Adverse Selection, Uncertainty Shocks and Business Cycles*

Daisuke Ikeda†

Bank of Japan

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Abstract

I study two dynamic economies in which imperfect financial markets materialize uncertainty shocks. In financial markets borrowers not only can divert a fraction of returns but also have better information about the riskiness of their projects than do lenders, resulting in adverse selection as well as moral hazard. Uncertainty shocks change the degree of uncertainty about borrowers riskiness. I show analytically that the uncertainty shocks emerge either as (i) financial shocks which change a wedge between a return to capital and a risk-free rate, or nearly as (ii) shocks to the marginal efficiency of investment. In both cases quantitative analysis suggests that the uncertainty shocks and mechanisms studied here may constitute important elements of business cycles.

(*JEL*) E30, E44, E52.

Key words: Adverse selection, uncertainty shocks.

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†E-mail: daisuke.ikeda@boj.or.jp. Address: 2-1-1 Nihonbashi-Hongokucho,Chuo-ku,Tokyo 103-8660 Japan.

1. Introduction

The great recession starting in late 2007, accompanied by a sharp rise in interest rate spreads, has shifted researchers' focus to financial factors. In an attempt to understand the causes of the great recession in a dynamic general equilibrium framework, two groups of researchers explore the role of two different shocks. The first group focuses on financial shocks which change a wedge between a return to capital and a risk free rate (an interest rate spread) exogenously. It shows that the financial shocks can explain the key features of the great recession (Hall, 2010, Gilchrist, Yankov and Zakrajsek, 2009, and Gilchrist and Zakrajsek, 2010 among others).[1] The second group focuses on shocks to the marginal efficiency of investment (MEI shocks). It shows that the MEI shocks constitute the main driving force of business cycles in the U.S. including the great recession (Justiniano, Primiceri and Tambalotti, 2009a,b). Justiniano, et al (2009b) make a conjecture that the MEI shocks appear as reduced-form shocks related with financial intermediation.

While both the financial shocks and the MEI shocks deepen the understanding of the great recession, both the two shocks share a common drawback. They appear in a reduced-form in a dynamic general equilibrium model. The financial shocks change an interest rate spread exogenously. The MEI shocks may have a connection with financial factors, but the detail remains unspecified in a model. Consequently economic mechanisms behind the two shocks remain unknown.

Apart from those two shocks, shocks to the degree of uncertainty have got attention as a source of business cycles. Uncertainty, measured by various second moments, appears to increase after major economic and political incidents (Bloom, 2009) and during recessions in the U.S. including the great recession (Bloom, Floetotto and Jaimovich, 2010).[2] In an informal level, the great recession saw an economy face a lot of uncertainty about the quality and the riskiness of assets, which might be reflected to a sharp rise in interest rate spreads.

Motivated by the above observations, I build and study two dynamic general equilibrium economies in which imperfect financial markets materialize uncertainty shocks. In the two economies both an agency problem and asymmetric information make financial markets incomplete. Specifically, entrepreneurs (borrowers) not only can divert a fraction of returns

[1] Gilchrist, et al (2009) and Gilchrist and Zakrajsek (2010) call the financial shocks as credit supply shocks. They use the stripped-down model of Bernanke, Gertler and Gilchrist (1999, BGG hereafter) in which they abstract loan rates from the original BGG model. According to Gilchrist and Zakrajsek (2010) one interpretation of the credit supply shocks is the risk shocks considered by Christiano, Motto and Rostagno (2010, CMR hereafter). Though the two shocks become similar after log-linearization, the risk shocks do not exactly coincide with the credit supply shocks.

[2] Also, using firm-level data or macro data, Gilchrist, Sim and Zakrajsek (2010) and Kiley and Sim (2011) provide empirical evidence on uncertainty shocks as a source of business cycles. Kehrig (2011), using plant-level data, shows that the dispersion of total factor productivity is procyclical.

but also have better information about the riskiness of their projects than do intermediaries (lenders), resulting in adverse selection in financial markets as well as moral hazard. Uncertainty shocks I consider change the degree of uncertainty about an entrepreneur's riskiness. I show analytically that the uncertainty shocks emerge as the financial shocks in one economy and nearly as the MEI shocks in another economy after log-linearization.

Thus, the uncertainty shocks, combined with imperfect financial markets, go beyond reduced-form shocks, providing micro foundations for both the financial shocks and the MEI shocks. Also, imperfect financial markets serve as another mechanisms through which a change in uncertainty affects real economic activities, in addition to real factors considered by Bloom (2009) and Bloom, et al (2010).

In modeling imperfect financial markets I focus on one specific source of asymmetric information in financial markets: the riskiness of projects run by entrepreneurs, as in Stiglitz and Weiss (1981).[3] Each entrepreneur has a project with different degree of riskiness, which is private information to the entrepreneur. Intermediaries provide funds to entrepreneurs taking into account the distribution of entrepreneurs riskiness while the intermediaries do not know the riskiness of specific entrepreneur's project. This asymmetric information results in adverse selection in which some safe entrepreneurs do not get funded.

