Economic Growth with Locked-in Childbirth: From Under- to Over-Investment in Education*

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Abstract

This research argues that a quantitative change in the irreversible fertility decisions triggers the transition from under- to over-investment in education in the growth process. In the early stage of economic development, parents place greater importance on the quantity than the quality of children. The resulting large sunk cost of child rearing makes unaffordable education investment for children who are unexpectedly competent. Hence, a population control policy that reduces the sunk cost advances the growth process and prevents the emergence of a poverty trap. In the later stage, by contrast, the quantity of children is locked into a small size, leading to over-investment in education. An expansion of child care subsidies will promote the accumulation of aggregate human capital and improve the welfare of future generations.

Keywords: Over-investment, Under-investment; Education; Fertility; Growth.

JEL Classification: D10; J13; J24; O15.

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1 Introduction

It is widely recognized that investment in human capital is one of the steady means to improve the welfare of individuals and of an economy as a whole. Despite the recognition, under-investment in education is widespread over developing countries. A large fraction of households cannot afford the expenses for schooling in the presence of borrowing constraints. On the other hand, the recognition brought the issue of over-investment in higher education on developed countries. Students of these countries do not necessarily reward the financial aid from their parents.[1] Using the data on UK graduates between 1985 to 1990, an empirical analysis by Chevalier (2003) calculated that the wage loss of over-educated workers was amount to 22% to 26%.[2]

A plausible conjecture from these facts is that the developed economies went through a transition from under- to over-investment in education in the growth process. It appears, however, that no single theory has shed light on the underlying mechanism of the phenomenon. Existing theories are only partially satisfactory in this respect. The literature on inequality and growth, which has flourished since the 1990s, asserts the possibility of under-investment in human capital in the presence of capital market imperfections (cf. Galor and Zeira, 1993; Moav, 2002; Mookherjee and Ray, 2003). By contrast, the other theoretical literature argues that information imperfections, along with market imperfections, may induce precautionary reactions of individuals toward over-investment in human capital (cf. Gould et al., 2001; Aiyagari et al., 2002).[3] None of these studies encompass the aforementioned transition.

Motivated by these observations, this research develops a theory to analyze the optimality of private education, which varies with the process of economic development. It argues that the optimality is determined through the dynamic interaction between education investment and fertility decisions in the growth process. Although imperfect, parents adjust the quantity of children they intend to raise in order to prepare for future education expenses.[4] The resulting accumulation

[1] For the 2009 academic year in Japan, for instance, no less than 39.3 percent of high school dropouts was due to the failure of adaptation to school life and study (the Japanese Ministry of Education, Culture, Sports, Science and Technology. `http://www.mext.go.jp/b_menu/houdou/22/12/1300746.htm`).

[2] In this line of empirical research is Sicherman (1991), who relies on the PSID (Panel Study of Income Dynamics) data for the late 1970s.

[3] Gould et al., (2001) considers the eroding effect of technological progress, which is biased and random across sectors, on human capital. Aiyagari et al. (2002) highlight the lack of insurance markets for ability as well as that of loan markets.

[4] Goldstein et al. (2003, p. 487, Table 2) compare mean personal ideal family size and mean personal expected family size for young women by using the Eurobarometer 2001 survey. They report that the former measure is smaller

of human capital alters technological progress, the return on education investment, and fertility decisions of the next generation. This research also aim to offer some policy implications for welfare improvement, by examining policy reforms that affect the incentive of childbirth.

The growth model presented later features four key elements. First, the educational attainment of children, a determinant of human capital, is the fruit of their own efforts and parental support. Second, there may arise conflict over private education policy within households, because of the difference in their motivations.[5] Children make efforts to acquire skills at the cost of leisure time, whereas parents face the trade-off between the quantity and the quality of their children.[6] Second, childbirth is the irreversible investment in the quantity of children, as proposed by Fraser (2001) and Doepke and Zilibotti (2005).[7] Once determined, the number of children to raise is not adjustable in either directions and hence is interpreted as a sunk cost of child rearing. Third, childbirth is accompanied by idiosyncratic, unexpected ability shocks on children. While these shocks may induce parents to revise their initial education plan, their ex-post reactions may be constrained by the irreversibility of childbirth.

Figure 1 gives insight into the lock-in effects of childbirth on education investment, which are generated by the second and the third elements above.[8] Panel (a) shows optimization by a household whose children are, in fact, more competent than expected. At the time of childbirth, the households does not observe the true ability of children and its ex-ante (before the observation of true ability) optimal choice occurs at $(\bar{n}, e^P)$, where the ex-ante indifference curve is tangent to

than the latter by 0.2 to 0.4 points in major European countries (p. 486). A similar pattern applies to the United States (Hagewen and Morgan, 2005, p. 509, Figure 1). These disparities are consistent with this paper's view that the budget constraint is binding in child rearing; i.e., parents can raise funds for desirable education at the cost of the family size.

[5] There is a strand of literature analyzing child-rearing strategies and parent-child conflict in various contexts. Weinberg (2001) develops a static agency model in which an altruistic parent motivates her child to make efforts by a pecuniary means. The author finds that in the presence of the subsistence level of consumption, the effort level increases with parental income only at low income levels. Akabayashi (2006) develops a dynamic theory that explains child maltreatment by a parent who imperfectly observes the accumulation of the child's human capital. The tough love model of Bhatt and Ogaki (2008) shows that parents leave little transfers to impatient children so that poor consumption in childhood makes them more patient.

[6] See Becker and Lewis (1973) for the formulation of the quantity-quality trade-off faced by parents.

[7] In relation to schooling, a recent study by de la Croix and Doepke (2009) focused on the lock-in effect of fertility decisions on individuals' voting preferences, in accounting for the differences in public education systems across countries. The assumption of perfect irreversibility would be relaxed by dividing the period of childbirth into two so that unexpected ability shock occurs between them. This approach is taken by Iyigun (2000) for different research objectives from the present paper. The author develops a growth model with no uncertainty (i.e., no lock-in effect of childbirth) and demonstrates that the timing of childbearing is delayed by the accumulation of human capital.

[8] The model presented in Figure 1 is not identical to the one introduced in Section 2. In particular, education investment is a discrete choice in the latter model. Figure 1 nonetheless conveys the essence of the lock-in effects of childbirth that are analyzed later.

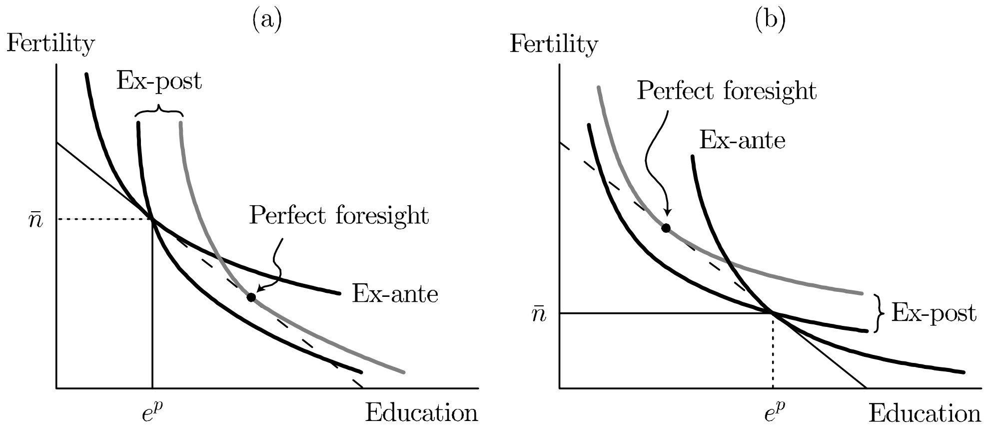

Figure 1. The Lock-in Effects of Childbirth on Education.

the budget line including the broken line. The quantity of children, $\bar{n}$, is locked into a large level in prospect of small education investment indicated by e^P.

In the diagram, "Perfect foresight" indicates the point at which the ex-post indifference curve (i.e., the indifference curve under perfect foresight) is tangent to the original budget line. However, this point is beyond the actual budget constraint, which is kinked at $\bar{n}$. That is, because the household observes the true ability after childbirth, it is not possible to reduce the quantity of children to achieve any higher education level than e^P. The ex-post optimal decision is therefore to carry out the initial education plan, e^P, with no change in the family size. The opposite case is explained by panel (b), in which children are in fact less competent than expected.

Taking the two types of lock-in effects into account, the theory developed below demonstrates the following scenario of economic development. In the early stage of development, where technological progress is sluggish, households place greater importance on the quantity than the quality of children at the time of childbirth. The resulting sunk cost of child rearing, dominant in the household budget, makes unaffordable education investment for children who are unexpectedly competent. Although these children find skill acquisition advantageous, borrowing constraints prevent them from making education loans.[9] In this situation, a policy reform that discourages childbirth mitigates under-investment in education and advances technological progress.

In the later stage of development, by contrast, households place more importance on the quality

[9] Given this result, one may interpret locked-in childbirth as an implicit assumption in the aforementioned literature on inequality, human capital, and growth.

of children at the time of childbirth. Now that the quantity of children is locked into a smaller size in the household budgets, the burden of education expenses is not as heavy as before. As a result, households invest in education as planned unless their children are significantly incompetent. The problem here is that children receiving parental support are not necessarily inclined to engage in skill acquisition. Under the circumstance, an expansion of child care subsidies at the cost of the elderly's welfare, will mitigate over-investment in education with no adverse effect on human capital accumulation. It will also enrich the government by accelerating growth in the taxpaying population, leaving the possibility of welfare improvement of future generations.

The rest of the present paper consists of the following sections. Section 2 describes the structure of the baseline model and considers optimal fertility decisions and individuals' attitudes toward education. The last part of the section reveals the determination of macroeconomic variables. Section 3 demonstrates the transition from under- to over-investment in education as a result of fertility decline induced by exogenous technological progress. Section 4 incorporates endogenous technological progress, which permits the interaction between fertility and education, and then investigates policy reforms that affect the incentive for childbirth. The proofs of mathematical results are given in the Appendix.

