Japanese Yield Curves In and Out of a Zero Rate Environment: A Macro-Finance Perspective

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October 3, 2011
Revised: November 3, 2011

Abstract

This paper applies a tractable two-regime macro-finance affine term structure model to empirically investigate macroeconomic effects on Japanese government bond (JGB) yields in and out of a zero interest rate environment. The estimated results qualitatively assess how differently deflation and low growth contribute to lowering longer-term JGB yields between the normal and zero rate regimes.

1 Introduction

In light of more-than-a-decade of lasting low Japanese government bond (JGB) yields in and out of a zero rate environment with prolonged deflation and low growth, this paper empirically investigates macroeconomic effects on JGB yields by applying a no-arbitrage affine term structure model (ATSM) with macro structure. To date little work has applied such a framework to a zero rate environment due to complications arising from the zero lower bound of nominal interest rates. This paper attempts to fill this gap by incorporating a regime-dependent monetary policy rule into an ATSM with macro structure.

*I thank Kazuo Ueda and seminar participants at the Graduate School of International Corporate Strategy of Hitotsubashi University, Institute for Monetary and Economic Studies of Bank of Japan, the University of Tokyo, and Yokohama National University for their helpful comments.

The Japanese policy interest rate process (Figure 1) appears to have at least two regimes: a regime during which the policy interest rate is near zero and flat (the zero rate regime) and the other regime consisting of the remaining periods (the normal regime). I thus construct a model with two regimes. Exploiting information from Bank of Japan public policy announcements, I treat the regime as observable.[1] In short, this paper considers a two-regime process of the short-term interest rate (the policy interest rate), with the regime defined by an observable monetary policy regime.

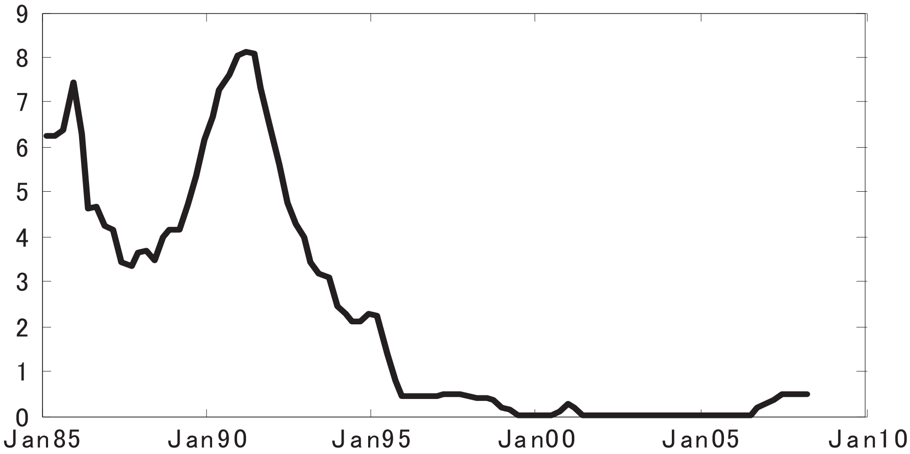

Figure 1. Uncollateralized overnight call rate (annualized rate in percent).

How can one model a regime dependent monetary policy rule in and out of a zero rate environment? One satisfactory approach is to directly impose a non-negativity constraint on the standard monetary policy rule, or the Taylor rule. This approach, however, cannot be handled by ATSM, and thereby the existing macrofinance term structure models applied to the Japanese zero rate environment mostly lie outside of the affine family (e.g., Oda and Ueda (2007) and Ichiue and Ueno (2006)).[2][3] This paper's model, on the other hand, lies within the affine family where a regime dependent rule

[1] Another approach to model the regime process is to treat it as unobservable. For example, see Fujiwara (2006) and Inoue and Okimoto (2008) for Markov-switcing models applied to the Japanese policy interest rate process.

[2] An exception is Bernanke, Reinhart, and Sack (2005) who use an ATSM that does not involve modeling of a zero lower bound.

[3] For more applications of ATSMs with no macro structure, see for example, Singleton and Kim

is modeled using a Markov chain with two regimes.

The use of tractable ATSMs with a zero lower bound is gaining more attention in light of the continued zero rate environment in Japan and the United States. Hamilton and Wu (2011, HW henceforth) examine US yield curves using a two-regime ATSM with a zero lower bound. Their model features include (i) the two regimes—the zero rate and the normal regimes—treated as observable to the econometrician, unlike the existing regime switching ATSMs (e.g., Bansal and Zhou (2002), Dai, Singleton, and Yang (2007), Ang, Bekaert, and Wei (2008)), (ii) regime dependent coefficients in the short rate dynamics ensuring that the short end of the yield curve is non-negative, and (iii) a constant probability of exiting the zero rate regime.

I extend HW in three directions. First, I introduce macro structure into HW's model given the importance of macro factors in explaining yield curve dynamics discussed in the literature (e.g., Ang and Piazessi (2003), Hördahl, Tristani, and Vestin (2006), and Rudebush and Wu (2008)). Specifically, I extend the Ang and Piazessi (2003) macro structure by allowing the short-term interest rate to follow a regime-dependent monetary policy rule and letting the dynamics of macro variables depend on the lagged short term interest rate. Second, given that Japan has experienced a zero rate environment more than once, I introduce a Markov chain to allow regimes to shift repeatedly. The model can thus explain shifts from the normal to the zero rate regime, as well as shifts in the other direction. Third, to intuitively interpret the probability measures in the model, I solve the model under the physical measure instead of under the risk-neutral measure.[4]

This paper's estimated results quantitatively assess how much prolonged deflation or low growth contributes to lowering longer-term JGB yields. The results also indicate that such macroeconomic effects weaken under the zero rate environment due to the invariability of the short-term interest rate to macroeconomic fluctuations with a high zero rate commitment. Furthermore, the term premium component of bond yield is estimated via two-regime three-variable VAR forecasting: the estimated component

(2010). They investigate JGB yield curves using the model of Cox, Ingersoll, and Ross (1985, CIR henceforth) with the intention of comparing its performance with those under non-affine models.

[4] For details, see Appendix A.

drove fluctuations in long term yields under the zero rate regime and contributed to a decline in long term yields in the late 1990s. Lastly, the paper's results are consistent with the previous findings that a zero-rate commitment effectively brings down the market participants' expectations of future policy rates and the yield curves flatten on average in the zero rate environment (e.g., Okina and Shiratsuka (2004), Baba et. al. (2005), Oda and Ueda (2007), and Nakazono and Ueda (2011)).[5]

This paper proceeds as follows. Section 2 describes the macro-finance term structure model. Section 3 sets out the estimation strategy, and Section 4 discusses estimated results and robustness checks. Section 5 concludes.

2 The Model

I consider a discrete-time affine term structure model of the sort employed by Ang and Piazzesi (2003) as a point of departure and generalize it in two directions. First, I allow the short-term interest rate to follow a regime dependent monetary policy rule. Thus the model can consider changes in yield dynamics in and out of the zero rate environment. Second, I allow the dynamics of macro variables to depend on the lagged short term interest rate as well as their own lagged variables, in a spirit similar to Ang, Piazzesi, and Wei (2006) and Hördahl, Tristani, and Vestin (2006). Thus the policy interest rate can directly influence future macro variables. In the next estimation-strategy section, similar to Koeda and Kato (2010), I will explain that the inclusion of the lagged short term interest rate requires modifying the Ang and Piazzesi-type specification of the system of equations.

2.1 The Markov chain

I consider an observable Markov chain with two regimes: the zero rate regime ($d_t = 0$) and the normal regime ($d_t = 1$). The i, j element of the 2×2 transition matrix is given

[5] See Ugai (2007) for a comprehensive survey on the empirical work on the effects of quantitative easing policy in Japan.

by $\pi_{ij} = \Pr(d_t = j | d_{t-1} = i)$. Thus, π_{00} (π_{11}) is the probability that the zero rate regime (the normal regime) continues on to the next period.