In contrast with Stiglitz and Weiss (1981) and the others who fix the scale of investment, I extend their model allowing for the variable scale of investment. In doing so I introduce an agency problem, similar to Gertler and Karadi (2010), in which entrepreneurs can divert a fraction of returns. This agency problem puts a balance sheet constraint, limits the amount of loans provided by intermediaries and help endogenize the scale of investment.[4]

I solve a non-trivial problem featuring both adverse selection and moral hazard. To ensure the existence of equilibrium, I consider the following timing of events in making a loan arrangement. First, competitive intermediaries provide the schedule of contracts. Second, observing the other intermediaries' schedules an intermediary decides to exit or stay in the market. Third, entrepreneurs choose an intermediary among those staying in the market and choose a contract from the schedule provided by the intermediary. Unlike the standard timing of events, my model features the second step in which intermediaries have an option to withdraw their loan arrangements. Without this option an intermediary may take advantage of the other intermediaries schedules which screen entrepreneurs by their private information. This feature reminds of the non-existence of equilibrium in competitive insurance markets analyzed by Rothchild and Stiglitz (1976). The second step in a loan making process in the model is inspired by Wilson (1977) and Hellwig (1987)

[3] Mankiw (1986) and Bernanke and Gertler (1990) extend Stiglitz and Weiss (1981) model and analyze the effect of adverse selection from a macroeconomic perspective in a static partial equilibrium framework.

[4] If one allowed the variable scale of investment while keeping the framework of Stiglitz and Weiss (1981), only the riskiest entrepreneur would invest and the resulting equilibrium would be efficient, as mentioned by Christiano and Ikeda (2011).

who propose a similar idea to resolve the non-existence problem of Rothchild and Stiglitz (1976).

I embed credit frictions, characterized by both adverse selection and moral hazard, into a real business cycle model. I consider two real business cycle models which differ in a financial market subject to credit frictions. In the first model (Model-I), I embed credit frictions into the demand side of capital. Entrepreneurs own, trade and rent out capital, whose activities constitute the aggregate demand for capital. The uncertainty shocks play a role as a shifter of the demand curve. In Model-I, the uncertainty shocks emerge as the financial shocks.

In the second model (Model-II), I embed credit frictions into the supply side of investment. Entrepreneurs produce investment goods, whose activities constitute the aggregate supply of investment. In Model-II, the uncertainty shocks emerge nearly as the MEI shocks.

I show quantitatively that the uncertainty shocks generate significant fluctuations consistent with business cycles in both the two models, as long as the models have amplification mechanisms. The amplification mechanisms consist of a counter-cyclical markup in wages and variable capital utilization rates. I embed the markup exogenously to focus on the role of amplification mechanisms. With the markup the marginal product of labor is equated to the markup times the marginal rate of substitution between consumption and hours. The model overcomes a co-movement problem, first pointed by Barro and King (1984), if the counter-cyclicality of markup is great enough.[5].

The negative uncertainty shocks increase the degree of overall uncertainty about the riskiness of projects. With imperfect financial markets, an increase in uncertainty implies an increase in the degree of asymmetric information, aggravating adverse selection in financial markets. Intermediaries respond by raising loan rates offered to entrepreneurs, resulting in a spike of external finance premium and a drop in aggregate loans. In Model-I, the demand for capital and the price of capital decrease initially and decrease further through a balance sheet channel. In Model-II, the supply of investment decreases and also decreases further through a balance sheet channel. Enhanced by the amplification mechanisms, investment, output, consumption and hours all fall, while the external finance premium rises, consistent with the U.S. business cycles. A slight difference appears in the price of capital and the net worth. While the price of capital and the net worth decrease in Model-I consistent with business cycles, they increase in Model-II.

The uncertainty shocks I consider in my model share the same spirits with Williamson (1987) who considers shocks to the riskiness of the entrepreneur's return in the framework

[5]In standard neoclassical environments where the marginal rate of substitution between consumption and hours is equated to the marginal product of labor, consumption and hours move to an opposite direction without a change in aggregate technologies or preferences. The uncertainty shocks change neither aggregate technologies nor preferences, so that the uncertainty shocks cannot overcome the co-movement problem by themselves

of Townsend (1983) costly state verification model. CMR (2010) consider similar shocks named risk shocks in a dynamic stochastic general equilibrium model with financial frictions a la BGG (1999). I show that the uncertainty shocks in Model-I share similar quantitative implications to the risk shocks while the two shocks are notionally different.

The two dynamic general equilibrium models I present in this paper have a close connection with a growing literature on adverse selection in macroeconomic settings (Eisfeldt 2004, Kurlat 2010, Bigio 2010, 2011). The two models in this paper differ from those papers with respect to the source of asymmetric information. While I focus on asymmetric information on the riskiness of projects as in Stiglitz and Weiss (1981), they focus on asymmetric information on the quality of assets (projects). House (2005) studies Stiglitz and Weiss (1981) model in an overlapping generations framework, keeping the fixed scale of investment. I allow the variable scale of investment and embed adverse selection into a dynamic general equilibrium model in a reasonable manner.