2 The Model

The economy has an overlapping-generations structure and operates over an infinite discrete time horizon $t \geq 0$.[10] One single homogeneous good is produced in one sector by using human capital. The formation of human capital is the fruit of the efforts by children as well as education investment by their parents. As a result of parent-child conflict over private education policy, either under- or over-investment in education arises in the growth process.

2.1 Firms

In perfectly competitive environments, producers generate a single homogeneous good by employing human capital (i.e., efficiency unit of labor) with a linear technology. The level of output per worker

[10] This is an extension of the model developed by Galor and Weil (2000), who explore the mechanism underlying the demographic transition in the long-term growth process.

in period t, denoted as y_t, is determined through the production function

$$y_t = A_t H_t / N_t, \tag{1}$$

where A_t, H_t, and N_t are the technology level, the employed amount of aggregate human capital, and the size of the working population, respectively, in period t.

For the sake of simplicity, the price of the final good is normalized to unity. As a result of profit maximization by competitive producers, who are price takers, H_t maximizes the aggregate profit $A_t H_t - w_t H_t$, where w_t is the market wage rate per unit of human capital in period t. In the competitive labor markets considered herein, w_t is adjusted so that the resulting profit is neither negative nor infinity large. It follows that

$$w_t = A_t.$$

Thus, the wage rate w_t increases proportionally with the technology level A_t.

2.2 Households

A new generation is born at the beginning of each period and lives for two periods. Generation t, born in period $t-1$, consists of a continuum of individuals existing on the interval $[0, N_t]$.

2.2.1 Environment

Consider the lifetime of an individual $i \in [0, N_t]$ of generation t. In the first period (childhood), the individual enjoys leisure, l^i_{t-1}, and may also engage in skill acquisition. In the second period (adulthood or parenthood), the individual acquires h^i_t efficiency units of labor and allocates them between child rearing and working. The individual raises n^i_t units of identical children by spending $(\delta + e^i_t)$ efficiency units of labor per child, where $\delta > 0$ and $e^i_t \geq 0$ are the fixed and the education cost, respectively.[11] The remaining labor is supplied to producers to earn wages, that are used up for consumption, c^i_t. It follows that

$$c^i_t = w_t[h^i_t - (\delta + e^i_t)n^i_t]. \tag{2}$$

Utility of an individual i of generation t, u^i_t, depends on leisure in childhood, consumption in adulthood, and the quantity and quality of his/her children. Each of these children, indexed by

[11] Parents may either train their children on their own or hire a teacher from the outside by paying $w_t e^i_t$.

$j(i) \in [0, N_{t+1}]$, acquires $h_{t+1}^{j(i)}$ efficiency units of labor in period $t+1$. Taking these into account, the utility function is given by

$$u_t^i = (1-\beta)\ln l_{t-1}^i + \beta\left\{(1-\alpha)\ln c_t^i + \alpha \ln\left[n_t^i h_{t+1}^{j(i)}\right]\right\}, \tag{3}$$

where $(\alpha, \beta) \in (0,1) \times (0,1)$.

2.2.2 Production of Human Capital

While children may differ in educational attainment and innate ability across households, there is no heterogeneities among siblings. The quantity of efficiency units of labor obtained by a child $j(i)$, born from a parent i in period t, is determined according to the production function

$$h_{t+1}^{j(i)} = h(\epsilon_t^i, a_t^i, g_{t+1}). \tag{4}$$

where $a_t^i \geq 0$ and $\epsilon_t^i \geq 0$ denote the levels of his/her ability and education attainment, respectively, and g_{t+1} is the growth rate of technology between periods t and $t+1$.[12] One of the key assumptions of the model is that ϵ_t^i does not necessarily coincide with the level of parental education investment, e_t^i, which may be wasted by children.

The function h is assumed to satisfy three key properties. First, educational attainment has a discrete impact on the formation of human capital. Children become either educated (skilled) or uneducated (unskilled) labor, depending on whether their educational attainment reach a threshold level $\bar{\epsilon} > 0$. That is, $\forall (a_t^i, g_{t+1}) \in \mathbb{R}_+^2$,[13]

$$h(\epsilon_t^i, a_t^i, g_{t+1}) = \begin{cases} h(0, a_t^i, g_{t+1}) > 0 & \text{if } \epsilon_t^i < \bar{\epsilon}; \\ h(\bar{\epsilon}, a_t^i, g_{t+1}) > 0 & \text{if } \epsilon_t^i \geq \bar{\epsilon}. \end{cases} \tag{5}$$

Second, a higher ability level makes skill acquisition more gainful (i.e., ability-education complementarity).[14] That is,

$$\begin{aligned} &\eta_a(a_t^i, g_{t+1}) > 0, \quad \forall (a_t^i, g_{t+1}) \in \mathbb{R}_{++} \times \mathbb{R}_+; \\ &\eta(0, g_{t+1}) \leq 1 \quad \text{and} \quad \lim_{a_t^i \to \infty} \eta(a_t^i, g_{t+1}) = \infty, \quad \forall g_{t+1} \geq 0. \end{aligned} \tag{6}$$

[12] As for the notation of e_t^i and a_t^i, note that there is no need to use the superscript $j(i)$ instead of i, because children raised by a same parent are identical.

[13] Throughout the present paper, $f_x(x, y)$ denotes the partial derivative of a function f with respect x.

[14] The condition $\eta(0, g_{t+1}) \leq 1$, which is not indispensable, means that education investment does not enhance human capital of children who are significantly incompetent. Because education investment is costly, these children are raised as unskilled labor. On the ground that this is a minor case under the sufficient condition, any uneducated worker is simply referred to as unskilled labor throughout the present paper.

where the function η gives the relative skill level for given ability and the growth rate of technology:

$$\eta(a_t^i, g_{t+1}) \equiv \frac{h(\bar{\epsilon}, a_t^i, g_{t+1})}{h(0, a_t^i, g_{t+1})}. \tag{7}$$

Third, the acceleration of technological progress makes skilled labor more productive than unskilled labor (i.e., skill-biased technological progress). That is,

$$\begin{aligned} &\eta_g(a_t^i, g_{t+1}) > 0, \quad \forall (a_t^i, g_{t+1}) \in \mathbb{R}_+ \times \mathbb{R}_{++}; \\ &\eta(1,0) \leq 1 \quad \text{and} \quad \lim_{g_{t+1}\to\infty} \eta(1, g_{t+1}) = \infty. \end{aligned} \tag{8}$$

As will become evident, the conditions in the second line of Eqs. (6) and (8), which can be relaxed at the cost of exposition, ensure the existence of some critical values for education decisions.

2.2.3 Optimization by Parents

Optimization by parents, who are price takers with perfect foresight, is divided into two steps. At the time of childbirth (ex-ante optimization), parents plan for future education investment in the belief that their newborn children have average ability. After childbirth (ex-post optimization), parents *unexpectedly* find the true ability level of their children and thus may be inclined to alter their initial plans.

Ex-Ante Optimization: Childbirth and Education Planning An individual i of generation t (a parent i in period t) decides the quantity of children, n_t^i, and the planned level of education investment, e_t^p. This decision making builds on the belief that these children will have average ability, whose level is normalized to unity, and fully make use of parental education support (i.e., $a_t^i = 1$ and $e_t^p = \epsilon_t^i$). Then, in light of Eqs. (2), (3), and (4),

$$\{n_t^i, e_t^p\} = \arg\max \left\{ (1-\alpha)\ln[h_t^i - (\delta + e_t^p)n_t^i] + \alpha \ln[n_t^i h(e_t^p, 1, g_{t+1})] \right\}, \tag{9}$$

subject to $(n_t^i, e_t^p) \geq 0$. The first-order condition for n_t^i yields

$$n_t^i = \frac{\alpha}{\delta + e_t^p} h_t^i. \tag{10}$$

As will become apparent, the education planning determines the quantity of children, which in turn binds actual education investment.

Substituting Eq. (10) into Eq. (9) reveals that

$$e_t^p = \arg\max \frac{h(e_t^p, 1, g_{t+1})}{\delta + e_t^p},$$

subject to $e_t^p \geq 0$. In view of Eq. (5), e_t^p takes either 0 and $\bar{\epsilon}$, and there is a critical level of g_{t+1}, denoted as $\tilde{g} > 0$, for which parents are indifferent between them; that is,

$$\eta(1, \tilde{g}) = d. \tag{11}$$

where $d \equiv (\delta + \bar{\epsilon})/\delta$. By assuming for simplicity that the lower education level is chosen at $\tilde{g}$,

$$\begin{aligned} e_t^p &= \begin{cases} 0 & \text{if } g_{t+1} \leq \tilde{g}; \\ \bar{\epsilon} & \text{if } g_{t+1} > \tilde{g}. \end{cases} \\ &\equiv e^p(g_{t+1}). \end{aligned} \tag{12}$$

Hence, the planned level of education investment is identical among the members of each generation.

Ex-Post Optimization: Education Investment After having babies all at once, the adult individual i in period t unexpectedly observes the ability level of his/her children, a_t^i. In reconsidering the education plan, two important assumptions are imposed. First, regardless of the level of observed ability, the individual continues to believe that these children will fully make use of parental education support (i.e., $e_t^i = \epsilon_t^i$). Second, the predetermined fertility choice is unadjustable. At this timing, it is not possible to change the quantity of children in either direction. Namely, the individual takes n_t^i in Eq. (10) as given in deciding the *actual* level of education investment, e_t^i. Thus in light of Eqs. (9) and (12),

$$\begin{aligned} e_t^i &= \arg\max \left\{ (1-\alpha) \ln \left[1 - (\delta + e_t^i) \frac{\alpha}{\delta + e_t^p} \right] + \alpha \ln h(e_t^i, a_t^i, g_{t+1}) \right\} \\ &\equiv \arg\max V(e_t^i, a_t^i, g_{t+1}), \end{aligned} \tag{13}$$

subject to $e_t^i \geq 0$.