This Markov chain is more general than HW's, as it allows regimes to shift repeatedly. In HW, a two-regime setting is introduced only when the current regime is the zero rate regime; shifts from the normal to the zero rate regime are, then, not explained.

2.2 Short-term interest rate and macro-variable dynamics

While HW omit macro variables from the state vector, I include them in the vector together with the short-term interest rate. I employ the standard Taylor rule that includes the lagged short-term interest rate. The baseline dynamics of short-term interest rate and macro variables are given by

$$r_{1,t} = \underset{1\times 1}{\mu_0^{d_t}} + \underset{1\times 1}{\mu_1^{d_t}} r_{1,t-1} + \underset{1\times 2}{\boldsymbol{\mu}_2^{d_t}} X_t + \sigma_r^{d_t} \upsilon_t, \tag{1}$$

$$X_t = \underset{2\times 1}{\gamma_0} + \underset{2\times 1}{\gamma_1} r_{1,t-1} + \underset{2\times 2}{\rho} X_{t-1} + \underset{2\times 2}{\Sigma} \varepsilon_t, \tag{2}$$

$$X_t = [x_{1t}, x_{2t}]'. \tag{3}$$

where r_1 is the short rate and X is the 2×1 vector of inflation (x_1) and output gap (x_2) following an autoregressive process.[6] The Taylor-rule coefficients are regime dependent, and those coefficients under the zero rate regime are restricted to being $\mu_0^0 = c$, $\mu_1^0 = 0$, $\boldsymbol{\mu}_2^0 = [0, 0]$; thus, the short-term interest rate under the zero rate regime is close to a near-zero positive constant, c,[7] if σ_r^0 is sufficiently small. A scalar random shock υ and a 2×1 random shock vector ε are assumed to be standard normal and independent to each other and over time. Σ is an upper triangular matrix.

[6] This two-regime specification allows short-rate volatility to be time dependent. For a discussion on the role of short rate volatility in macro-finance term structure models, see for example, Koeda and Kato (2010).

[7] A positive value of c is supported theoretically, for example, see the optimal monetary policy rule proposed by Jung, Teranishi, and Watanabe (2005).

Substituting (2) into (1) to obtain

$$
\begin{aligned}
r_{1,t} &= \mu_0^{d_t} + \mu_1^{d_t} r_{1,t-1} + \boldsymbol{\mu}_2^{d_t} X_t + \sigma_r^{d_t} \upsilon_t, \\
&= \mu_0^{d_t} + \mu_1^{d_t} r_{1,t-1} + \boldsymbol{\mu}_2^{d_t} \left(\gamma_0 + \gamma_1 r_{1,t-1} + \rho X_{t-1} + \Sigma \varepsilon_t\right) + \sigma_r^{d_t} \upsilon_t, \\
&= \underbrace{\left(\mu_0^{d_t} + \boldsymbol{\mu}_2^{d_t} \gamma_0\right)}_{\overline{\mu}_0^{d_t}} + \underbrace{\left(\mu_1^{d_t} + \boldsymbol{\mu}_2^{d_t} \gamma_1\right)}_{\overline{\mu}_1^{d_t}} r_{1,t-1} + \underbrace{\left(\boldsymbol{\mu}_2^{d_t} \rho\right)}_{\bar{\boldsymbol{\mu}}_2^{d_t}} X_{t-1} + \sigma_r^{d_t} \upsilon_t + \boldsymbol{\mu}_2^{d_t} \Sigma \varepsilon_t.
\end{aligned} \tag{4}
$$

where $\overline{\mu}_0^{d_t} \equiv \mu_0^{d_t} + \boldsymbol{\mu}_2^{d_t} \gamma_0$, $\overline{\mu}_1^{d_t} \equiv \mu_1^{d_t} + \boldsymbol{\mu}_2^{d_t} \gamma_1$, and $\bar{\boldsymbol{\mu}}_2^{d_t} \equiv \boldsymbol{\mu}_2^{d_t} \rho$.

The above short-term interest rate and macro-variable dynamics can be rewritten in more concise form

$$
\underbrace{\begin{bmatrix} r_{1,t} \\ X_t \end{bmatrix}}_{f_t} = \underbrace{\begin{bmatrix} \bar{\mu}_0^{d_t} \\ \gamma_0 \end{bmatrix}}_{c_f^{d_t}} + \underbrace{\begin{bmatrix} \bar{\mu}_1^{d_t} & \bar{\boldsymbol{\mu}}_2^{d_t} \\ \gamma_1 & \rho \end{bmatrix}}_{\rho_f^{d_t}} f_{t-1} + \underbrace{\begin{bmatrix} \sigma_r^{d_t} & \boldsymbol{\mu}_2^{d_t} \Sigma \\ 0 & \Sigma \end{bmatrix}}_{\Sigma_f^{d_t}} \underbrace{\begin{bmatrix} \upsilon_t \\ \varepsilon_t \end{bmatrix}}_{e_t}, \tag{5}
$$

$$
\text{or,} \qquad f_t = \underset{3\times 1}{c_f^{d_t}} + \underset{3\times 3}{\rho_f^{d_t}} f_{t-1} + \underset{3\times 3}{\Sigma_f^{d_t}} e_t. \tag{6}
$$

2.3 The pricing kernel and the prices of risk

With the state (d, f), the no-arbitrage condition under the physical measure is given by

$$
E\left[\left. \frac{M_{t+1} P_{t+1}^{n-1}}{P_t^n} \right| d_t, f_t \right] - 1 = 0, \tag{7}
$$

where P^n is the n-period bond price and M is the pricing kernel. Following the convention in the ATSM literature, the pricing kernel is assumed to be

$$
M_{t+1} = \exp\left(-r_{1,t} - \frac{1}{2} \lambda_t' \Sigma_f^{d_{t+1}} \Sigma_f^{d_{t+1}\prime} \lambda_t - \lambda_t' \Sigma_f^{d_{t+1}} e_{t+1} \right), \tag{8}
$$

thus the stochastic discount factor depends on d as well as f. The prices of risk (λ) are an affine function of the factors and the regime

$$
\lambda_t = \underset{3\times 1}{\lambda_0^{d_t}} + \underset{3\times 3}{\lambda_1^{d_t}} f_t, \tag{9}
$$

thus the stocastic discount factor depends on d as well as f. By the law of iterated expectation, the LHS of (7) can be rewritten as

$$E\left\{E\left[\frac{M_{t+1}P_{t+1}^{n-1}}{P_t^n}-1\middle| d_{t+1},d_t,f_t\right]\middle| d_t,f_t\right\} \tag{10}$$
$$=\pi_{d_t1}\left\{E\left[\frac{M_{t+1}P_{t+1}^{n-1}}{P_t^n}-1\middle| d_{t+1}=1,d_t,f_t\right]\right\}+\pi_{d_t0}\left\{E\left[\frac{M_{t+1}P_{t+1}^{n-1}}{P_t^n}-1\middle| d_{t+1}=0,d_t,f_t\right]\right\},$$

with the Markov chain being independent of f in the sense that the transition probability does not depend on f. Using the approximation used by Hamilton and Wu (2011), I show that the n-period, zero-coupon bond yield is given by (see Appendix A for derivation)

$$\begin{aligned} r_{n,t} &= \underset{1\times1}{a_n^{d_t}}+\underset{1\times1}{b_n^{d_t}}r_{1,t}+\underset{1\times2}{\mathbf{c}_n^{d_t}}X_t, && (11)\\ &= a_n^{d_t}+\left[b_n^{d_t},\mathbf{c}_n^{d_t}\right]f_t, \\ a_1^{d_t} &= 0,\ b_1^{d_t}=1,\ \mathbf{c}_1^{d_t}=[0,0]. && (12)\end{aligned}$$