I organize the rest of the paper as follows. In Section 2, I present a partial equilibrium model with asymmetric information and an agency problem. I start from a symmetric information model with an agency problem as a baseline and proceed to an asymmetric information model. In Section 3, I embed the partial equilibrium model into dynamic general equilibrium models and present two models. In each model I make clear the role of uncertainty shocks analytically. In Section 4, I conduct simulations to explore the quantitative effects of uncertainty shocks. In Section 5, I discuss two issues. First, I compare the uncertainty shocks with the risk shocks analyzed by CMR (2010). Second, I consider another shocks to make clear the role of uncertainty shocks. In Section 6, I conclude the paper.

2. Adverse Selection: A Partial Equilibrium Model

I study a financing problem between entrepreneurs (borrowers) and intermediaries (lenders), taking as given returns. I solve for the optimal contract between entrepreneurs and intermediaries. In this section I omit time subscript t for notational simplicity. In the subsequent section I embed this partial equilibrium financing problem into a dynamic general equilibrium model and explore the impact of uncertainty shocks.

2.1. Environment

Overview: There exist many entrepreneurs and intermediaries. A financing problem is static in that it involves only one time borrowing and lending. The financing problem evolves in the following five steps. Initially, nature draws and assigns entrepreneur's type (private information) which characterizes the riskiness of investment project. In the first step, an intermediary provides a schedule of contracts to entrepreneurs without knowing the entrepreneurs' private information. In the second step, after observing the other intermediaries' schedules of contracts an intermediary decides whether to stay in the market or to leave the market. In the third step, an entrepreneur chooses an intermediary and its specific contract, and invests in project. In the fourth step, after realizing the outcome of project an entrepreneur decides whether to divert its return or not. Finally, an entrepreneur and an intermediary receive returns depending on the previous actions.

The financing problem features both adverse selection and moral hazard. Adverse selection occurs due to entrepreneur's private information about the riskiness of project. Moral hazard occurs due to entrepreneur's possible action of diverting its return.

Entrepreneurs: There exist many risk neutral entrepreneurs whose objective is to maximize their return. Initially an entrepreneur starts its business with net worth N_n in unit of goods, indexed by n. An entrepreneur has a project with its success probability p, which is private information to the entrepreneur and is drawn from distribution function $F : [\underline{p}, \bar{p}] \rightarrow [0, 1]$ with $0 < \underline{p} < \bar{p} \leq 1$, independently and identically across entrepreneurs. I assume that distribution $F(p)$ has full support.

A set of index, (n, p), characterizes an entrepreneur. The type-(n, p) entrepreneur has net worth N_n and has a project with probability of success p. I assume that for given n there exist many entrepreneurs so that distribution $F(\cdot)$ coincides with the distribution of the type-n entrepreneurs.

An entrepreneur chooses an intermediary and a contract among those offered by the intermediary. The type-(n, p) entrepreneur receives loan $B_n(p)$ from the intermediary. The sum of loan $B_n(p)$ and net worth N_n constitutes the entrepreneur's asset. The entrepreneur invests its asset in its project.

After the entrepreneur invests, the type-p project results in a success or a failure. In case of success the project yields the gross return $\theta(p)R^e$ per unit of goods invested. In case of failure the project yields zero return. I assume $\theta(p) = 1/p$ for analytical tractability so that the expected gross return, just after the realization of the project's outcome (success or failure), becomes the same for all projects. This simplification allows me to focus only on the riskiness of project as private information.

I assume that the limited liability law protects entrepreneurs. Entrepreneurs do not have any liability after paying to intermediaries. The limited liability assumption implies that intermediaries can not force entrepreneurs to pay when the entrepreneurs fail in their projects because the entrepreneurs have nothing to pay at hand.

Under the limited liability assumption, an intermediary offers a schedule of contracts specifying the amount of loan and payment $\{B_n(p), X_n(p)\}_p$, where $B_n(p)$ denotes the amount of loan and $X_n(p)$ denotes the amount of payment conditional on the success of project. Without loss of generality I consider a truth telling schedule of contracts such that type-(n, p) entrepreneur chooses $\{B_n(p), X_n(p)\}$ voluntarily and such that the entrepreneur does not divert its return. The payment conditional on the failure of project must be zero because of the limited liability assumption.