Note that the term $\alpha/(\delta + e_t^p)$ in Eq. (13) indicates the burden of the locked-in fertility decision on ex-post optimization. If δ is so small that $\alpha(\delta + \bar{\epsilon})/\delta \geq 1$, for example, *any* households choosing $e_t^p = 0$ beforehand cannot afford $e_t^i = \bar{\epsilon}$ to secure a positive amount of consumption. Such a case is beyond the scope of the present paper and thus is excluded by assuming

$$\alpha d < 1, \tag{14}$$

where $d \equiv (\delta + \bar{\epsilon})/\delta > 1$.

Now two points deserve special attention regarding ex-post optimization in Eq. (13). First, the degree of the lock-in effect of childbirth varies with the growth rate of technology through the change in the education plan, e_t^p. Second, there is no income effect on the actual level of education investment, e_t^i. This is because a rise in h_t^i proportionally increases the quantity of children, n_t^i, and thus has no impact on the budget constraint.

Let $\tilde{a}_t$ be a critical ability level for which parents in period t are indifferent between ex-post education decisions. That is,

$$V(0, \tilde{a}_t, g_{t+1}) = V(\bar{\epsilon}, \tilde{a}_t, g_{t+1}). \tag{15}$$

Then, as will become apparent, $\tilde{a}_t$ is given by a single-valued, *noncontinuous*, decreasing function $\tilde{a}(g_{t+1})$ such that

$$\begin{aligned} e_t^i &= \begin{cases} 0 & \text{if } a_t^i \leq \tilde{a}(g_{t+1}); \\ \bar{\epsilon} & \text{if } a_t^i > \tilde{a}(g_{t+1}). \end{cases} \\ &\equiv e(a_t^i, g_{t+1}). \end{aligned} \tag{16}$$

where a_t^i is the ability level of children raised by a parent i in period t. Thus, unlike in the ex-ante case, the ex-post education decision is heterogeneous across households of each generation.

A Benchmark Case: Perfect Foresight Environment As a benchmark case, suppose tentatively that parents have perfect foresight into children's ability, while they believe that their children will not waste parental education support (i.e., $e_t^i = \epsilon_t^i$). Using Eqs. (9) and (10), the preferred (and first-best) level of education investment for a parent i in period t, denoted as e_t^{i*}, is

$$e_t^{i*} = \arg\max \frac{h(e_t^{i*}, a_t^i, g_{t+1})}{\delta + e_t^{i*}}.$$

Note that e_t^{i*} is optimal for maximizing $n_t^i h_{t+1}^{j(i)}$, which is the aggregate amount of human capital produced by a parent i in period t. As will become clear, there is no reason that e_t^{i*} generally coincides with the actual level of education, e_t^i.

Noting Eq. (5) implying that e_t^{i*} takes either 0 and $\bar{\epsilon}$, let a_t^* be a critical ability level for which parents in period t are indifferent between the two education decisions. That is, by using Eq. (7),

$$\eta(a_t^*, g_{t+1}) = d > 1, \tag{17}$$

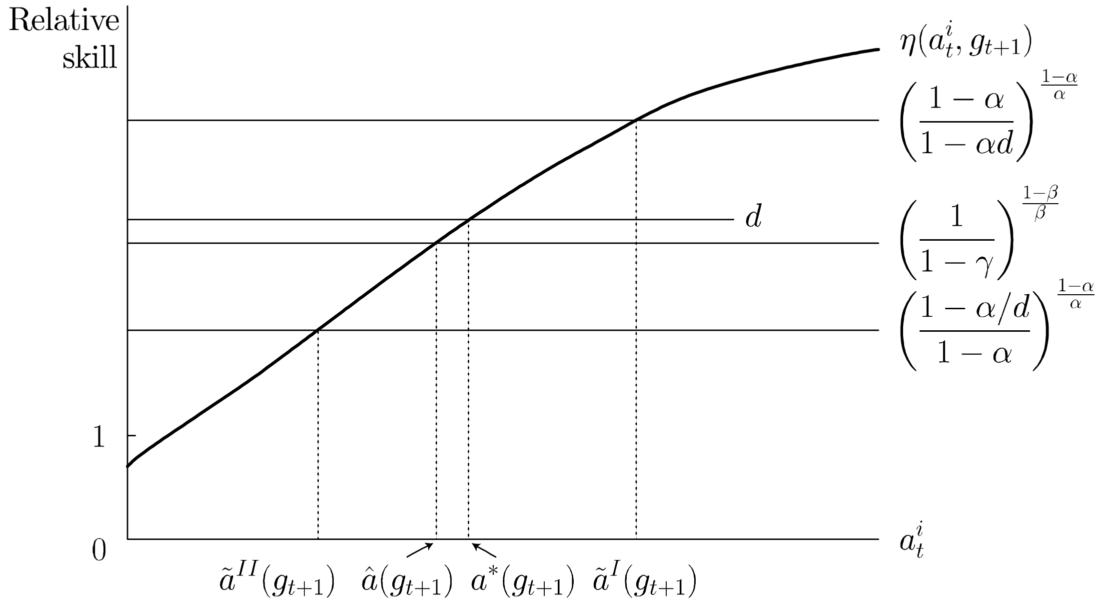

Figure 2. The Determination of the Critical Ability Levels for Education.

where $d \equiv (\delta+\bar{\epsilon})/\delta$ as defined earlier. This condition means that the relative advantage of education investment is equal to the relative disadvantage in the quantity of children.

Figure 2 is useful to understand the determination of a_t^*. The line indicating the value d lies between the top and the third horizontal lines because[15]

$$\frac{1-\alpha}{\alpha}\ln\frac{1-\alpha/d}{1-\alpha} < \ln d < \frac{1-\alpha}{\alpha}\ln\frac{1-\alpha}{1-\alpha d}. \tag{18}$$

It follows that there is a unique ability level for which Eq. (17) is satisfied for any $g_{t+1} \geq 0$. Furthermore, an increase in g_{t+1} shifts the function $\eta(a_t^i, g_{t+1})$ upward, raising the critical ability level. These are the intuition about Lemma 1.

Lemma 1 *There exists a single-valued function* $a_t^* = a^*(g_{t+1}) > 0$ *that satisfies Eq.* (17) *for any* $g_{t+1} \geq 0$. *Furthermore,*

(*a*) $a^{*\prime}(g_{t+1}) < 0\ \forall g_{t+1} > 0$;

(*b*) $a^*(\tilde{g}) = 1$.

[15] Eq. (10) implies that given $g_{t+1} > 0$ and $e_t^p = 0$, an adult individual i in period t obtains more utility by choosing $n_t^i = \frac{\alpha}{\delta}h_t^i$ than choosing $n_t^i = \frac{\alpha}{\delta+\bar{e}}h_t^i$. This result along with Eq. (9) reveals the first inequality in Eq. (18). On the other hand, Eq. (10) implies that given $g_{t+1} > 0$ and $e_t^p = \bar{e}$, an adult individual i in period t obtains more utility by choosing $n_t^i = \frac{\alpha}{\delta+\bar{e}}h_t^i$ than by choosing $n_t^i = \frac{\alpha}{\delta}h_t^i$. Using this result for Eq. (9) reveals the second inequality in Eq. (18).

Proof. See the Appendix. □

Recalling that the function η in Eq. (17) is strictly increasing in the ability level,

$$e_t^{i*} = \begin{cases} 0 & \text{if } a_t^i \leq a^*(g_{t+1}); \\ \bar{\epsilon} & \text{if } a_t^i > a^*(g_{t+1}). \end{cases} \tag{19}$$

The properties of $a^*(g_{t+1})$ presented in Lemma 1 are interpreted as follows. First, the existence of $a^*(g_{t+1}) > 0$ implies that in the absence of uncertainty, there are both skilled and unskilled workers in the economy regardless of the growth rate of technology. Second, the negative reaction of $a^*(g_{t+1})$ implies that education investment e_t^{i*} is promoted by the acceleration of technological progress, which is assumed to be skill-biased. Finally, the result $a^*(\tilde{g}) = 1$ means that whether they have perfect foresight or not, if $g_{t+1} = \tilde{g}$, parents whose children have average ability are indifferent between the education decisions [cf. Eq. (11)]. This is straightforward because these parents indeed receive no unexpected ability shock.

2.2.4 Optimization by Children

Faced with a trade-off between leisure and skill acquisition, children do not necessarily come up to the expectations of their parents. They enjoy leisure if the parental education support is burdensome on them. If it is insufficient for them, on the other hand, the absence of capital markets prevents anyone from taking out education loans to cover the shortage. Given one unit of time, children determine the optimal time allocation with the accurate information on their own abilities.

Consider a child $j(i)$ in period t, who is born from a parent i. The leisure time left for the child, $l_t^{j(i)}$, is

$$l_t^{j(i)} = 1 - \gamma s_t^i,$$

where $\gamma \in (0, 1)$ by assumption and $s_t^i \in [0, 1]$ is the effort level or, more precisely, the child's time devoted to skill acquisition.[16] The condition $\gamma < 1$ implies that a process of skill acquisition is perceived in part as leisure. On the other hand, achieving a certain education level requires not only parental support but also the child's own efforts. That is,

$$\epsilon_t^i = s_t^i e_t^i. \tag{20}$$

[16] The restriction on γ ensures the existence of two corner solutions, $s_t^i = 1$ and $s_t^i = 0$. As is evident from Eq. (21), no one chooses $s_t^i = 0$ if $\gamma = 0$, whereas no one chooses $s_t^i = 1$ if $\gamma = 1$.

Given Eqs. (2) and (10), the child knows that his/her consumption in adulthood, $c_{t+1}^{j(i)}$, has a linear relationship with his/her skills, $h_{t+1}^{j(i)}$. Thus it follows from Eqs. (3) and (4) that the skill-acquisition time chosen by the child, s_t^i, is

$$s_t^i = \arg\max \left\{(1-\beta)\ln(1-\gamma s_t^i) + \beta \ln h(s_t^i e_t^i, a_t^i, g_{t+1})\right\}, \tag{21}$$

subject to $s_t^i \in [0,1]$. Because children are identical within households, one may index their optimal choices by the superscript i rather than $j(i)$.