with the yield-equation coefficients $(a_n^{d_t},b_n^{d_t},\mathbf{c}_n^{d_t})$ derived recursively by

$$\bar{a}_n^{d_t}=\sum_{j=0,1}\pi_{d_tj}\left(\bar{a}_{n-1}^j+\bar{b}_{n-1}^j\bar{\mu}_0^j+\bar{\mathbf{c}}_{n-1}^j\gamma_0+\frac{1}{2}K_{n-1}^jK_{n-1}^{j\prime}-K_{n-1}^j\Sigma_f^{j\prime}\lambda_0^{d_t}\right), \tag{13}$$

$$\left[\bar{b}_n^{d_t},\bar{\mathbf{c}}_n^{d_t}\right]=\sum_{j=0,1}\pi_{d_tj}\left[\delta_{n-1}^j-K_{n-1}^j\Sigma_f^{j\prime}\lambda_1^{d_t}\right], \tag{14}$$

where

$$a_n^{d_t}=-\bar{a}_n^{d_t}/n,\ b_n^{d_t}=-\bar{b}_n^{d_t}/n,\ \mathbf{c}_n^{d_t}=-\bar{\mathbf{c}}_n^{d_t}/n, \tag{15}$$
$$\delta_{n-1}^j=\left[\bar{b}_{n-1}^j\bar{\mu}_1^j+\bar{\mathbf{c}}_{n-1}^j\gamma_1-1,\bar{b}_{n-1}^j\bar{\boldsymbol{\mu}}_2^j+\bar{\mathbf{c}}_{n-1}^j\rho\right], \tag{16}$$
$$K_{n-1}^j=\left[\bar{b}_{n-1}^j\sigma_r^{d_t},\bar{b}_{n-1}^j\boldsymbol{\mu}_2^j\Sigma+\bar{\mathbf{c}}_{n-1}^j\Sigma\right]. \tag{17}$$

The recursive equations reduce to those derived by HW if (i) the d_t process is a Markov chain under the risk-neutral measure, (ii) the prices of risk coefficients do not vary across regimes (i.e., setting $\lambda_0^1=\lambda_0^0$ and $\lambda_1^1=\lambda_1^0$), and (iii) the restrictions on the short-end yield equation coefficients (i.e., $a_1^{d_t},b_1^{d_t},\mathbf{c}_1^{d_t}$) differ across regimes.

3 Estimation strategy

I use quarterly data on interest rates and macro variables of inflation and output gap from 1985Q1 to 2008Q2. I use quarterly data because readily available monthly real activity measures in Japan, for example, industrial production, unemployment, the machinery orders, may not reflect the overall economic activity. The sample period starts from 1985Q1 in the benchmark estimation because reliable zero coupon bond yield data are available from that quarter; it ends in 2008Q2 examining the period prior to the Lehman shock. In Section 4.3, I discuss estimated results with alternative sample periods and monthly data frequency.

The 1-quarter zero coupon bond yields are used for the short-term interest rate and zero coupon bond yields of 2, 8, 20, and 40 quarter maturities are used for longer maturities; These bond yields are obtained from Wright's (2011) dataset. All bond yields are expressed at annualized rates in percent.

Regarding the macro variables, inflation is measured by quarterly percentage change in the seasonally adjusted GDP deflator from the main economic indicators of the Organization for Economic Cooperation and Development (OECD); real activity is measured by output gap estimated by applying the Hodrik-Prescott filter on the logs of the seasonally adjusted GDP at 2000 prices from the Japan Cabinet office. Output gap is expressed in percentage points.

The regime series is constructed based on public announcements. It takes value 0 under the so-called zero interest rate policy (ZIRP) in the period 1999Q2–2000Q3 and under the quantitative monetary easing policy (QMEP) including a short zero rate period that followed in the period 2001Q2–2006Q2.

The model consists of macro dynamics and static yield equations. The macro dynamics are summarized by equation (5) and the static yield equations are as follows:

$$R_t = A^{d_t} + \left[B^{d_t}, \mathbf{C}^{d_t}\right] f_t + \Sigma^m \varepsilon_t^m,$$

where $R_t = [r_t^2, r_t^8, r_t^{20}, r_t^{40}]'$ is a 4×1 vector of bond yields with maturities corresponding to the superscript numbers (in months). The yield equations are an affine function of the state variables with 4×1 coefficient vectors of A^{d_t} and B^{d_t} and a 4×2 coefficient matrix of $\mathbf{C}^{d_t}$ corresponding to (i) the constant term, (ii) the short-term interest rate term, and (iii) the macro-variable term, respectively. The subscript numbers in A^{d_t}, B^{d_t}, and $\mathbf{C}^{d_t}$ correspond to maturities (i.e., $A^{d_t} = \left[a_2^{d_t}, a_8^{d_t}, a_{20}^{d_t}, a_{40}^{d_t}\right]'$, $B^{d_t} = \left[b_2^{d_t}, b_8^{d_t}, b_{20}^{d_t}, b_{40}^{d_t}\right]'$ and $\mathbf{C}^{d_t} = \left[\mathbf{c}_2^{d_t}, \mathbf{c}_8^{d_t}, \mathbf{c}_{20}^{d_t}, \mathbf{c}_{40}^{d_t}\right]'$.) The elements in A^{d_t}, B^{d_t}, and $\mathbf{C}^{d_t}$ are derived from the recursive equations with the subscript numbers corresponding to maturities. Measurement errors ε^m are assumed to have constant variance and Σ^m is a diagonal matrix.

The system of equations to be estimated can be summarized as follows:

$$\underbrace{\begin{bmatrix} f_t \\ R_{t-1} \end{bmatrix}}_{Y_t} = \underbrace{\begin{bmatrix} c_f^{d_t} \\ A^{d_{t-1}} \end{bmatrix}}_{A_Y^{d_{t-1}d_t}} + \underbrace{\begin{bmatrix} \rho_f^{d_t} \\ \left[B^{d_{t-1}}, \mathbf{C}^{d_{t-1}}\right] \end{bmatrix}}_{B_Y^{d_{t-1}d_t}} f_{t-1} + \underbrace{\begin{bmatrix} \Sigma_f^{d_t} & 0 \\ 0 & \Sigma^m \end{bmatrix}}_{\Sigma_Y^{d_t}} \begin{bmatrix} e_t \\ \varepsilon_{t-1}^m \end{bmatrix}, \qquad (18)$$

where e and ε^m are iid standard normal and e_t and ε_s^m are independent for all (t, s). Thus, the observation equation linking R_{t-1} to the state (f_t) is appended to the VAR equations describing the state dynamics. I estimate this system using the maximum likelihood method (for details, see Appendix B).[8]

4 Estimation results

4.1 Benchmark estimation

The parameter estimates of the model are reported in Table 1. The Taylor rule coefficients are statistically significant with the correct signs. Short-rate volatility under the zero-rate regime is notably less than that under the normal regime (i.e., $\sigma_r^0 > \sigma_r^1$). The term-structure risks arise from the short-term interest rate, inflation, and real activity, and they are regime dependent. The prices of risk coefficients other than the constant term of the prices of risk equation are set to zero (i.e., $\lambda_1^1 = \lambda_1^0 = 0$) as they have very

[8] The sample here is $(y_1, ..., y_T) = (r_{1,1}, X_1, R_0; r_{1,2}, X_2, R_1; ..., r_{1,T}, X_T, R_{T-1})$. It may appear more natural to consider the sample $(r_{1,1}, X_1, R_1; r_{1,2}, X_2, R_2; ..., r_{1,T}, X_T, R_T)$, but the usual factorization argument can be more readily applied to the former. If the sample size T is large, the choice of the sample would not matter for the point estimation.

large standard errors. Thus the term structure risks affect only the recursive equations of $a_n^{d_t}$ through the constant term of the prices of risk equation.