In addition to asymmetric information, an entrepreneur has a moral hazard problem such that an entrepreneur can steal a fraction of the gross return made from a project. Given a pair of loan and payment, $\{B_n(p), X_n(p)\}$, chosen by the type-(n, p) entrepreneur, and given the success of its project, the return when the entrepreneur abides by the contract is given by

$$\begin{aligned} \text{Return}|\text{abide} &= \theta(p)R^e(B_n(p) + N_n) - X_n(p), \\ &= (1/p)\left\{R^e N_n + [R^e - pX_n(p)/B_n(p)]\, B_n(p)\right\}, \end{aligned} \tag{1}$$

where $\theta(p) \equiv 1/p$ denotes the productivity of the type-p project conditional on the success of the project, and I have used $\theta(p) \equiv 1/p$ in the second equality. The first line of equation (1) implies that the entrepreneur's return is given by the gross return from investment minus the payment to an intermediary. The term in bracket in the second line of equation (1) refers to an expected excess return per unit of loan. If the excess return were negative, the entrepreneur would not borrow. If the excess return were positive, the entrepreneur would borrow as much as possible.

The entrepreneur can break the contract and steal a fraction, $0 < \phi < 1$, of the gross return from investment, $\theta(p)R^e(B_n(p) + N_n)$. Then, the entrepreneur's return when the entrepreneur steals is given by

$$\text{Return}|\text{steal} = \phi(1/p)R^e(B_n(p) + N_n). \tag{2}$$

An intermediary collects the rest of the return when the entrepreneur steals.[6]

Intermediaries: There exist a small number of risk neutral intermediaries relative to entrepreneurs. An intermediary takes in deposits from households with risk-free rate R^f and

[6]Another agency problem generates the same incentive problem. Instead of stealing the return suppose that an entrepreneur can declare the failure of its project even when the entrepreneur actually succeeds in the project. When the entrepreneur declares the failure intermediaries monitor the entrepreneur and detect the false claim with probability $1 - \phi$ without no costs. An entrepreneur realizes the project's outcome (success or failure) and reports the outcome to an intermediary.

In this case, given the project's success, the return when the entrepreneur abides by the contract is given by

$$\text{Return}|\text{abide} = \theta(p)R^e(B_n(p) + N_n) - X_n(p),$$

where $\theta(p) \equiv 1/p$. When the entrepreneur misbehaves the return is given by

$$\text{Return}|\text{steal} = \phi\theta(p)R^e(B_n(p) + N_n).$$

The incentive constraint requiring that entrepreneur does not misbehave turns out to be the same as the one when the entrepreneur can steal a fraction of return.

makes loans to entrepreneurs. In the process of making a loan contract, first, the intermediary provides a schedule of contracts$\{B_n(p), X_n(p)\}_p$ as a function of entrepreneur's type p. Since the intermediary can observe the amount of net worth owned by entrepreneurs the menu also depends on the net worth indexed by n. In the second step, the intermediary observes the other intermediaries' schedules and decides whether to leave the market or not. The intermediary's profits become zero when the intermediary leaves the market. In the third step, entrepreneurs choose an intermediary. Because I restrict my attention to a truth telling schedule without loss of generality, the type-(n, p) entrepreneur chooses contract $\{B_n(p), X_n(p)\}$ offered by its chosen intermediary

I assume that intermediaries compete in lending. That is, intermediaries compete in providing a schedule of contracts to attract entrepreneurs. I also assume that entrepreneurs choose intermediaries randomly if the schedule of contracts offered by the intermediaries are indifferent.

2.2. Symmetric Information Benchmark

Before solving the model with asymmetric information I consider a model with symmetric information as a benchmark to make clear the role of asymmetric information. In an economy with symmetric information intermediaries observe the riskiness of project, p, while entrepreneurs still can misbehave and steal a fraction of return.

I consider, without loss of generality, an intermediary's loan arrangement in which an entrepreneur does not steal its return voluntarily. Given a unit of loan, an intermediary sets the borrowing interest rate, $R^b(p)$, in a competitive manner, resulting in the following equation:

$$pR^b(p) = R^f. \tag{3}$$

The left-hand-side of equation (3) denotes the expected payment from one unit of loan, while the right-hand-side of equation (3) denotes the cost of one unit of loan. The perfect competition among intermediaries results in the intermediary's zero profits condition, (3).

The intermediary faces an agency problem: an entrepreneur has an incentive to steal a fraction of return if the intermediary provides too much loans. The intermediary limits the amount of loan to ensure that the entrepreneur does not misbehave. Specifically, the loan provided by the intermediary has to satisfy an incentive constraint such that the return when the entrepreneur does not misbehave, given by (1), is equal or greater than the return when the entrepreneur misbehaves, given by (2). Noting that the payment becomes $X_n(p) = R^b(p)B_n(p) = R^f B_n(p)/p$ from the intermediary's zero profit condition, (3), I express the incentive constraint as

$$R^e N_n + (R^e - R^f)B_n(p) \geq \phi R^e(B_n(p) + N_n),$$

or

$$B_n(p) \leq \frac{(1-\phi)(R^e/R^f)}{1-(1-\phi)(R^e/R^f)} N_n. \tag{4}$$

Constraint (4) implies that the amount of loan is limited by the amount of net worth held by the entrepreneur.

