
- We have $\lambda_j$: *monthly* infrequency of price change from micro studies.
- Panel version of KM’s benchmark case (i.i.d. money growth)

\[
AR(1) \quad q_{i,t}^j = \lambda_j^{12} q_{i,t-1}^j + u' \tilde{D}_t + \zeta_i^j + \nu_{i,t}^j
\]

- $i$: cross-border city pair (e.g., NY and Toronto)
- If the Calvo model is correct,
- our estimate of AR(1) coef. $\hat{\alpha}_j = \lambda_j^{12}$ with annual data
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Persistence Puzzle: KM’s benchmark case

Calvo model with $\rho = 0$: AR(1)

$\alpha_j = \lambda_j^{12}$

$\lambda_j^{12} = (1 - f_j)^{12}$

Blue line: Calvo model’s prediction
Persistence Puzzle: KM’s benchmark case

\[ \lambda_j^{12} = (1-f_j)^{12} \]

Black dashed line: OLS line from red points.
Good-level Real Exchange Rates
- Persistence
- Dual Stickiness model

Dual stickiness model’s prediction ($\omega_j > 0$)

► Panel version of good-level RER under dual stickiness model

\[
AR(4) \quad q_{i,t}^j = \sum_{r=1}^{4} \psi_r q_{i,t-r} + u^t \tilde{D}_t + \zeta_i^j + \nu_{i,t}^j
\]

► In a general AR($p$) model, a persistence measure is the sum of autoregressive coefficients (SAR).

► If dual stickiness model is correct, $\alpha_j = \sum_{r=1}^{4} \psi_r$ must be

\[
\hat{\alpha}_j = 1 - (1 - \chi_j^{12})(1 - \rho^{12})(1 - \omega_j^{12})(1 - \omega_j^{12} \rho^{12})
\]

So, $\omega_j \uparrow \implies$ persistence $\uparrow$!!
Good-level Real Exchange Rates

Persistence

Dual Stickiness model

**Dual stickiness model’s prediction \((\omega_j > 0)\)**

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So, \(\omega_j \uparrow \iff \text{persistence} \uparrow!!\)
Persistence Puzzle: dual stickiness model

\[ \alpha_j = 1 - (1-\lambda_j^{12})(1-\rho^{12})(1-\omega_j^{12})(1-\omega_j^{12} \rho^{12}) \]

Purple line: No info. delay, Green line: 50 month info. delay
Persistence Puzzle: dual stickiness model

Black dashed line: OLS line from red points.
How much should be information stickiness needed to explain persistence?

$$\text{median}(\frac{\alpha_j^{\text{theory}}}{\alpha_j^{\text{data}}})$$

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- Our model can explain 100% of the median of persistence if
- $\omega = 0.93$ with Bils and Klenow’s data
- $(\omega = 0.89$ with Nakamura and Steinsson’s data)
- Avg. duration between info. updates is 14 and 9.5 months, resp.
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2. Our dual stickiness model \((\omega_j \geq 0)\)
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Calvo model’s predictions (No information delay)

We compute std ratio of the theory to the data in terms of a time varying component.

$$median \left[ \frac{std(q_{i,t}^{j,\text{theory}})}{std(q_{i,t}^{j,\text{data}})} \right] = 0.13 \quad \text{from Bils & Klenow}$$
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Avg. duration btwn info. updates is 17 and 12 months.
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Conclusion

1. The Kehoe and Midrigan’s findings are robust to the use of highly disaggregate panel data.
   - Calvo model fails to explain persistence and volatility of good-level RER.
2. One possible explanation is the dual stickiness model.
   - The dual stickiness model solves persistence and volatility puzzles.
   - Implied durations between info. updates are comparable to estimates of previous studies.
Reconciling monthly models with annual data

- e.g., the monthly Calvo model with iid money growth:
  \[ q_{it}^j = \lambda_j q_{it-1}^j + \lambda_j \eta_t + \tilde{\zeta}_i^j. \]
  where \( \eta_t \): difference between money growth rates of two countries.

- Annual transformation
  \[
  q_{it}^j = \lambda_j q_{it-1}^j + \lambda_j \eta_t + \tilde{\zeta}_i^j \\
  = \lambda_j^2 q_{it-2}^j + \lambda_j \eta_t + \lambda_j^2 \eta_{t-1} + (1 + \lambda_j) \tilde{\zeta}_i^j \\
  \ldots \\
  = \lambda_j^{12} q_{it-12}^j + \lambda_j \sum_{r=0}^{11} \lambda^j \eta_{t-r} + \sum_{r=0}^{11} \lambda^j \tilde{\zeta}_i^j \\
  \underbrace{u' \tilde{D}_t}_{- \zeta_i^j} \\
  \]
How much should be information stickiness needed to explain persistence? Good-specific $\omega_j$ rather than $\omega$

Distribution of information delay implied by persistence

$\text{median} = 12.9$ months (Bils and Klenow), 8.9 months (Nakamura and Steinsson)
How much should be information stickiness needed to explain volatility? Good-specific $\omega_j$ rather than $\omega$

median = 16.6 months (Bils and Klenow), 11.9 months (Nakamura and Steinsson)
KM find pricing complementarities do not help for solving puzzles

- KM also consider pricing complementarities.
- Production function of firms

\[ y = m^a n^{1-a}, \quad m = \left( \int m_j^{\theta-1} dj \right)^{\theta-1}, \quad m_j = \left( \int m_{j,z}^{\theta-1} dz \right)^{\theta-1} \]

- Input costs of firms in all good \( j \) move together.
- They set an extreme value of \( a = 0.99 \). They find....
- The persistence can be explained somewhat better than otherwise.
- The volatility cannot be explained by pricing complementarities.