Probability coefficients

π_{11}	π_{00}
0.957	0.965
(0.027)	(0.033)

Short-rate dynamics

$\overline{\mu}_0^1$	$\overline{\mu}_1^1$	$\overline{\mu}_2^1$		σ_r^1	σ_r^0
0.091	0.975	0.239	0.002	0.536	0.101
(0.182)	(0.029)	(0.058)	(0.008)	(0.072)	(0.014)

Dynamics of macro variables

γ_0	γ_1	ρ		Σ	
-0.251	0.107	0.497	0.027	0.173	---
(0.035)	(0.015)	(0.061)	(0.015)	(0.014)	---
-0.251	-0.001	-0.747	0.964	0.006	0.882
(0.263)	(0.111)	(0.439)	(0.060)	(0.101)	(0.099)

Prices of risk (x 1/100)

	λ_0^1	λ_0^0
short rate	-0.009	-0.031
	(0.023)	(0.300)
inflation	-0.186	-0.410
	(0.486)	(4.901)
real activity	-0.376	-0.066
	(0.204)	(0.074)

Measurement error

2 quarters	8 quarters	20 quarters	40 quarters
0.113	0.354	0.489	0.781
(0.017)	(0.126)	(0.198)	(0.378)

Table 1. Estimated Parameters. This table reports estimated coefficients in the benchmark estimation. Numbers in parenthesis indicate standard errors. Measurement error is the estimated standard error of measurement error corresponding to each maturity.

The probability that the zero rate regime continues into the next period (π_{00}) captures the effect of the zero rate commitment on market participants' expectations of future policy rate; it is estimated to be relatively close to 1 (0.965).[9] The high value of π_{00} implies that interest rate expectations are effectively brought down to a low level. I also discuss estimated results allowing π_{00} to be time dependent on Section 4.2.

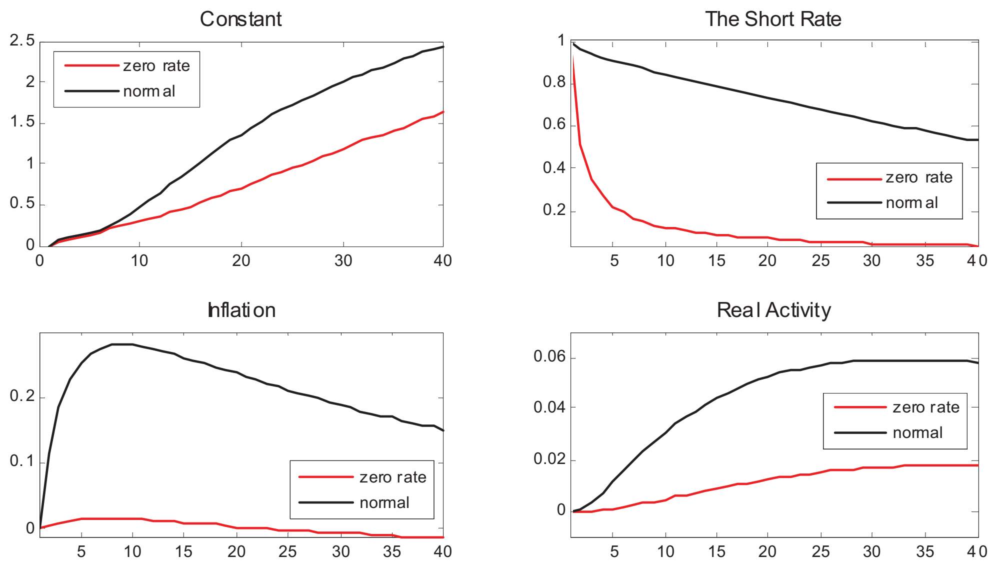

Figure 2. Factor weights against maturity in the benchmark estimation. This figure plots the coefficients of the yield equation against maturity (in quarters).

Figure 2 shows how the yield-equation coefficients change against maturity under the normal regime (dashed lines) and the zero rate regime (red solid lines). The constant, short-term interest rate, inflation, and real activity terms correspond to $a_n^{d_t}$, $b_n^{d_t}$, and $\mathbf{c}_n^{d_t}$ respectively with the model implied yields (given by eq(11)) are expressed at the annualized rate in percent. The upward slopes of a_n^1 and a_n^0 represent the shapes of average yield curves under the normal and zero rate regimes. They imply that yield curves flatten on average under the zero rate regime. The downward slopes of b_n^1 and b_n^0 imply that an increase in the short-term interest rate has a more positive impact on the shorter-end of yield curves. The shape of b_n^0 implies that under the zero rate regime, the

[9] Dai, Singleton, and Yang (2006) find that constant regime-shift probabilities under the physical measure lead to high regime persistency. Their conclusion, however, is not readily applicable to the model, because the model allows (i) the factor coefficients in the short rate dynamics to vary across regimes and (ii) the factor dynamics depends on the Markov process governing regime changes.

short-rate level itself has little impact on the long end of yield curves. The shapes of $\mathbf{c}_n^1$ and $\mathbf{c}_n^0$ capture the positive impact of macro variables on yield curves. The flatter shape of $\mathbf{c}_n^0$ implies that macroeconomic effects on JGB yields weaken under the zero rate regime. A closer look at the recursive equations for $\mathbf{c}_n^{d_t}$[10] indicates that this delinkage under the benchmark estimation is caused largely by the short rate's invariability to macroeconomic variables with a high value of π_{00}.

The bottom two charts in Figure 2 thus demonstrate how differently deflation and low growth contribute to lowering longer-term JGB yields between the normal and zero rate regimes. Under the normal regime, 1-percent deflation lowers 2- and 10-year JGB yields by 28 and 15 basis points, respectively, and 1-percent output gap increase raises 2- and 10-year JGB yields by 2 and 6 basis points, respectively. On the other hand, under the zero rate regime, the macroeconomic effects on the JGB yields are not apparent with wide standard deviation bands (Figure AC-1).

4.2 Term premia

The long term bond yields can be decomposed into the expectations and term premium components. I define the term premium of n-period bond yield as the actual n-period bond yield minus the average of the expected future short-term interest rates (i.e., $\frac{1}{n}E_t\left\{\sum_{j=0}^{n-1} r_{1,t+j}\right\}$), and calculate the expectations components via two-regime three-variable VAR (eq. (6)) forecasting. Figure 3 reports the model implied term premia of 5-year bonds, the corresponding averages of expected future short-term interest rates, and the actual yields. It indicates that a large bond yield decline in early 1990s was driven by the expectations components; whereas that in late 1990s was driven by both expectations and term premium components. It also suggests that the long rate fluctuations under the zero rate regime were driven by term-premium dynamics which declined after the

[10] In the benchmark estimation, the recursive equations of $\bar{b}_n^{d_t}$ and $\bar{\mathbf{c}}_n^{d_t}$ are simplified as $\bar{b}_n^{d_t} = \sum_{j=0,1} \pi_{d_t j}\left[\bar{b}_{n-1}^j \bar{\mu}_1^j + \bar{\mathbf{c}}_{n-1}^j \gamma_1 - 1\right]$ and $\bar{\mathbf{c}}_n^{d_t} = \sum_{j=0,1} \pi_{d_t j}\left[\bar{b}_{n-1}^j \bar{\boldsymbol{\mu}}_2^j + \bar{\mathbf{c}}_{n-1}^j \rho\right]$.

QMEP introduction.