I assume positive premium $R^e/R^f > 1$, otherwise an entrepreneur has no incentive to borrow from intermediaries.[7] I also assume that the degree of moral hazard controlled by ϕ is great enough for the denominator in the right-hand-side of (4) to be positive. Then, entrepreneurs borrow the maximum amount subject to the incentive constraint, (4). Consequently, the incentive constraint, (4), binds in equilibrium.

The equilibrium contract offered by an intermediary consists of $\{B_n(p), X_n(p)\}$, where the amount of loan is given by the left-hand-side of (4) and the amount of repayment in case of success is given by $R^b B_n(p)$ with R^b given by (3). Because the intermediary earns zero profits under the contract, the intermediary has no incentive to exit the market, which would result in zero profits, after observing the other intermediaries' loan contracts. An entrepreneur chooses an intermediary randomly because all intermediaries offer the same contract. Because there is no profitable deviation for both intermediaries and entrepreneurs, the contract constitutes an equilibrium in a model with symmetric information.

The equilibrium contract has a convenient feature for aggregation. Because the right-hand-side of (4) is linear in net worth N_n and is independent of type p, I can aggregate the individual loan over n and p and express the aggregate loan as

$$B = \frac{(1-\phi)(R^e/R^f)}{1-(1-\phi)(R^e/R^f)} N, \tag{5}$$

where B and N denote the aggregate loan and the aggregate net worth respectively. The aggregate loan is increasing in the premium, R^e/R^f, and is increasing in the aggregate net worth, N. More importantly the aggregate loan is independent of the distribution of the riskiness of project, $F(p)$. A change in $F(p)$ affects neither the aggregate loan nor any other aggregate variables in a model with symmetric information.

2.3. Asymmetric Information

Now I consider a model with asymmetric information. I first define an equilibrium in a game-theoretic manner and then solve the model.

Subgame Perfection: A game I consider consists of two types of players: many entrepreneurs and a small number of intermediaries relative to entrepreneurs. The game evolves in three stages. Initially, nature draws and assigns type p, which characterizes

[7] In a general equilibrium model studied in the next section I endogenize the returns, R^e and R^f, which satisfy $R^e/E^f < 1$ in equilibrium around steady state.

the riskiness of project, to an entrepreneur. In the first stage, an intermediary, indexed by i, offers a schedule of contracts, $\sigma_{1,i} \equiv \{B_n^i(p), X_n^i(p)\}_p$, which specifies the amount of loan, $B_n^i(p)$, and the amount of payment in case of project's success, $X_n^i(p)$, as a function of p. In the second stage, the intermediary observes the other intermediaries' schedules offered in the first stage, and decides whether to leave or stay in the market. In the third stage, entrepreneurs choose an intermediary. If an entrepreneur chooses the i-th intermediary the entrepreneur chooses contract $\{B_n^i(p), X_n^i(p)\}$, because I limit my attention to a truth telling contract, without loss of generality. Finally, given that the contract does not make entrepreneurs misbehave, the actions from stage 1 to 3 determine the returns of entrepreneurs and intermediaries.

The i-th intermediary's strategy consists of $\{\sigma_{1,i,}, \sigma_{2,i}\}$ where $\sigma_{1,i} \equiv \{B_n^i(p), X_n^i(p)\}_p$ denotes a schedule of contracts, and $\sigma_{2,i}$ specifies a decision of whether to stay ($\sigma_{2,i}(\sigma_1) = 1$) or leave ($\sigma_{2,i}(\sigma_1) = 0$) as a function of a set of schedules, $\sigma_1 \equiv \{\sigma_{1,i}\}_i$. An entrepreneur's strategy specifies a decision of which intermediary to choose as a function of σ_1 and $\sigma_2 \equiv \{\sigma_{2,i}\}_i$. I limit my attention to a deterministic strategy, yet I assume that an entrepreneur chooses an intermediary randomly if the entrepreneur finds some intermediaries indifferent.

The i-th intermediary competes in lending and chooses schedule $\sigma_{1,i}$ in the first stage using backward induction. In the third stage, given σ_1 and σ_2, entrepreneurs choose an intermediary which offers the most profitable contract. In the second stage, given σ_1 and the other intermediaries' strategy of whether to stay or leave, $\sigma_{2,-i}(\sigma_1) \equiv \{\sigma_{2,j}(\sigma_1)\}_{j \neq i}$, the i-th intermediary chooses whether to stay or leave. If the intermediary chooses to stay, set $\{\sigma_1,\, \sigma_{2,-i}(\sigma_1)\}$ determines the distribution of entrepreneurs choosing the intermediary, denoted by $F^i(p; \sigma_1, \sigma_{2,-i})$. The intermediary chooses to stay, $\sigma_{2,i}(\sigma_1) = 1$ if and only if

$$V_n^i(\sigma_1, \sigma_{2,-i}) \equiv \int_{\underline{p}}^{\bar{p}} [pX_n^i(p) - R^f B_n^i(p)] dF^i(p; \sigma_1, \sigma_{2,-i}) \geq 0, \tag{6}$$

where $V_n^i(\sigma_1, \sigma_{2,-i})$ denotes the intermediary's profits when choosing to stay, given $\{\sigma_1,$ $\sigma_{2,-i}(\sigma_1)\}$. The bracket in (6) denotes the expected profits made from lending to a single type-p entrepreneur. With probability p the intermediary receives repayment $X_n^i(p)$ for amount of loan $B_n^i(p)$ with cost R^f per unit of loan. Integrating the profits over p yields the intermediary's profits, given by (6).