Recalling Eq. (16) showing that e_t^i is either 0 or $\bar{\epsilon}$, optimization by a child $j(i)$ in period t is divided into two cases. First, if $e_t^i = 0$, then $s_t^i = 0$. Because $\bar{\epsilon}$ is the minimum requirement of educational attainment to become skilled labor, it is wastefulness for the child to devote his/her time, which is at most one, to schooling if parental support is less than $\bar{\epsilon}$. Second, if $e_t^i = \bar{\epsilon}$, then $s_t^i = 0$ or 1. The child may engage in skill acquisition depending on his/her ability level and the growth rate of technology. Unless parental support is more than $\bar{\epsilon}$, it is necessary for the child sacrifice his/her entire time to become skilled labor.

Let $\hat{a}_t$ be a critical ability level for which the child given $e_t^i = \bar{\epsilon}$ is indifferent between working as skilled labor and as unskilled labor. That is, in view of Eq. (7),

$$\beta \ln \eta(\hat{a}_t, g_{t+1}) = (1-\beta)\ln\frac{1}{1-\gamma} > 0, \tag{22}$$

meaning that the relative advantage of working as skilled labor is just equal to the relative disadvantage in leisure. Figure 2 illustrates the determination of $\hat{a}_t$ graphically. It shows that there is a unique ability level, indicated by $\hat{a}(g_{t+1})$, for which Eq. (22) is satisfied for any $g_{t+1} \geq 0$. Furthermore, an increase in g_{t+1} shifts the function $\eta(a_t^i, g_{t+1})$ upward, raising the critical ability level. These are the intuition about Lemma 2.

Lemma 2 *There exists a single-valued function* $\hat{a}_t = \hat{a}(g_{t+1}) > 0$ *that satisfies Eq. (22) for any* $g_{t+1} \geq 0$. *Furthermore,* $\hat{a}'(g_{t+1}) < 0 \ \forall g_{t+1} > 0$.

Proof. The lemma is proven in a similar way to Lemma 1. □

Now suppose that children choose to work as unskilled labor when they are indifferent. It follows that the skill acquisition time chosen by a child of a parent i in period t is, for $e_t^i = \bar{\epsilon}$ and for all $g_{t+1} \geq 0$,

$$s_t^i = \begin{cases} 0 & \text{if } a_t^i \leq \hat{a}(g_{t+1}); \\ 1 & \text{if } a_t^i > \hat{a}(g_{t+1}), \end{cases} \tag{23}$$

noting that the function η in Eq. (22) is strictly increasing in the ability level. As mentioned above, $s_t^i = 0$ if $e_t^i = 0$, regardless of the growth rate of technology.

Figure 2 provides the case that $-\frac{1-\beta}{\beta}\ln(1-\gamma)$ is in the neighbor of $\ln d$, which occurs if and only if

$$\hat{a}(g_{t+1}) \approx a^*(g_{t+1}) \qquad \forall g_{t+1} \geq 0, \tag{A1}$$

noting Eqs. (17) and (22). In comparison of Eqs. (19) and (23), this condition means that given perfect foresight, parents would take similar stances toward education policy to their children, who fully observe their own abilities. In other words, parent-child conflict barely exists in the absence of asymmetric information on ability. The analysis below focuses on this case, in order to exclude some possible but irrelevant scenarios.[17]

Figure 3 draws the function $\hat{a}(g_{t+1})$ using the results of Lemma 1. The negative reaction of $\hat{a}(g_{t+1})$ implies that more children are induced to acquire skills as skill-biased technological progress accelerates.

2.3 Education Attainment

Educational attainment ϵ_t^i reaches $\bar{\epsilon}$ only if both parents and their children take positive stances to private education. Combining the results of Eqs. (16) and (23) for Eq. (20), the level of educational attainment by a child $j(i)$ in period t is

$$\begin{aligned} \epsilon_t^i &= \begin{cases} 0 & \text{if } a_t^i \leq \max[\tilde{a}(g_{t+1}), \hat{a}(g_{t+1})]; \\ \bar{\epsilon} & \text{if } a_t^i > \max[\tilde{a}(g_{t+1}), \hat{a}(g_{t+1})]. \end{cases} \\ &\equiv \epsilon(a_t^i, g_{t+1}). \end{aligned} \tag{24}$$

As depicted by Figure 3, $\hat{a}_t = \hat{a}(g_{t+1})$ may or may not be greater than $\tilde{a}_t = \tilde{a}(g_{t+1})$, depending on the level of g_{t+1}. When $\hat{a}_t < \tilde{a}_t$, for instance, children born with $a_t^i \in (\hat{a}_t, \tilde{a}_t]$ find skill acquisition

[17] The explicit condition for Eq. (A1) is

$$\frac{1-\alpha}{\alpha}\ln\frac{1-\alpha/d}{1-\alpha} < \frac{1-\beta}{\beta}\ln\frac{1}{1-\gamma} < \frac{1-\alpha}{\alpha}\ln\frac{1-\alpha}{1-\alpha d}.$$

Noting Eq. (18), the condition above holds if $-\frac{1-\beta}{\beta}\ln(1-\gamma) \approx \ln d$.

advantageous whereas their parents are not inclined to support them. That is, the borrowing constraint is binding on them, and education investment is insufficient from the viewpoint of the young generation as a whole. Throughout the present paper, this case is referred to as *under*-investment in education. Conversely, *over*-investment occurs when $\tilde{a}_t < \hat{a}_t$. Children born with $a_t^i \in (\tilde{a}_t, \hat{a}_t]$ waste education investment they receive, growing up with unskilled labor.

Comparing Eqs. (9) and (21) reveals the mechanism through which the relative position of $\tilde{a}_t$ to $\tilde{a}_t$ may change over time. In deciding their education policies, adult individuals take into account the quantity of children they intend to raise, whereas these children do not care about how many siblings they have. As a result, only the formers are constrained by the irreversibility of childbirth. Fertility decline in the growth process may therefore reverse the relationship between $\tilde{a}_t$ and $\tilde{a}_t$.

2.4 Macroeconomic Variables

Suppose that ability levels are continuously distributed on $\mathbb{R}_+$ according to a stationary probability distribution function (PDF). They are independent of each other not only within generations but also across generations. In other words, ability is not inherited within dynasties, although it is identical among children born from a same household.

Let $G(a_t)$ denote the corresponding cumulative distribution function, where a_t denotes an ability level observed in period t. Substituting Eq. (24) into Eq. (4), the average level of efficiency units of labor in period t, denoted as h_t, is

$$\begin{aligned} h_t &= \int_0^{N_t} h_t^i di / N_t \\ &= \int_0^{\infty} h(\epsilon(a_{t-1}, g_t), a_{t-1}, g_t) dG(a_{t-1}). \end{aligned} \tag{25}$$

Then, it follows from Eq. (10) that the ratio of the child population to the adult population in period t, denoted as n_t, is

$$n_t \equiv \frac{N_{t+1}}{N_t} = \int_0^{N_t} n_t^i di / N_t = \frac{\alpha}{\delta + e^p(g_{t+1})} h_t. \tag{26}$$

It follows that the evolution of the working population is

$$N_{t+1} = n_t N_t = \prod_{j=0}^{t} n_j N_0, \tag{27}$$

where the initial size N_0 is historically given. Then, in light of Eqs. (12), (13), and (16), the aggregate level of human capital employed by firms in period t, H_t, is

$$H_t = N_t h_t \left\{ 1 - \frac{\alpha}{\delta + e^p(g_{t+1})} \left[\delta + \int_0^\infty e(a_t, g_{t+1}) dG(a_t) \right] \right\},$$

where the negative term is, as a whole, interpreted as the time cost of child rearing per worker.

These results show that all aggregate variables—n_t, N_t, h_t, H_t, and y_t in Eq. (1)—are determined for all $t \geq 0$ once the initial size of the working population, N_0, and the dynamic path of the growth rate of technology, $\{g_t\}_{t=0}^{\infty}$, are given.

3 Exogenous Technological Progress

This section demonstrates the transition from under- to over-investment in education in the process of economic development driven by exogenous technological progress. It focuses on the dynamic path on which the growth rate of technology g_t eventually exceeds the critical level $\tilde{g}$ and remains above $\tilde{g}$ afterwards, although it does not necessarily increase monotonically. Then, there is a critical period $\tilde{t}$ after which g_t exceeds $\tilde{g}$ for the first time; i.e., $g_t \leq \tilde{g}\ \forall t \leq \tilde{t}$ and $g_t > \tilde{g}\ \forall t > \tilde{t}$. The growth process is then divided into the following two stages.

Stage I ($0 \leq t < \tilde{t}$): This underdevelopment stage involves *under*-investment in education. Because the quantity of children is locked into a large level in the household budget, parents whose children are unexpectedly competent cannot afford education investment. These children have no access to capital markets to make education loans.

Stage II ($t \geq \tilde{t}$): This developed stage is characterized by *over*-investment in education. Now that the quantity of children is locked into a small level in the household budget, even parents whose children are unexpectedly incompetent have spare resources for education investment.

3.1 Stage I: Under-Investment in Education

In Stage I, where $0 \leq t < \tilde{t}$ and $0 < g_{t+1} \leq \tilde{g}$, all households concentrate their resources on the quantity, rather than the quality, of children before the true ability level is observed. That is, their planned level of education and fertility decisions are

$$e_t^p = 0 \quad \text{and} \quad n_t^i = \frac{\alpha}{\delta} h_t^i. \tag{28}$$

It follows from Eq. (13) that for any $g_{t+1} \in [0, \tilde{g}]$,

$$V(e_t^i, a_t^i, g_{t+1}) = \begin{cases} (1-\alpha)\ln(1-\alpha) + \alpha \ln h(0, a_t^i, g_{t+1}) & \text{if } e_t^i = 0; \\ (1-\alpha)\ln(1-\alpha d) + \alpha \ln h(\bar{\epsilon}, a_t^i, g_{t+1}) & \text{if } e_t^i = \bar{\epsilon}, \end{cases} \tag{29}$$

where $0 < 1 - \alpha d < 1 - \alpha$ from Eq. (14). The difference between $1 - \alpha$ and $1 - \alpha d$, indicating the adverse effect of revising the initial education plan on consumption, is a result from the irreversibility of childbirth. The difference would not exist if the quantity of children was adjustable at the time of education investment.