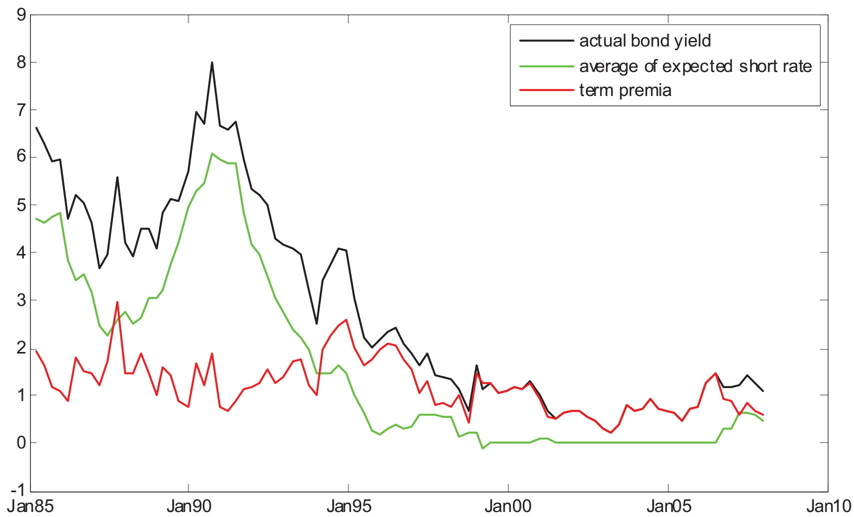

Figure 3. Estimation of expectations and term premium components of 5 year bond yields (in annualized rate in percent).

4.3 Robustness checks

The robustness checks of these benchmark results are three fold aiming at testing (i) different degrees of zero rate commitment (π_{00}) across zero rate periods, (ii) monthly data, and (iii) alternative sample periods. The estimated results discussed in this subsection are available upon request.

Allowing π_{00} to be time dependent So far, as in HW, π_{00} is assumed to be constant in our model. However, some may wonder if such a degree of commitment changes across different zero rate periods. To address this concern, I reestimate the model allowing π_{00} to take two different values, p_1 and p_2; market participants are assumed to use p_1 as the basis of their bond yield projections for the period 1985Q1–2001Q1, and p_2 thereafter. Thus this specification of π_{00} assumes that the π_{00} perceived by market participants will not be updated with a new value until the next zero rate period begins. The estimated p_1 is smaller than p_2 (0.95 and 0.97 respectively). Accordingly, after the

QMEP introduction, the estimated degree of zero rate commitment increased and the effect of macroeconomic variables on JGB yield curves weakened (Figure AC-2).

Monthly data Some may be interested in examining results using monthly data. I thus reestimate the model with monthly data replacing the macro variables with the consumer price index (excluding food and energy) from the Japan Statistics Bureau and industrial production from the Ministry of Economy, Trade and Industry. These macro variables are expressed as the year-on-year difference in logs of the original series. The key results from the benchmark estimation remain broadly unchanged.

Alternative sample periods Some may be concerned about possible structural breaks, for example a structural break in mid-90s suggested by several researchers (e.g., Miyao (2000), Fujiwara (2006), and Inoue and Okimoto (2008)). I thus reestimated the model choosing 1995Q1 as the new starting date. Given this shorter sample periods, I estimated the model using both monthly and quarterly data. The magnitude of macroeconomic effects on the bond yields under the normal regime is unchanged on the short and middle part of yield curves, but it declines on the long end of yield curve (Figure AC-3). Other main results from the benchmark estimation remain broadly unchanged with this shorter sample period.

Next, I extend the sample period to 2010Q4[11] to include the recent global financial crisis. In December 2008, the Bank of Japan announced a policy to induce the uncollateralized overnight call rate to 0.1 percent. Since then the call rate has remained at around 0.1 percent. I thus set $d_t = 0$ from 2009Q1 and re-estimate the model using the extended sample period. The key results from the benchmark estimation remain broadly unchanged.

[11] Wright's (2011) bond yield data ends in May 2009. We thus extend his data by estimating zero-coupon bond yields using data on bond prices, coupon rates, and issue and redemption dates for all available 5, 10, and 20 year government bonds outstanding on the given date; these data were taken from e-AURORA database from the Nomura Research Institute. A cubic spline is fitted each month to the yields on all sample bonds with maturities up to 20 years. All bond yields are continuously compounded and expressed at annualized rates in percent.

5 Conclusion

This paper applies a standard macrofinance ATSM with a two-regime setting to a zero rate environment, providing formal empirical evidence on Japan's experience with zero interest rates. The estimated results quantitatively assess how deflation and low growth contribute to lowering longer-term JGB yields, revealing how differently macroeconomic variables affect the JGB yields across regimes. Furthermore, the results suggest that the time-varying term premium component of bond yields played an important role particularly in the late 1990s and under the zero rate regime.

Looking forward, when Japan finally emerges from a zero rate environment, the increased macrofinance linkage under the normal regime will imply that channels that can steeply raise macroeconomic variables could pose a risk to the JGB markets.

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A Recursive bond prices under the physical measure

Recall that the no-arbitrage condition under the P measure is given by[12]

$$E\left[\left.\frac{M_{t+1}P_{t+1}^{n-1}}{P_t^n}\right| d_t, f_t\right] - 1 = 0, \tag{A-1}$$

By the law of iterated expectation, the LHS of (A-1) can be rewritten as

$$\begin{aligned}
&E\left\{\left. E\left[\left.\frac{M_{t+1}P_{t+1}^{n-1}}{P_t^n} - 1\right| d_{t+1}, d_t, f_t\right]\right| d_t, f_t\right\} \\
&= \pi_{d_t 1}\left\{\underbrace{E\left[\left.\frac{M_{t+1}P_{t+1}^{n-1}}{P_t^n} - 1\right| d_{t+1} = 1, d_t, f_t\right]}_{J_1}\right\} + \pi_{d_t 0}\left\{\underbrace{E\left[\left.\frac{M_{t+1}P_{t+1}^{n-1}}{P_t^n} - 1\right| d_{t+1} = 0, d_t, f_t\right]}_{J_2}\right\}.
\end{aligned} \tag{A-2}$$

with the Markov chain being independent of f in the sense that the transition probability does not depend on f. Using the basic relationship between bond yields and prices

[12] The no-arbitrage condition under the Q measure is given by $E^Q\left\{\left.\frac{P_{t+1}^{n-1}}{P_t^n}\right| d_t, f_t\right\} - 1 = 0$.

(i.e., $r_t^n = -\log P_t^n$), (2), (4), (8), and (11), J_1 can be rewritten as

$$J_1 = E\left[\begin{array}{c} \exp(-r_{t,1} - \frac{1}{2}\lambda_t'\Sigma_f^1\Sigma_f^{1\prime}\lambda_t - \lambda_t'\Sigma_f^1 e_{t+1}) \\ \times\exp(\bar{a}_{n-1}^1 + \bar{b}_{n-1}^1 r_{1,t+1} + \bar{\mathbf{c}}_{n-1}^1 X_{t+1}) \\ \times\exp(-\bar{a}_n^{d_t} - \bar{b}_n^{d_t} r_{1,t} - \bar{\mathbf{c}}_n^{d_t} X_t) - 1 \end{array}\middle| d_{t+1} = 1, d_t, f_t\right],$$