Let $\{\sigma_2^*(\sigma_1)\}$ denote a set of Nash equilibria in the second stage game, given σ_1. Given one of the Nash equilibria, set $\{\sigma_1, \sigma_2^*(\sigma_1)\}$ determines the distribution of entrepreneurs choosing the i-th intermediary in the third stage, denoted by $F^i\left(p; \sigma_1, \sigma_2^*(\sigma_1)\right)$. In the first stage, the i-th intermediary chooses its schedule $\sigma_{1,i}$ to maximize its profits, given by

$$V_n^i\left(\sigma_1, \sigma_2^*(\sigma_1)\right) \equiv \int_{\underline{p}}^{1} [pX_n^i(p) - R^f B_n^i(p)] dF^i\left(p; \sigma_1, \sigma_2^*(\sigma_1)\right), \tag{7}$$

where $\sigma_1 \equiv \{\sigma_{1,i}, \sigma_{1,-i}\}$.

Now I define an equilibrium of a model with asymmetric information.

Definition 1: A *subgame-perfect equilibrium* for a model with asymmetric information consists of the schedule of contracts offered by intermediaries in the first stage, $\sigma_1^* \equiv \{\sigma_{1,i}^*\}_i$, the intermediaries' strategies whether to leave or stay in the market in the second stage, $\sigma_2^* \equiv \{\sigma_{2,i}^*\}_i$, and the entrepreneurs' strategies choosing an intermediary in the third stage, satisfying (i) entrepreneurs choose an intermediary providing the most profitable contract, (ii) given σ_1 and $\sigma_{2,-i}^*$, the i-th intermediary chooses to stay, $\sigma_{2,i}^*(\sigma_1) = 1$, if and only if the profits from doing to are non-negative, for all i, and (iii) given $\sigma_{1,-i}^*$ and σ_2^*, the i-th intermediary chooses $\sigma_{1,i}^*$ to maximize its profits, for all i. An equilibrium is *symmetric* if and only if the strategy in the first stage is the same for all entrepreneurs: $\sigma_{1,i}^* = \sigma_{1,j}^*$ for all i, j.

Symmetric Equilibrium: In deriving an equilibrium I limit my attention to a symmetric equilibrium in which all intermediaries employ the same strategy. I have already characterized an intermediary's equilibrium strategy in the second stage as in Definition 1. In the following I derive an equilibrium strategy in the first stage, that is, I solve problem (iii) in Definition 1, given the assumption of symmetric equilibrium: $\sigma_{1,i}^* = \{B_n(p), X_n(p)\}$ for all i.

My procedure to derive an equilibrium consists of three steps. First, I characterize a set of strategies which satisfy incentive and participation constraints. Second, I make a conjecture that one of the strategies constitutes an equilibrium strategy. Third, I show that the candidate for an equilibrium actually constitutes a unique symmetric equilibrium.

Step 1: I characterize a set of equilibrium schedules of contracts, $\{B_n(p), X_n(p)\}_p$. An equilibrium schedule of contracts has to satisfy the following four constraints. First, I restrict my attention to a truth-telling contract so that an equilibrium schedule has to satisfy an incentive constraint:

$$R^e B_n(p) - pX_n(p) \geq R^e B_n(\tilde{p}) - pX_n(\tilde{p}), \quad \forall p, \tilde{p}. \tag{IC1}$$

Constraint (IC1) ensures that the type-p entrepreneur chooses $\{B_n(p), X_n(p)\}$ voluntarily. The left-hand-side of (IC1) denotes the entrepreneur's return from getting loans when choosing pair $\{B_n(p), X_n(p)\}$ and the right-hand-side of (IC1) denotes the return when choosing the other pair.

Second, an equilibrium schedule has to satisfy another incentive constraint:

$$R^e[N_n + B_n(p)] - pX_n(p) \geq \phi R^e[N_n + B_n(p)], \quad \forall p, \tag{IC2}$$

The second incentive constraint, (IC2), ensures that an entrepreneur does not misbehave. The left-hand-side of (IC2) denotes the entrepreneur's return when the entrepreneur does not misbehave, while the right-hand-side of (IC2) denotes the return when the entrepreneur misbehaves and walks way with some returns.

Third, an equilibrium schedule has to satisfy an entrepreneur's participation constraint:

$$W_n(p) \equiv R^e B_n(p) - pX_n(p) \geq 0, \quad \forall p. \tag{PCe}$$

The entrepreneur's return has to be non-negative, otherwise the entrepreneur would not participate in this loan arrangement.