It follows from Eqs. (7) and (29) that in Stage I, the critical ability level $\tilde{a}_t$ in Eq. (15) satisfies

$$\alpha \ln \eta(\tilde{a}_t, g_{t+1}) = (1-\alpha) \ln \frac{1-\alpha}{1-\alpha d} > 0. \tag{30}$$

This condition means that the relative advantage of education investment is equal to the relative disadvantage in consumption. Figure 2 illustrates the determination of $\tilde{a}_t$ in Stage I. It shows that there is a unique ability level, indicated by $\tilde{a}^I(g_{t+1})$, for which Eq. (30) is satisfied for any $g_{t+1} \geq 0$. An increase in g_{t+1} shifts the function $\eta(a_t^i, g_{t+1})$ upward, raising the critical ability level. Furthermore, $\tilde{a}^I(g_{t+1}) > a^*(g_{t+1})$ as follows from the parameter relationship in Eq. (18). These are the intuition about Lemma 3.

Lemma 3 *There exists a single-valued function $\tilde{a}_t = \tilde{a}^I(g_{t+1})$ that satisfies Eq.* (30) *for any $g_{t+1} \geq 0$. Furthermore,*

(*a*) $\tilde{a}^{I\prime}(g_{t+1}) < 0 \ \forall g_{t+1} > 0$;

(*b*) $\tilde{a}^I(g_{t+1}) > 1 \ \forall g_{t+1} \in [0, \tilde{g}]$;

(*c*) $\tilde{a}^I(g_{t+1}) > a^*(g_{t+1}) \ \forall g_{t+1} \geq 0$.

Proof. See the Appendix. □

Thus in Stage I, where $g_{t+1} \in [0, \tilde{g}]$, the preferred education level for a parent i in period t is

$$e_t^i = \begin{cases} 0 & \text{if } a_t^i \leq \tilde{a}^I(g_{t+1}); \\ \bar{\epsilon} & \text{if } a_t^i > \tilde{a}^I(g_{t+1}), \end{cases} \tag{31}$$

recalling that the function η in Eq. (30) is strictly increasing in the ability level. Figure 3 graphically represents the properties of $\tilde{a}^I(g_{t+1})$ based on the results of Lemma 3. The solid line portion indicates that $\tilde{a}_t$ is given by $\tilde{a}^I(g_{t+1})$ only on the interval $[0, \tilde{g}]$. Because $\tilde{a}^I(g_{t+1})$ is decreasing

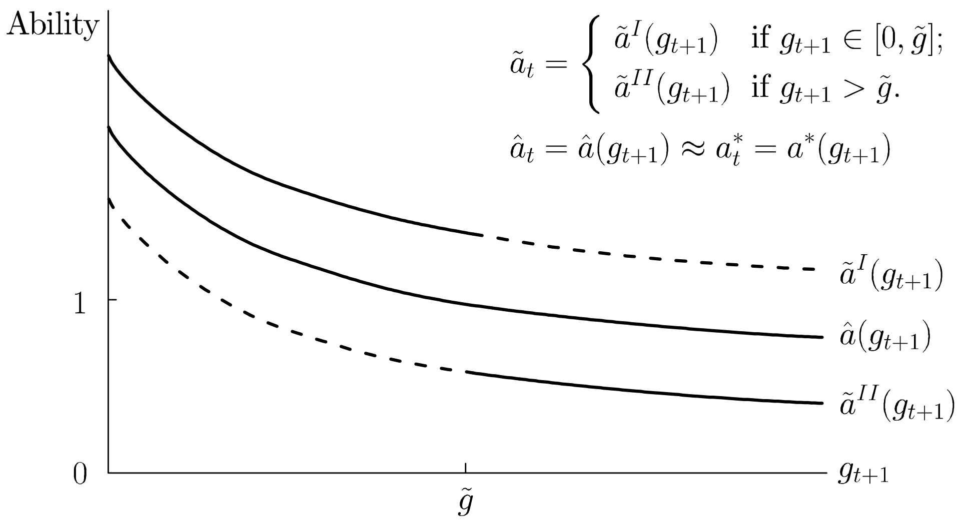

Figure 3. The Relationship Between the Growth Rate of Technology and the Critical Ability Levels for Education.

in g_{t+1}, education investment is promoted by the acceleration of technological progress, which is assumed to be skill-biased.

Furthermore, $\tilde{a}^I(g_{t+1})$ is greater than 1 on $[0, \tilde{g}]$, implying that in Stage I, parents whose children have average ability do not invest in education. This result is explained by noting Eq. (28). In prospect of no education investment, the quantity of children in Stage I is locked into a large level that magnifies the education expenses per household. In order to offset the adverse effect and induce education investment, the ability level needs to be sufficiently higher than the expected level at childbirth (i.e., $a^i_t \geq \tilde{a}^I(g_{t+1}) > 1$).

The diagram depicts $\tilde{a}^I(g_{t+1})$ above $a^*(g_{t+1})$, which represents the critical ability level for the perfect-foresighted parents considered in Section 2.2.3. Given the ability information in advance, parents would adjust childbirth so as to achieve their preferred level of education investment. That is, unlike imperfect-foresighted parents, their education decisions would not be constrained by the irreversibility of childbirth. This is why $a^*(g_{t+1})$ is lower than $\tilde{a}^I(g_{t+1})$ for any $g_{+1} \geq 0$.[18]

Under Eq. (A1), the gap between $\tilde{a}^I(_{t+1})$ and $a^*(g_{t+1})$ implies parent-child conflict over education policy, because children behave as if they were the aforementioned perfect foresighted parents. This is the intuition of Proposition 1 below.

[18] Note that $\tilde{a}^I(g_{t+1})$ is greater than $a^*(g_{t+1})$ for all $g_{t+1} \geq 0$. Technically speaking, this is because $\tilde{a}^I(g_{t+1})$ is derived, regardless of g_{t+1}, from the utility comparison under the child-rearing plan in Eq. (28).

Proposition 1 (Under-Investment in Education) *Under Eq. (A1),*

$$\tilde{a}^I(g_{t+1}) > \hat{a}(g_{t+1}) \quad \forall g_{t+1} \geq 0.$$

Proof. See the Appendix. □

In Figure 3, adult individuals whose children have $a_t^i \in (\hat{a}_t, \tilde{a}_t)$ do not invest in education against the demand from their children. In such a circumstance, childbirth-discouraging policy, which locks the quantity of children into a smaller level, mitigates under-investment in education and may thereby prevent the emergence of a poverty trap in Stage I. This possibility will be examined later in Section 4.2.

3.2 Stage II: Over-Investment in Education

In Stage II, where $t \geq \tilde{t}$ and $g_{t+1} > \tilde{g}$, all households prepare for future education by adjusting the quantity of children they intend to raise. That is, their planned level of education and corresponding fertility decisions are

$$e_t^p = \bar{\epsilon} \quad \text{and} \quad n_t^i = \frac{\alpha}{\delta + \bar{\epsilon}} h_t^i. \tag{32}$$

It follows from Eq. (13) that for any $g_{t+1} \geq \tilde{g}$,

$$V(e_t^i, a_t^i, g_{t+1}) = \begin{cases} (1-\alpha)\ln(1-\alpha/d) + \alpha \ln h(0, a_t^i, g_{t+1}) & \text{if } e_t^i = 0; \\ (1-\alpha)\ln(1-\alpha) + \alpha \ln h(\bar{\epsilon}, a_t^i, g_{t+1}) & \text{if } e_t^i = \bar{\epsilon}, \end{cases} \tag{33}$$

where $1-\alpha/d > 1-\alpha$ from Eq. (14). The difference between $1-\alpha$ and $1-\alpha/d$ indicates the adverse effect of revising the initial education plan on consumption. Because the quantity of children is unadjustable at the time of education investment, the remaining income after choosing no education is spent entirely on consumption.

It follows from Eqs. (7) and (29) that in Stage II, the critical ability level $\tilde{a}_t$ in Eq. (15) satisfies

$$\alpha \ln \eta(\tilde{a}_t, g_{t+1}) = (1-\alpha) \ln \frac{1-\alpha/d}{1-\alpha} > 0. \tag{34}$$

This condition means that the relative advantage of education investment is equal to the relative disadvantage in consumption. Figure 2 illustrates the determination of $\tilde{a}_t$ in Stage I. It shows that there is a unique ability level, indicated by $\tilde{a}^{II}(g_{t+1})$, for which Eq. (30) is satisfied for any $g_{t+1} \geq 0$. An increase in g_{t+1} shifts the function $\eta(a_t^i, g_{t+1})$ upward, raising the critical ability level. Furthermore, $\tilde{a}^{II}(g_{t+1}) < a^*(g_{t+1})$ as follows from the parameter relationship in Eq. (18). These are the intuition about Lemma 3.

Lemma 4 *There exists a single-valued function* $\tilde{a}_t = \tilde{a}^{II}(g_{t+1}) > 0$ *that satisfies Eq.* (34) *for any* $g_{t+1} \geq 0$. *Furthermore,*

(*a*) $\tilde{a}^{II\prime}(g_{t+1}) < 0 \ \forall g_{t+1} > 0$;

(*b*) $\tilde{a}^{II}(g_{t+1}) < 1 \ \forall g_{t+1} \geq \tilde{g}$;

(*c*) $\tilde{a}^{II}(g_{t+1}) < a^*(g_{t+1}) \ \forall g_{t+1} \geq 0$.