$$= E\left[\begin{array}{c} \exp(-r_{t,1} - \frac{1}{2}\lambda_t'\Sigma_f^1\Sigma_f^{1\prime}\lambda_t - \lambda_t'\Sigma_f^1 e_{t+1}) \\ \times\exp\left(\begin{array}{c} \bar{a}_{n-1}^1 + \bar{b}_{n-1}^1\left[\bar{\mu}_0^1 + \bar{\mu}_1^1 r_{1,t} + \bar{\boldsymbol{\mu}}_2^1 X_t + \sigma_r^1 \upsilon_{t+1} + \boldsymbol{\mu}_2^1\boldsymbol{\Sigma}\varepsilon_{t+1}\right] \\ +\bar{\mathbf{c}}_{n-1}^1\left[\gamma_0 + \gamma_1 r_{1,t} + \rho X_t + \Sigma\varepsilon_{t+1}\right] \end{array}\right) \\ \times\exp(-\bar{a}_n^{d_t} - \bar{b}_n^{d_t} r_{1,t} - \bar{\mathbf{c}}_n^{d_t} X_t) \end{array}\middle| d_{t+1} = 1, d_t, f_t\right] - 1,$$

$$= E\left[\begin{array}{c} \exp(-\frac{1}{2}\lambda_t'\Sigma_f^1\Sigma_f^{1\prime}\lambda_t + \bar{a}_{n-1}^1 + \bar{b}_{n-1}^1\bar{\mu}_0^1 + \bar{\mathbf{c}}_{n-1}^1\gamma_0) \\ \times\exp(\underbrace{\left[\bar{b}_{n-1}^1\bar{\mu}_1^1 + \bar{\mathbf{c}}_{n-1}^1\gamma_1 - 1\right]}_{\delta_{1,n-1}^1} r_{1,t} + \underbrace{\left[\bar{b}_{n-1}^1\bar{\boldsymbol{\mu}}_2^1 + \bar{\mathbf{c}}_{n-1}^1\rho\right]}_{\boldsymbol{\delta}_{2,n-1}^1} X_t) \\ \times\exp\left(\left\{\underbrace{\left[\bar{b}_{n-1}^1\sigma_r^1, \bar{b}_{n-1}^1\boldsymbol{\mu}_2^1\Sigma + \bar{\mathbf{c}}_{n-1}^1\Sigma\right]}_{K_{n-1}^1} - \lambda_t'\Sigma_f^1\right\} e_{t+1}\right) \\ \times\exp(-\bar{a}_n^{d_t} - \bar{b}_n^{d_t} r_{1,t} - \bar{\mathbf{c}}_n^{d_t} X_t) \end{array}\middle| d_{t+1} = 1, d_t, f_t\right] - 1,$$

$$= \left[\begin{array}{c} \exp(-\frac{1}{2}\lambda_t'\Sigma_f^1\Sigma_f^{1\prime}\lambda_t + \bar{a}_{n-1}^1 + \bar{b}_{n-1}^1\bar{\mu}_0^1 + \bar{\mathbf{c}}_{n-1}^1\gamma_0) \\ \times\exp(\underbrace{\left[\delta_{1,n-1}^1, \boldsymbol{\delta}_{2,n-1}^1\right]}_{\delta_{n-1}^1} f_t)\exp(\frac{1}{2}K_{n-1}^1 K_{n-1}^{1\prime} + \frac{1}{2}\lambda_t'\Sigma_f^1\Sigma_f^{1\prime}\lambda_t - K_{n-1}^1\Sigma_f^{1\prime}\lambda_t) \\ \times\exp(-\bar{a}_n^{d_t} - \bar{b}_n^{d_t} r_{1,t} - \bar{\mathbf{c}}_n^{d_t} X_t) \end{array}\right] - 1,$$

$$= \left[\begin{array}{c} \exp(\bar{a}_{n-1}^1 + \bar{b}_{n-1}^1\bar{\mu}_0^1 + \bar{\mathbf{c}}_{n-1}^1\gamma_0 + \frac{1}{2}K_{n-1}^1 K_{n-1}^{1\prime}) \\ \times\exp(\delta_{n-1}^1 f_t)\exp\left(-K_{n-1}^1\Sigma_f^{1\prime}\left(\lambda_0^{d_t} + \lambda_1^{d_t} f_t\right)\right) \\ \times\exp(-\bar{a}_n^{d_t} - \bar{b}_n^{d_t} r_{1,t} - \bar{\mathbf{c}}_n^{d_t} X_t) \end{array}\right] - 1,$$

$$= \left[\exp\left(\begin{array}{c} \bar{a}_{n-1}^1 + \bar{b}_{n-1}^1\bar{\mu}_0^1 + \bar{\mathbf{c}}_{n-1}^1\gamma_0 + \frac{1}{2}K_{n-1}^1 K_{n-1}^{1\prime} - K_{n-1}^1\Sigma_f^{1\prime}\lambda_0^{d_t} \\ + \left[\delta_{n-1}^1 - K_{n-1}^1\Sigma_f^{1\prime}\lambda_1^{d_t}\right] f_t \\ -\bar{a}_n^{d_t} - \bar{b}_n^{d_t} r_{1,t} - \bar{\mathbf{c}}_n^{d_t} X_t \end{array}\right)\right] - 1. \qquad (19)$$

If the expression inside exponential form in (19) is small enough,[13] using the approximation used by Hamilton and Wu (2011) (i.e., $x \simeq \exp(x) - 1$), J_1 can be approximately

[13] In benchmark estimation, the absolute value of this expression varies between 1.37E-07 and 0.146.

rewritten as

$$J_1 \simeq \bar{a}^1_{n-1} + \bar{b}^1_{n-1}\bar{\mu}^1_0 + \bar{\mathbf{c}}^1_{n-1}\gamma_0 + \frac{1}{2}K^1_{n-1}K^{1\prime}_{n-1} - K^1_{n-1}\Sigma^{1\prime}_f\lambda^{d_t}_0 \tag{A-3}$$
$$+ \left[\delta^1_{n-1} - K^1_{n-1}\Sigma^{1\prime}_f\lambda^{d_t}_1\right] f_t - \bar{a}^{d_t}_n - \bar{b}^{d_t}_n r_{1,t} - \bar{\mathbf{c}}^{d_t}_n X_t.$$

Similarly J_2 can approximated by

$$J_2 \simeq \bar{a}^0_{n-1} + \bar{b}^0_{n-1}\bar{\mu}^0_0 + \bar{\mathbf{c}}^0_{n-1}\gamma_0 + \frac{1}{2}K^0_{n-1}K^{0\prime}_{n-1} - K^0_{n-1}\Sigma^{0\prime}_f\lambda^{d_t}_0$$
$$+ \left[\delta^0_{n-1} - K^0_{n-1}\Sigma^{0\prime}_f\lambda^{d_t}_1\right] f_t - \bar{a}^{d_t}_n - \bar{b}^{d_t}_n r_{1,t} - \bar{\mathbf{c}}^{d_t}_n X_t.$$

Thus the bond price equation can be approximately rewritten as

$$P^n_t = \exp\left(\bar{a}^{d_t}_n + \left[\bar{b}^{d_t}_n, \bar{\mathbf{c}}^{d_t}_n\right] f_t\right)$$

where

$$\bar{a}^{d_t}_n = \sum_{j=0,1} \pi_{d_t j}\left(\bar{a}^j_{n-1} + \bar{b}^j_{n-1}\bar{\mu}^j_0 + \bar{\mathbf{c}}^j_{n-1}\gamma_0 + \frac{1}{2}K^j_{n-1}K^{j\prime}_{n-1} - K^j_{n-1}\Sigma^{j\prime}_f\lambda^{d_t}_0\right),$$

$$\left[\bar{b}^{d_t}_n, \bar{\mathbf{c}}^{d_t}_n\right] = \sum_{j=0,1} \pi_{d_t j}\left[\delta^j_{n-1} - K^j_{n-1}\Sigma^{j\prime}_f\lambda^{d_t}_1\right].$$