Finally, an equilibrium schedule has to satisfy an intermediary's zero profit condition:

$$0 = V_n(p^*) = \int_{\underline{p}}^{p^*} [pX_n(p) - R^f B_n(p)] dF(p). \tag{8}$$

Here I make a conjecture that there exists $p^* < \bar{p}$ such that (PCe) holds with equality for $p = p^*$ and the amount of payment is strictly positive, $X_n(p^*) > 0$. The conjecture parallels with Stiglitz and Weiss (1981) model in which entrepreneurs with p above a certain threshold do not get loans because of adverse selection. Given threshold p^*, the intermediary's profits consist of a repayment from the type-p entrepreneur, $pX_n(p)$, and the cost of funds, $-R^f B_n(p)$, integrated over distribution $F(p)$. Because I restrict my attention to a symmetric equilibrium, all intermediaries offer the same schedule of contracts in equilibrium. Entrepreneurs are assumed to choose an intermediary randomly if intermediaries are indifferent. Then, an intermediary faces distribution $F(p)$ up to a constant scaling factor in a symmetric equilibrium. The zero profit condition follows from the assumption of competitive intermediaries.

Step 2: I narrow down a set of equilibrium schedules and pick up one schedule as a candidate for an equilibrium schedule. I sketch the procedure in five steps.

First, I start from a guess that there exists threshold p^* above which entrepreneurs do not get loans. This guess, combined with (IC1), implies that

$$\begin{aligned} &\text{for } p > p^* \quad R^e B_n(p) - pX_n(p) = 0, \\ &\text{for } p < p^* \quad R^e B_n(p) - pX_n(p) > 0. \end{aligned} \tag{9}$$

That is, (PCe) holds with equality for $p > p^*$ and (PCe) holds with strict inequality for $p < p^*$. From (IC1) and (9), for $p > p^*$ I obtain:

$$0 \geq R^e B_n(p) - p^* X_n(p) = (p - p^*) X_n(p). \tag{10}$$

I have derived the inequality in (10) from (IC1) for $p = p^*$. I have derived the equality in (10) from (9). Since $p - p^* > 0$ the payment has to be zero for $p > p^*$, otherwise condition (10) would not hold, violating either (IC1) or (9). Therefore, for $p > p^*$ both the payment and the loan become zero: $X_n(p) = B_n(p) = 0$. For $p < p^*$ it is straightforward to show that both the payment and the loan has to be positive. Given threshold p^*, this first step ensures that a schedule satisfies (PCe) automatically.

Second, I replace (IC1) by the local incentive compatibility constraint and the monotonicity constraint as in a standard mechanism design problem:

$$R^e[dB_n(p)/dp] - p[dX_n(p)/dp] = 0, \quad \forall p \tag{11}$$

$$dX_n(p)/dp \leq 0, \tag{12}$$

Intuitively, constraints (11) and (12) correspond to the first and second order conditions of an entrepreneur's problem respectively. In the problem the type-p entrepreneur maximizes its profits by choosing pair $\{B_n(p), X_n(p)\}$.

Third, I express the profits of entrepreneurs, $W_n(p) \equiv R^e B_n(p) - pX_n(p)$, as a function of payment schedule $X_n(p)$ using local incentive compatibility constraint (11) and the envelope theorem as follows:

$$W_n(p) = \int_p^{p^*} X_n(x)dx. \tag{13}$$

Expression (13) implies that the type-p entrepreneur's profits are increasing in payment $X_n(p')$ for $p' \geq p$. The intuition behind (13) has to do with the fact that risky entrepreneurs with lower p repay less in expected values than safer entrepreneurs with higher p. For example, if the highest type-p^* entrepreneur re-payed more, it would have to receive more loan to satisfy (PCe). This new pair of loan and payment for $p = p^*$ is also available to the other entrepreneurs. For $p < p^*$, an increase in loan would increase the type-p entrepreneur's expected gross return by the same amount as the type-p^* entrepreneur, while the expected repayment would be less for the type-p entrepreneurs than the type-p^* entrepreneur. Therefore, risky entrepreneurs with lower p can enjoy rents accrued from a lower repayment in expected values.

Using (13) I express loan schedule $B_n(p)$ as a function of $X_n(p)$:

$$B_n(p) = \left[pX_n(p) + \int_p^{p^*} X_n(x)dx\right] / R^e. \tag{14}$$

In this procedure I have substituted out local incentive compatibility constraint (11). Constraint (11) is satisfied as long as a loan schedule is given by (14).