Proof. See the Appendix. □

Thus in Stage II, where $g_{t+1} > \tilde{g}$, the preferred education level for a parent i in period t is

$$e_t^i = \begin{cases} 0 & \text{if } a_t^i \leq \tilde{a}^{II}(g_{t+1}); \\ \bar{\epsilon} & \text{if } a_t^i > \tilde{a}^{II}(g_{t+1}), \end{cases} \tag{35}$$

recalling that the function η in Eq. (34) is strictly increasing in the ability level. Figure 3 graphically represents the properties of $\tilde{a}^{II}(g_{t+1})$ from Lemma 4. The solid line portion indicates that $\tilde{a}^{II}(g_{t+1})$ is equal to $\tilde{a}_t$ only on the interval $(\tilde{g}, \infty)$. Like the other functions in the diagram, the negative slope of $\tilde{a}^{II}(g_{t+1})$ shows that education investment is promoted by the acceleration of skill-biased technological progress.

The property that $\tilde{a}^{II}(g_{t+1})$ is smaller than 1 on $(\tilde{g}, \infty)$ implies that in Stage II, parents whose children have average ability invest in education. This result is explained by noting Eq. (32). In prospect of education investment, the quantity of children in this stage is locked into a small level that lowers the sunk cost of child rearing. Parents are consequently inclined to follow their initial education plans, unless their children are significantly incompetent as opposed to the initial expectation (i.e., $a_t^i \leq \tilde{a}^{II}(g_{t+1}) < 1$).[19]

The diagram depicts $\tilde{a}^{II}(g_{t+1})$ below $a^*(g_{t+1})$, which represents the critical ability level for the perfect foresighted parents considered in Section 2.2.3. Unlike imperfect-foresighted parents considered herein, they would adjust the quantity of their children in accordance with the ability of their children, so that the child-rearing cost would not become unexpectedly small. This is why $a^*(g_{t+1})$ is higher than $\tilde{a}^{II}(g_{t+1})$ for any $g_{+1} \geq 0$.[20]

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[19] As shown by Lemma 4(b), this is also true for the case $g_{t+1} = \tilde{g}$, in which parents are indifferent between $e_t^p = 0$ and $e_t^p = \bar{\epsilon}$. The reason is that $\tilde{a}^{II}(g_{t+1})$ is the critical level for education investment when n_t^i is locked into a small level, $\frac{\alpha}{\delta+\bar{\epsilon}} h_t^i$.

[20] Moreover, this result and Lemma 3(c) show that $\tilde{a}(g_{t+1})$ given a positive value for any $g_{t+1} \geq 0$. Namely, both skilled and unskilled labor exist at any stage of economic development (recall that ability levels are distributed on $\mathbb{R}_+$).

In fact, the gap between $\tilde{a}^{II}(g_{t+1})$ and $a^*(g_{t+1})$ implies parent-child conflict over private education policy, because children behave as if they were the aforementioned perfect foresighted parents. This is the intuition of Proposition 2 below.

Proposition 2 (Over-Investment in Education) *Under Eq. (A1),*

$$\tilde{a}^{II}(g_{t+1}) < \hat{a}(g_{t+1}) \quad \forall g_{t+1} \geq 0.$$

Proof. See the Appendix.. □

Thus in Stage II, there exists the ability range $(\tilde{a}_t, \hat{a}_t)$, on which children waste education support provided by their parents. The following section extends the baseline model to an endogenous growth model by assuming that these children do not contribute to technological innovation.

4 An Extension: Endogenous Growth with Public Policy

This section endogenizes technological progress and investigates the resulting evolution of the economy. It demonstrates the possibility of multiple steady-state equilibria and then examines how the government's policy allows the economy to develop beyond a a poverty trap.

4.1 Technological Progress

In the economy considered herein, technological progress is driven by skilled workers, who are willing, not forced, to engage in skill acquisition in childhood. Specifically,

$$g_{t+1} = g(\theta_t), \tag{36}$$

where $\theta_t \in (0, 1)$ is the fraction of skilled workers in generation t.[21] $g(\theta_t)$ is a continuous function characterized by the following properties. First, no new technology is created without skilled workers; i.e., $\lim_{\theta_t \to 0} g(\theta_t) = 0$. Second, in line with Galor and Moav (2000), technological progress is accelerated by an increase in the fraction of skilled workers; i.e., $g'(\theta_t) > 0 \ \forall \theta_t \in (0, 1)$. Third, technological progress is bounded above; i.e., $\lim_{\theta_t \to 1} g(\theta_t) < \infty$.

Members of generation $t-1$ face the same probability of drawing a certain ability level a_{t-1} according to the cumulative distribution function $G(a_{t-1})$. In light of Eq. (24), the probability that

[21] This formulation implies that the two important factors of technological progress are offset by each other: the scale effect of population and the fishing out effect (i.e., reaching a higher technology level increases the difficulty in finding new ideas). See for example Jones (1998, Ch. 5) and Weil (2009, Ch. 9)

any member of generation $t-1$ becomes unskilled labor in period t is $p_t \equiv G(\max[\tilde{a}(g_t), \hat{a}(g_t)])$, where

$$\max[\tilde{a}(g_t), \hat{a}(g_t)] = \begin{cases} \tilde{a}^I(g_t) & \text{if } g_t \in [0, \tilde{g}]; \\ \hat{a}(g_t) & \text{if } g_t > \tilde{g}, \end{cases}$$

noting Propositions 1–2. It then follows from Eq. (26) that

$$\begin{aligned} \theta_t &= \int_0^{N_{t-1}} (1-p_t) n_{t-1}^i di / N_t \\ &= (1-p_t) n_{t-1} N_{t-1} / N_t \\ &= 1 - G(\max[\tilde{a}(g_t), \hat{a}(g_t)]), \end{aligned} \tag{37}$$

where $G'(a_{t-1}) > 0 \; \forall a_{t-1} > 0$.

Substituting Eq. (37) into Eq. (36), the evolution of g_t is governed by the dynamical system

$$\begin{aligned} g_{t+1} &= \begin{cases} g(1 - G(\tilde{a}^I(g_t))) \equiv \phi^I(g_t) & \text{if } g_t \in [0, \tilde{g}]; \\ g(1 - G(\hat{a}(g_t))) \equiv \phi^{II}(g_t) & \text{if } g_t > \tilde{g}. \end{cases} \\ &\equiv \phi(g_t), \end{aligned} \tag{38}$$

where g_0 is historically given. Figure 4 represents the dynamical system characterized by four key properties. First, $\phi^I(0) > 0$ because even in the prospect of no technology growth, households whose children are sufficiently competent find education investment desirable; i.e., $\tilde{a}^I(0) < \infty$. Second, $\phi^{I\prime}(g_t)$ and $\phi^{II\prime}(g_t) > 0 \; \forall g_t > 0$ because the acceleration of skill-biased technological progress attracts both parents and children toward education; i.e., $\tilde{a}'(g_t) < 0$ and $\hat{a}'(g_t) < 0 \; \forall g_t > 0$. Third, $\phi^I(g_t) < \phi^{II}(g_t) \; \forall g_t \geq 0$ because parental education investment is constrained by the quantity of children, which is locked into a large level (cf. Eq. (28), whereas children's education decisions are not; i.e., $\tilde{a}^I(g_t) > \hat{a}(g_t) \; \forall g_t \geq 0$. Forth, $\phi(g_t)$ is discontinuous at $g_t = \tilde{g}$, above which the economy enters Stage II. Lastly, $\lim_{g_t \to \infty} \phi(g_t) < \infty$ because the function $g(\theta_t)$ is bounded above.

In the diagram, $\bar{g}^l$ and $\bar{g}^h$ indicate *potential* steady-state levels of g_t, for which $\phi^I(\bar{g}^l) = \bar{g}^l$ and $\phi^{II}(\bar{g}^h) = \bar{g}^h$, respectively.[22] Suppose that the initial condition is given by

$$0 \leq g_0 < \bar{g}^l. \tag{A2}$$

[22] While the existence of $\bar{g}^h$ and $\bar{g}^l$ is guaranteed by the properties of $\phi^I(g_t)$ and $\phi^{II}(g_t)$, their uniqueness is generally ambiguous because these functions may not be concave. To avoid confusion, these values are defined as $\bar{g}^l \equiv \min\{\bar{g} \geq 0 \mid \phi^I(\bar{g}) = \bar{g}\}$ and $\bar{g}^h \equiv \min\{\bar{g} \geq 0 \mid \phi^{II}(\bar{g}) = \bar{g}\}$, implying that $\bar{g}^l < \bar{g}^h$.

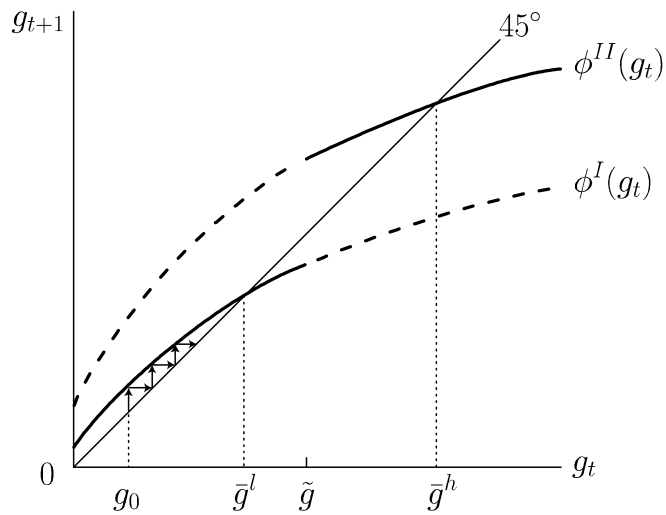

Figure 4. The Evolution of the Growth Rate of Technology.

Note that steady-state equilibrium $g_t = \phi(g_t)$ occurs at either $\bar{g}^l$ or $\bar{g}^h$, yet may not at both of them. Multiple steady-state equilibria occur if and only if

$$\phi^I(\tilde{g}) \leq \tilde{g} < \phi^{II}(\tilde{g}). \tag{A3}$$

This multiplicity condition holds depending on the quantitative properties of the underlying functions ϕ, G, $\tilde{a}^I$, and $\hat{a}$ as well as on the level of $\tilde{g}$. In particular, the gap between $\phi^I(\tilde{g})$ and $\phi^{II}(\tilde{g})$ is due in part to the deviation of $\tilde{a}^I(\tilde{g})$ from $\hat{a}(\tilde{g})$ or, in other words, the prevalence of under-investment in education at the end of Stage I.