B The log likelihood function

In preparation for the following discussion, define $Z_t = [d_t, Y_t]$ and recall that the model can be summarized as (18). I wish to describe the joint density of $(Z_t, Z_{t-1}, ..., Z_1)$ given Z_0 with the parameters to be estimated given by[14]

$$\theta = \left[\pi_{11}, \pi_{00}, \bar{\mu}^1_0, \bar{\mu}^1_1, \bar{\mu}^1_2, \sigma^0_r, \sigma^1_r, \gamma_0, \gamma_1, \rho, \Sigma, \Sigma^m, \lambda^0_0, \lambda^0_1, \lambda^1_0, \lambda^1_1\right].$$

The joint density of observations 1 through t conditioned on Z_0 satisfies

$$\begin{aligned} & f\left(Z_t, Z_{t-1}, ..., Z_1 | Z_0; \theta\right) && \text{(B–1)} \\ = \;& f\left(Z_{t-1}, ..., Z_1 | Z_0; \theta\right) \times f\left(Z_t | Z_{t-1}, ..., Z_0; \theta\right), \\ = \;& f\left(Z_{t-1}, ..., Z_1 | Z_0; \theta\right) \times f\left(Z_t | Z_{t-1}; \theta\right), \end{aligned}$$

[14] The short rate level under the zero rate regime (c) is set to the corresponding sample average.

where the last equality holds by Markov property. Through the usual sequential substitution, the joint density satisfies

$$
\begin{aligned}
f\left(Z_t, Z_{t-1}, ..., Z_1 | Z_0; \theta\right) &= \prod_{t=1}^{T} f\left(Z_t | Z_{t-1}; \theta\right), \\
&= \prod_{t=1}^{T} f\left(d_t, Y_t | d_{t-1}, Y_{t-1}; \theta\right), \\
&= \prod_{t=1}^{T} \Pr\left(d_t | d_{t-1}, Y_{t-1}\right) f\left(Y_t | d_t, d_{t-1}, Y_{t-1}; \theta\right), \\
&= \prod_{t=1}^{T} \Pr\left(d_t | d_{t-1}\right) f\left(Y_t | d_t, d_{t-1}, Y_{t-1}; \theta\right), \\
&= \prod_{t=1}^{T} \Pr\left(d_t | d_{t-1}\right) f\left(Y_t | d_t, d_{t-1}, f_{t-1}; \theta\right) \qquad \text{(B-2)}
\end{aligned}
$$

where the second last equality hold by Markov property. Now since $\left[e_t, \varepsilon_{t-1}^m\right]'$ in (18) is iid standard normal, the distribution of Y_t conditioned on d_t, d_{t-1}, f_{t-1} is given by

$$
\begin{aligned}
& Y_t | d_t, d_{t-1}, f_{t-1} \sim N\left(A_Y^{d_{t-1}d_t} + B_Y^{d_{t-1}d_t} f_{t-1}, \Sigma_Y^{d_t} \Sigma_Y^{d_t}{}'\right) \\
= \; & (2pi)^{-T/2} \left| \left(\Sigma_Y^{d_t} \Sigma_Y^{d_t}{}'\right)^{-1} \right|^{1/2} \\
& \times \exp\left[(-1/2) \left(Y_t - A_Y^{d_{t-1}d_t} - B_Y^{d_{t-1}d_t} f_{t-1}\right)' \left(\Sigma_Y^{d_t} \Sigma_Y^{d_t}{}'\right)^{-1} \left(Y_t - A_Y^{d_{t-1}d_t} - B_Y^{d_{t-1}d_t} f_{t-1}\right) \right].
\end{aligned}
$$

The log likelihood is the log of (B-2),

$$
\begin{aligned}
L(\theta) &= const \\
&\quad + \sum_{t=1}^{T} \left[\begin{array}{c} \ln \Pr\left(d_t | d_{t-1}\right) - \frac{1}{2} \log \left| \Sigma_Y^{d_t} \Sigma_Y^{d_t}{}' \right| \\ -\frac{1}{2} \left(Y_t - A_Y^{d_{t-1}d_t} - B_Y^{d_{t-1}d_t} f_{t-1}\right)' \left(\Sigma_Y^{d_t} \Sigma_Y^{d_t}{}'\right)^{-1} \left(Y_t - A_Y^{d_{t-1}d_t} - B_Y^{d_{t-1}d_t} f_{t-1}\right) \end{array} \right].
\end{aligned}
$$

which can be rewritten as

$$
\begin{aligned}
L(\theta) &= const \\
+\sum_{t=1}^{T} 1(d_t &= 1, d_{t-1}=1) \left[\begin{array}{c} \ln \pi_{11} - \frac{1}{2}\log\left|\Sigma_Y^1 \Sigma_Y^{1\prime}\right| \\ -\frac{1}{2}\left(Y_t - A_Y^{11} - B_Y^{11} f_{t-1}\right)' \left(\Sigma_Y^1 \Sigma_Y^{1\prime}\right)^{-1} \left(Y_t - A_Y^{11} - B_Y^{11} f_{t-1}\right) \end{array} \right] \\
+\sum_{t=1}^{T} 1(d_t &= 0, d_{t-1}=1) \left[\begin{array}{c} \ln\left(1-\pi_{11}\right) - \frac{1}{2}\log\left|\Sigma_Y^0 \Sigma_Y^{0\prime}\right| \\ -\frac{1}{2}\left(Y_t - A_Y^{10} - B_Y^{10} f_{t-1}\right)' \left(\Sigma_Y^0 \Sigma_Y^{0\prime}\right)^{-1} \left(Y_t - A_Y^{10} - B_Y^{10} f_{t-1}\right) \end{array} \right] \\
+\sum_{t=1}^{T} 1(d_t &= 1, d_{t-1}=0) \left[\begin{array}{c} \ln\left(1-\pi_{00}\right) - \frac{1}{2}\log\left|\Sigma_Y^1 \Sigma_Y^{1\prime}\right| \\ -\frac{1}{2}\left(Y_t - A_Y^{01} - B_Y^{01} f_{t-1}\right)' \left(\Sigma_Y^1 \Sigma_Y^{1\prime}\right)^{-1} \left(Y_t - A_Y^{01} - B_Y^{01} f_{t-1}\right) \end{array} \right] \\
+\sum_{t=1}^{T} 1(d_t &= 0, d_{t-1}=0) \left[\begin{array}{c} \ln \pi_{00} - \frac{1}{2}\log\left|\Sigma_Y^0 \Sigma_Y^{0\prime}\right| \\ -\frac{1}{2}\left(Y_t - A_Y^{00} - B_Y^{00} f_{t-1}\right)' \left(\Sigma_Y^0 \Sigma_Y^{0\prime}\right)^{-1} \left(Y_t - A_Y^{00} - B_Y^{00} f_{t-1}\right) \end{array} \right].
\end{aligned}
$$

where $1(.,.)$ is the indicator function.

C Robustness checks on the factor weights

Figure AC-1 reports one standard deviation bands corresponding to Figure 1. The red solid lines plot the yield-equation equation against maturity (in months) under the normal regime (left column) and the zero rate regime (right column). The dashed green lines show one standard deviation bands. The lower bands of $\mathbf{c}_n^1$ lie above zero, indicating that inflation and growth have a positive impact on JGB bond yields under the normal regime. On the other hand, the lower bands of $\mathbf{c}_n^0$ lie around or below zero, indicating that the macroeconomic effects on the JGB yields are not apparent under the zero rate regime.

Figure AC-2 reports the factor weights with time dependent π_{00} discussed in Section 4.3. The dash-dot, dash, and solid lines correspond to the factor weights under the ZIRP (i.e., when $\pi_{00} = p_1$), those under the QMEP (i.e., when $\pi_{00} = p_2$), and those under the normal regime. The figure indicates that the higher the π_{00} under the zero rate regime, the flatter the yield curves, and the weaker the macroeconomic effects on yield curves become.