Note that both of the steady-state equilibria are locally stable. In particular, one of them, which occurs at $\bar{g}^l$, works as a poverty trap in Stage I. Given the initial condition in Eq. (A2), the growth rate of technology increases monotonically over time and converges to the lower steady-state level $\bar{g}^l$ in Stage I. In this circumstance, only a small fraction of households are inclined to invest in education (i.e., $\tilde{a}^I(g_t)$ is substantially large), and technological progress becomes stagnant before g_t reaches the threshold level $\tilde{g}$.

g_t converges toward $\bar{g}^h$ if and only if $g_0 > \tilde{g}$ or, equivalently, if and only if the economy starts out with Stage II. This implies that, under Eq. (A3), there is no mechanism that permits the endogenous transition from Stage I to Stage II. The following section examines how public policy can bring the economy out of the poverty trap to the higher steady-state level.

4.2 Public Policy

Now suppose that individuals live three periods (childhood, adulthood, and elderhood), whereas no change is made on the production of final output. The economy has a central government that levies an income tax to provide child care for young parents and a public service for the aged. It is shown that a population control policy that discourages childbirth prevents the emergence of a poverty trap. Furthermore, an expansion of child care subsidies in developed stages will promote the accumulation of aggregate human capital and improve the welfare of future generations.

4.2.1 The Government

In every period t, the government levies a tax on potential income $w_t h_t^i$ with a constant rate $\tau \in (0,1)$. The resulting tax revenue, which is amount to $\tau w_t h_t N_t$, is allocated between generations living in the period. On the one side, each adult individual receives $w_t(\delta^0 - \delta)n_t$ as child case subsidies, where $\delta^0 > 0$ and $\delta > 0$ are the pre- and the post-subsidy cost of child rearing per child, respectively, in terms of efficiency units of labor. On the other side, each old individual receives x_t units of the elderly-related public service. Under the balanced budget condition, the government's budget constraint is,

$$\tau h_t = (\delta^0 - \delta)n_t + \frac{x_t}{w_t n_{t-1}}, \tag{39}$$

where $n_{t-1} = N_t/N_{t-1}$ is the ratio of the taxpaying population to the aged population in period t. Therefore, a rise in $w_t n_{t-1}$ allows, ceteris paribus, the government to increase either δ or x_t in period t.

4.2.2 Households

Consider the life of an individual i of generation t (a parent i in period t). As for the first and second periods of life, the environment is the same as before, except that the individual in the second period receives child care in exchange for the tax payment. In the third period, the individual retires and merely consumes the public service for the elderly, x_{t+1}. Then it follows from Eq. (2) that the budget constraint faced by the individual is replaced with

$$c_t^i = w_t[(1-\tau)h_t^i - (\delta + e_t^i)n_t^i].$$

In view of Eq. (3), the utility function is extended to

$$
\begin{aligned}
u_t^i &= (1-\beta)\ln l_{t-1}^i + \beta\left\{(1-\alpha)\ln c_t^i + \alpha \ln\left[n_t^i h_{t+1}^{j(i)}\right]\right\} \\
&\quad + \ln x_{t+1}.
\end{aligned}
$$

Individuals take not only the wage rate but also public policy as given. At the time of childbirth, they believe as before that their children will be born with average ability and fully make use of parental education support (i.e., $a_t^i = 1$ and $e_t^p = \epsilon_t^i$). Then the quantity of children raised by a parent i in period t is

$$
n_t^i = \frac{\alpha}{\delta + e_t^p}(1-\tau)h_t^i,
$$

where the only deviation from the baseline model is that n_t^i now depends on the tax rate τ. Because of the multiplicative relationship between $(1-\tau)$ and h_t^i, introducing the income tax has no effect on parents' attitudes toward education represented by e_t^p and e_t^i. These results and Eq. (26) reveal that the ratio of the taxpaying population to the aged population, n_t, is

$$
n_t = \frac{\alpha}{\delta + e_t^p}(1-\tau)h_t, \tag{40}
$$

where e_t^p and h_t are respectively given by Eqs. (12) and(25).

4.3 Analysis

This section explores the implications of policy reforms that mitigate either under- or over-investment in education. The following results, most of which are apparent from Figure 2, reveal how a change in child care subsidies affects individuals' attitudes toward education.

Lemma 5 *Let $\hat{a}(g_{t+1})$, $\tilde{a}^I(g_{t+1})$, and $\tilde{a}^{II}(g_{t+1})$ be expressed as $\hat{a}(g_{t+1}; d)$, $\tilde{a}^I(g_{t+1}; d)$, and $\tilde{a}^{II}(g_{t+1}; d)$, respectively, where $d \equiv 1 + \bar{\epsilon}/\delta > 1$. Then,*

(a) $\hat{a}_d(g_{t+1}; d) = 0 \ \forall g_{t+1} \geq 0$;

(b) $\tilde{a}_d^I(g_{t+1}; d) > 0$ *and* $\tilde{a}_d^{II}(g_{t+1}; d) > 0 \ \forall g_{t+1} \geq 0$;

(c) $\lim_{d\to 1}[\hat{a}(g_{t+1}; d) - \tilde{a}^I(g_{t+1}; d)] \geq 0 \ \forall g_{t+1} \geq 0$;

(d) $\partial \tilde{g}/\partial d > 0$.

Proof. See the Appendix. □

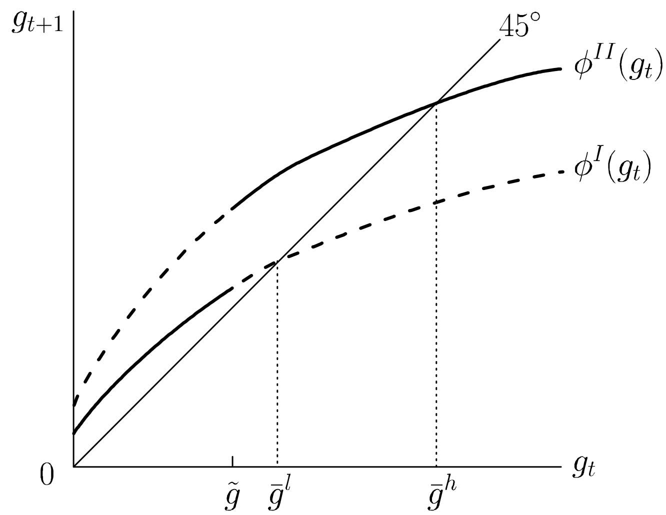

Figure 5. The Dynamical System for a Large δ.

Given the results of Lemma 5 and , consider an increase in δ (i.e., shift in the government budget from child care subsidies to the elderly-related service) during Stage I. The parameter change leads to the downward shift of $\tilde{a}^I(g_{t+1})$ in Figure 3, with no effect on $\hat{a}(g_{t+1})$. The reduction in the interval between $\hat{a}(g_{t+1})$ and $\tilde{a}^I(g_{t+1})$, on which children demand parental education investment, implies the mitigation of under-investment in education and the increase in the fraction of skilled labor in Stage I.[23] This result is straightforward. A rise in δ discourages childbirth and thereby weakens its lock-in effect on education investment. By contrast, child care subsidies have no impact on the education decisions of children, because they do not take into account the household budget that depends on the family size.

The rise in δ also brings about the downward shift of $\tilde{a}^{II}(g_{t+1})$ in Figure 3 for the same reason as $\tilde{a}^I(g_{t+1})$. The reduction in the interval between $\tilde{a}^{II}(g_{t+1})$ and $\hat{a}(g_{t+1})$, on which children waste parental education investment, implies the mitigation of over-investment in education in Stage II with no effect on the fraction of skilled labor.

Figure 5 depicts the resulting effect of the parameter change on the dynamical system under Eq. (A3). Increasing δ shifts up the function $\phi^I(g_t)$ toward $\phi^{II}(g_t)$ as a result of the increase in the fraction of skilled labor for given g_t. The change in δ also has no effect on $\phi^{II}(g_t)$ and an adverse effect on the critical level $\tilde{g}$, which divides Stages I and II. These structural changes in the

[23]Note that Lemma 5(c) implies the violation of Eq. (A1). The second inequality in Footnote 17 does not hold if δ is sufficiently large (and thus d is sufficiently close to 1). Nevertheless, Eq. (A1) is kept satisfied unless the relationship between $\hat{a}(g_t)$ and $\tilde{a}^I(g_t)$, which reflects the relationship between $\phi^I(g_t)$ and $\phi^{II}(g_t)$, is reversed.

dynamical system advance technological progress in Stage I. Furthermore, if δ is sufficiently small, the poverty trap dissipates as depicted in the diagram; that is,[24]

$$\phi^I(\tilde{g}) > \tilde{g}. \tag{A3'}$$

In this case, g_t converges toward the steady-state level in Stage II, $\bar{g}^h$, regardless of the initial condition.

Now two remarks deserve special attention. First, the government does not need to change δ permanently in order to keep the economy away from the poverty trap. It may decrease δ back to the original level once the growth rate of technology exceeds the threshold level $\tilde{g}$. Second, it is not clear whether the economy in the poverty trap is dynamically inefficient, because of the ambiguous overall effect of the policy reform on the government budget in Eq. (**??**). While the acceleration of technological progress boosts the wage rate w_t, it is to some extent countervailed by the deceleration of growth in the taxpaying population (i.e., a decrease in n_{t-1}).

These results are summarized below.

Proposition 3 (Population Control Policy in Stage I) *Under Eqs.* (*A*1) *and* (*A*2)*, consider a policy reform that shifts the government budget during Stage I, from child care subsidies to the elderly-related service.*

(*a*) *The reform mitigates under-investment in education and advances technological progress in Stage I.*

(*b*) *If Eq.* (*A*3) *is initially satisfied, the reform may prevent the economy from being trapped in Stage I, depending on its extent.*

Next, consider a temporary decrease in δ (i.e., shift in the government budget from the elderly-related service to child care subsidies) after the economy enters Stage II. The parameter change brings about the upward shifts of $\tilde{a}^I(g_{t+1})$ and $\tilde{a}^{II}(g_{t+1})$. The reduction in the interval between $\tilde{a}^{II}(g_{t+1})$ and $\hat{a}(g_{t+1})$, on which children waste parental education investment, implies the mitigation of over-investment in education in Stage II with no change in the fraction of skilled labor, θ_t, and in average human capital, h_t. This result is intuitive. As follows from Eq. (40), a reduction in δ

[24] Lemma 5(c) is of particular importance for the dissipation of the poverty trap. It ensures the existence of a small value of $d > 1$ for which $\phi^I(g_t)$ coincides with $\phi^{II}(g_t)$ for given g_t. This fact, along with the property $\phi^I(0) > 0$ and Lemma 5(d), reveals that $\phi^I(\tilde{g})$ is greater than $\tilde{g}$ if d is sufficiently small.

encourages childbirth and thereby raises the sunk cost of child rearing that constrains education investment.

The transition from Figures 5 to 4 illustrates the resulting effect of the policy reform on the dynamical system. A decline in δ shifts down the function $\phi^I(g_t)$ away from $\phi^{II}(g_t)$ and also increases the critical level $\tilde{g}$. Although this may lead to the re-emergence of the poverty trap, technological progress is intact as long as the increase in $\tilde{g}$ is moderate.

Because the wage rate and average human capital are unaffected by the policy reform, accelerated growth in the taxpaying population enlarges the government budget in the period after the policy reform.[25] The increment of the tax revenue can be allocated to child care subsidies in the period without curtailing the budget for the elderly-related service. Therefore, it is possible to improve the welfare of future generations by gradually raising child care subsidies over Stage II (to the extent that Eq. (A1) is satisfied). These results are summarized below.

Proposition 4 (Child Care Subsidies in Stage II) *Under Eqs.* (*A*1), (*A*3′), *and* (*A*2), *consider a policy reform that gradually shift the government budget during Stage II, from the elderly-related service to child care subsidies. The reform mitigates over-investment in education and improves the welfare of future generations with no harm on technological progress.*

As a final remark, it is notable that the proposition is silent about Pareto improvement. In the absence of capital markets, no compensation is possible for the squeezed elderly in the initial reform period.

5 Concluding Remarks

This research has elucidated the role of irreversible childbirth in economic growth. In the stage of underdevelopment, unexpected ability shocks on newborn children induce some households to revise their education plans upward. Their ex-post reactions, however, may be constrained by the predetermined family size, which squeezes the household budget as a sunk cost. Fertility decline in the growth process therefore lessens the sunk cost and promotes education investment, easing borrowing constraints on children who aim to raise the money for education. In the more developed

[25] By using Eq. (39), this means that n_{t-1} increases whereas h_t and w_t, both of which vary with technological progress, are unchanged by the policy reform.

stage, the irreversibility constraint is binding in the opposite direction. That is, households are unable to have children additionally even if they do not invest in education against the initial plan. For the second best, households tend to invest their resources on the existing children, leading to over-investment in education.

These results derive two notable policy implications. First, a policy reform that discourages childbirth lessens the sunk cost of child rearing and promotes education investment in the under-developed stage. Technological progress fueled by the increased share of skilled workers ultimately alters households' (ex-ante) stances toward education and brings about fertility decline. Second, raising child care subsidies in the developed stage will mitigate over-investment in education and accelerate growth in the working population. The resulting expansion of the government budget improves the welfare of future generations at the cost of the elderly-related service in the initial reform period.

While the central thesis of the present research is intuitive, the developed theory builds on a number of simplifying assumptions. A more general theory would allow for the following aspects. First, parental human capital or ability would be one of the key factors in the formation of the expectation of children's ability. In this case, ability shocks after childbirth would not be totally unexpected. Second, parents would more or less make a coordination of private education policy with their children, which would moderate under- and over-investment in education. Third, fertility adjustment to prepare for future education expenses should be bounded below (that is, the minimum number of children is 1) because in reality investment in the quantity of children is a discrete choice. Unbounded fertility adjustment would make over-investment in education more likely to occur in the developed stage. Lastly, it is desirable to examine the possibility of Pareto improvement in the presence of capital markets, through which the government can transfer resources across nonadjoining generations. These issues should be addressed in future research.

Appendix: Technical Discussions

Proof of Lemma 1. The properties of $\eta(a_t, g_{t+1})$ with respect to a_t^i, given by Eq. (6), ensure the existence of a unique value $a_t^* > 0$ such that $\eta(a_t^*, g_{t+1}) = d$ for any $g_{t+1} \geq 0$. This implies the first statement of Lemma 1. It then follows from Eq. (11) that $a_t^* = a^*(g_{t+1}) = 1$ if $g_{t+1} = \tilde{g}$.

Furthermore, the Implicit Function Theorem reveals that $a^{*\prime}(g_{t+1}) = -\eta_g(a_t^*, g_{t+1})/\eta_a(a_t^*, g_{t+1}) < 0\ \forall g_{t+1} > 0$, by noting Eqs. (6) and (8). $\square$

Proof of Lemma 3. The first statement and result (a) of the lemma are proven in a similar way to Lemma 1.

(b) Since, in view of Eq. (30), $\eta(a_t^i, g_{t+1})$ increases with a_t^i, the result holds if

$$\ln \eta(1, g_{t+1}) < \frac{1-\alpha}{\alpha} \ln \frac{1-\alpha}{1-\alpha d} \quad \forall g_{t+1} \in [0, \tilde{g}].$$

Noting Eqs. (8) and (11), one finds that $\forall g_{t+1} \in [0, \tilde{g}]$,

$$\begin{aligned} \eta(1, g_{t+1}) &\leq \eta(1, \tilde{g}) \\ &= d. \end{aligned}$$

These results and Eq. (18) prove Lemma 3(b).

(c) Noting Eq. (6), the result holds if, $\forall g_{t+1} \geq 0$,

$$\ln \eta(a^*(g_{t+1}), g_{t+1}) < \ln \eta(\tilde{a}^I(g_{t+1}), g_{t+1}).$$

In light of Eq. (18) and Lemma 1, $\forall g_{t+1} \geq 0$,

$$\ln \eta(a^*(g_{t+1}), g_{t+1}) < \frac{1-\alpha}{\alpha} \ln \frac{1-\alpha}{1-\alpha d}.$$

Then, the result follows from the first statement of Lemma 3, which shows that $\forall g_{t+1} \geq 0$,

$$\ln \eta(\tilde{a}^I(g_{t+1}), g_{t+1}) = \frac{1-\alpha}{\alpha} \ln \frac{1-\alpha}{1-\alpha d}; \tag{41}$$

$\square$

Proof of Proposition 1. Noting Eq. (6), the result holds if, $\forall g_{t+1} \in [0, \tilde{g}]$,

$$\ln \eta(\hat{a}(g_{t+1}), g_{t+1}) < \ln \eta(\tilde{a}^I(g_{t+1}), g_{t+1}).$$

Lemma 2 shows that $\forall g_{t+1} \geq 0$,

$$\ln \eta(\hat{a}(g_{t+1}), g_{t+1}) = \frac{1-\beta}{\beta} \ln \frac{1}{1-\gamma}. \tag{42}$$

Thus noting Eq. (41), the proposition holds under Eq. (A1), which is explicitly expressed in Footnote 17 $\square$

Proof of Lemma 4. The first statement and result (a) of the lemma are proven in a similar way to Lemma 1.

(b) Since, in view of Eq. (30), $\eta(a_t^i, g_{t+1})$ is increasing in a_t^i, the result holds if

$$\ln \eta(1, g_{t+1}) > \frac{1-\alpha}{\alpha} \ln \frac{1-\alpha/d}{1-\alpha} \quad \forall g_{t+1} \geq \tilde{g}.$$

Noting Eqs. (8) and (11), one finds that $\forall g_{t+1} \geq \tilde{g}$,

$$\begin{aligned} \eta(1, g_{t+1}) &\geq \eta(1, \tilde{g}) \\ &= d. \end{aligned}$$

These results and Eq. (18) prove Lemma 4(b).

(c) Noting Eq. (6), the result holds if, $\forall g_{t+1} \geq 0$,

$$\ln \eta(a^*(g_{t+1}), g_{t+1}) > \ln \eta(\tilde{a}^{II}(g_{t+1}), g_{t+1}).$$

In light of Eq. (18) and Lemma 1, $\forall g_{t+1} \geq 0$,

$$\ln \eta(a^*(g_{t+1}), g_{t+1}) > \frac{1-\alpha}{\alpha} \ln \frac{1-\alpha/d}{1-\alpha}.$$

On the other hand, Lemma 4 shows that $\forall g_{t+1} \geq 0$,

$$\ln \eta(\tilde{a}^{II}(g_{t+1}), g_{t+1}) = \frac{1-\alpha}{\alpha} \ln \frac{1-\alpha/d}{1-\alpha}. \tag{43}$$

The result therefore follows. □

Proof of Proposition 2. Noting Eq. (6), the result holds if, $\forall g_{t+1} \in [0, \tilde{g}]$,

$$\ln \eta(\tilde{a}^{II}(g_{t+1}), g_{t+1}) < \ln \eta(\hat{a}(g_{t+1}), g_{t+1}).$$

Comparing Eqs. (42) and (43) reveals that the proposition holds under Eq. (A1), which is explicitly expressed in Footnote 17. □

Proof of Lemma 5. (a) The result is obtained by noting that Eq. (22) is unaffected by d.

(b) A rise in d increases the term on the right side of Eq. (30) and that of Eq. (34). Then the results are obtained by applying the Implicit Function Theorem as in the proof of Lemma 1.

(c) If d is sufficiently close to 1,

$$\frac{1-\beta}{\beta} \ln \frac{1}{1-\gamma} \geq \frac{1-\alpha}{\alpha} \ln \frac{1-\alpha}{1-\alpha d}.$$

Then, the result is obtained by utilizing the proof of Proposition 1.

(d) A rise in d increases the term on the right side of Eq. (11). Then the result is obtained by applying the Implicit Function Theorem as in the proof of Lemma 1. $\square$

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