Figure AC-3 reports the factor weights with a shorter sampler period (January 1995–August 2008) taking into account a possible structural break in 1995. Given the shorter sample, I report estimated results using monthly data described in Section 4.3. The inflation effect on the bond yields under the normal regime somewhat declines on the longer end of yield curve.

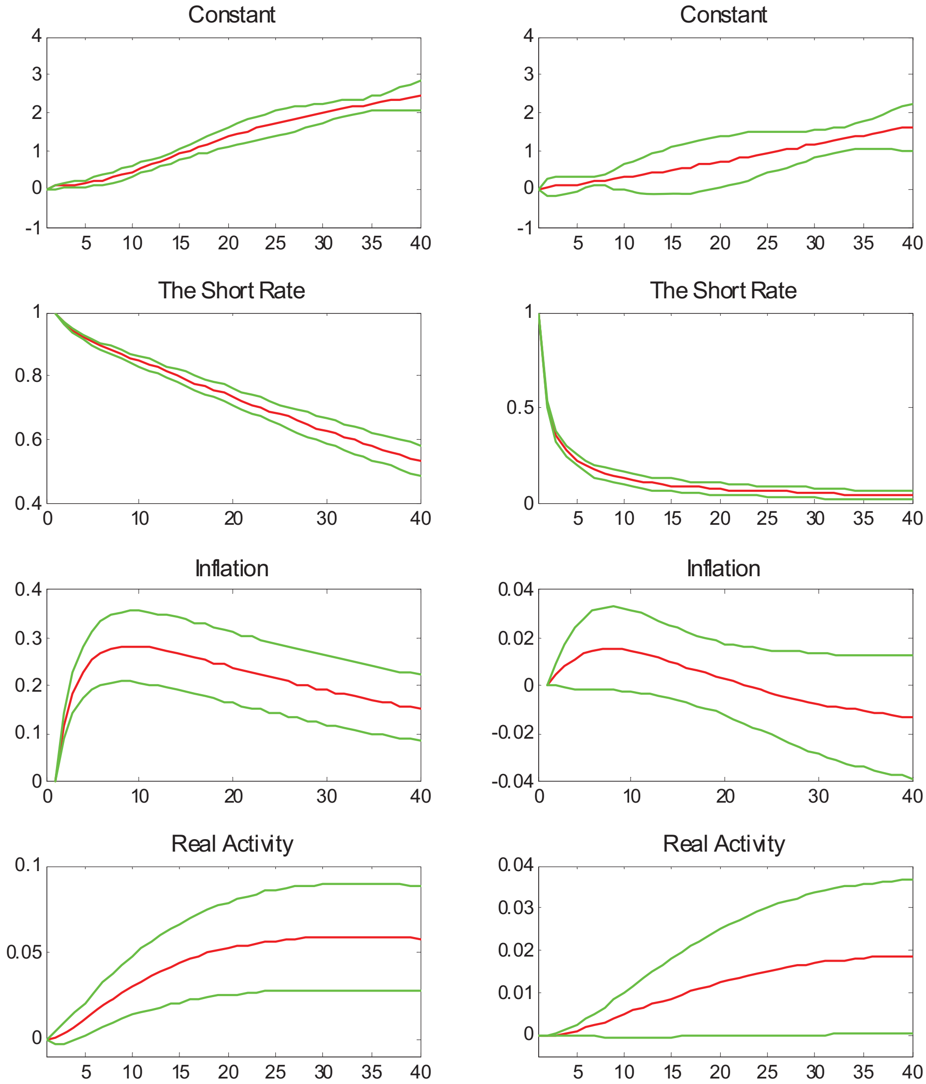

Figure AC-1. Factor weights against maturity with one standard deviation bands. This figure plots the coefficients of the yield equation against maturity (quarters) in the benchmark estimation.

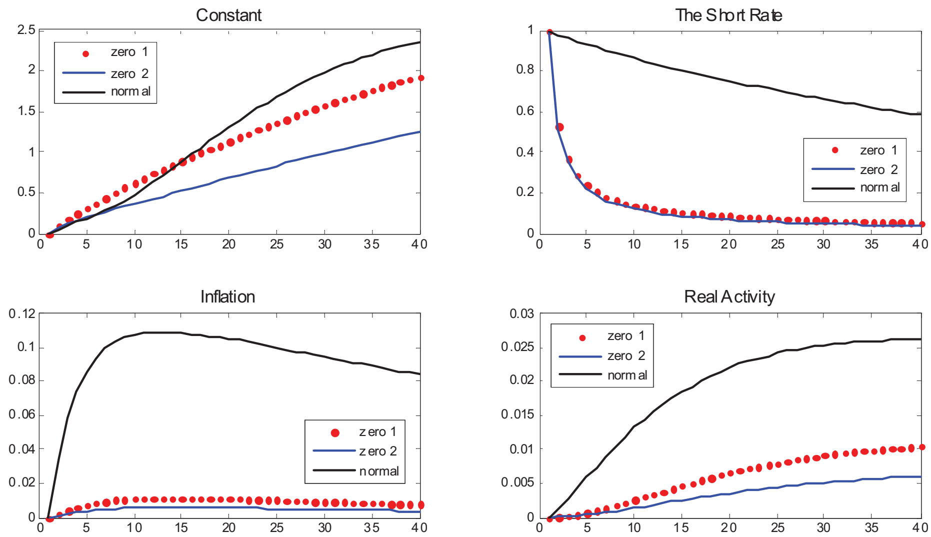

Figure AC-2. Factor weights against maturity with time dependent π_{00}**.** This figure plots the coefficients of the yield equation against maturity (quarters) under the normal regime (solid lines) and under the zero rate regime (red dotted lines correspond to the period before the QMEP introduction and blue dashed lines correspond to the period thereafter.)

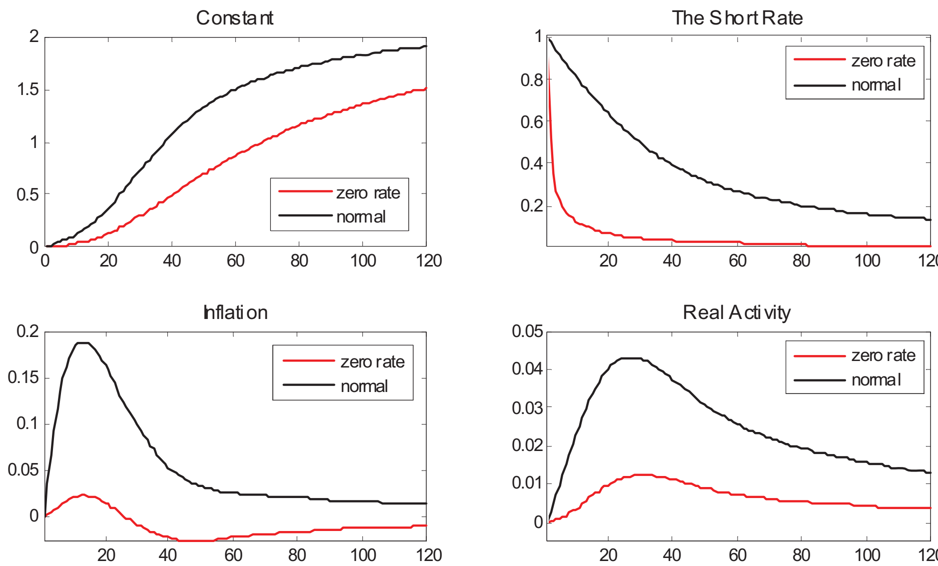

Figure AC-3. Factor weights against maturity with a shorter sample period using monthly data. This figure plots the coefficients of the yield equation against maturity (quarters) with the sample period of January 1995 – August 2008.