Fourth, using loan schedule (14) I rewrite (IC2) as

$$X_n(p) \leq \frac{(1-\phi)}{\phi p} R^e N_n + \frac{(1-\phi)}{\phi p} \int_p^{p^*} X_n(x)dx. \tag{15}$$

Here I make a conjecture that the inequality in (15) holds with equality for $p \leq p^*$. Under the conjecture equation (15) becomes an integral equation for unknown function $X_n(p)$. I solve the integral equation and obtain a schedule of contract as a candidate for an equilibrium:

$$X_n(p) = \left[\frac{(1-\phi)R^e}{\phi}(p^*)^{\frac{1-\phi}{\phi}}\right]\left(\frac{1}{p}\right)^{\frac{1}{\phi}} N_n, \tag{16}$$

$$B_n(p) = \frac{1-\phi}{\phi}\left\{1 + \frac{1}{1-\phi}\left[\left(\frac{p^*}{p}\right)^{\frac{1-\phi}{\phi}} - 1\right]\right\} N_n, \tag{17}$$

where I have derived equation (17) from equations (14) and (16). Note that $X_n(p)$ is strictly decreasing in p and satisfies monotonicity condition (12). Then, the schedule, (16) and (17), constructed so far satisfies (PCe), (IC1) and (IC2).

Fifth, I pin down threshold p^* using the intermediary's zero profits condition, (8). Using loan schedule (14), I express the intermediary's zero profit condition as follows:

$$0 = V_n(p^*) = \int_{\underline{p}}^{p^*} \omega(p) X_n(p) dp, \tag{18}$$

where $\omega : (\underline{p}, \bar{p}) \to \mathcal{R}$ is given by

$$\omega(p) = pf(p) - \frac{R^f}{R^e} pf(p) - \frac{R^f}{R^e} F(p). \tag{19}$$

At this point I simply assume that a solution to (18) uniquely exists. That is, I assume that there exists a unique $p^* \in (\underline{p}, \bar{p})$ such that $V(p^*) = 0$, given by (18).

The candidate for an equilibrium, given by (16), (17) and (18), has an important feature. The fact that the second incentive constraint, (IC2) (or (15)), holds with equality implies that the schedule maximizes payment $X_n(p)$ for $p \leq p^*$, which, in turn, implies that the schedule maximizes the entrepreneur's profits, given by (13), for all $p \leq p^*$. For entrepreneurs with $p > p^*$, profits are zero. I summarize the above argument in Lemma 1.

Lemma 1: Suppose that a solution to (18) uniquely exists. Then, among loan and payment schedules satisfying (IC1), (IC2), (PCe) and the intermediary's zero profit condition, (8), the candidate for an equilibrium, given by (16) and (17), maximizes the type-p entrepreneur's profits for all p.

Step 3: Finally, I show that the candidate for an equilibrium, given by (16), (17) and (18), constitutes a unique symmetric equilibrium. The statement consists of two parts: an equilibrium part and a uniqueness part. I summarize the result in the following proposition and provide the proof in the appendix.

Proposition 1: Suppose that a solution to (18) uniquely exists. Then, the candidate for an equilibrium, given by (17), (16) and (18), constitutes a unique symmetric equilibrium.

Intuitively, the candidate for an equilibrium maximizes the type-p entrepreneur's profits for all p, as shown in Lemma 1, and attracts all entrepreneurs, so that an intermediary has no incentive to deviate and the candidate actually constitutes an equilibrium. The condition in Proposition 1 is satisfied for some distributions and some values for $R^f/R^e < 1$. For example, as I employ in simulating a model later, the condition in Proposition 1 is satisfied for a uniform distribution with support $[\underline{p}, \bar{p}]$ with $0 < \underline{p} < \bar{p} \leq 1$ for $R^f/R^e < 1$ close to unity.

2.4. Equilibrium Loan Contract

I study the properties and the implications of the equilibrium loan contract. First, I study its micro properties. Second, I study its macro implications.

Micro properties: I summarize the simple properties of the equilibrium loan contract in the following Lemma.

Lemma 2: An equilibrium payment and loan schedules, (16) and (17), satisfies the following properties:

(i) Both loan schedule (17) and payment schedule (16) are decreasing in riskiness p and increasing in threshold p^*.

(ii) The borrowing interest rate, $R^b(p) = X_n(p)/B_n(p)$, is decreasing in riskiness p.

(iii) Threshold p^* is increasing in the premium, R^e/R^f.

Both Lemma 2 (i) and (ii) imply that entrepreneurs with higher risk project get more loans and pay back more conditional on the success of a project. Intuitively entrepreneurs face a trade-off between the amount of loans and the incentive to misbehave. On the one hand, entrepreneurs with $p < p^*$ would like to borrow as much as possible because the return from borrowing is positive. On the other hand, more borrowing induces entrepreneurs to misbehave. Entrepreneurs with lower p have less incentive to misbehave because they have lower expected payment, as is clear from (IC2). Therefore, the risky entrepreneurs can borrow and pay more in the equilibrium contract.

Lemma 2 (iii) implies that an intermediary offers more loans to entrepreneurs as the return earned by entrepreneurs gets higher. Given threshold p^* a rise in the premium increases the intermediary's profits. Consequently the intermediary loosens lending and increases p^* until the intermediary's zero profit condition holds.

Macro implications: The equilibrium loan contract, summarized in Proposition 1, has two nice properties for aggregation. First, threshold p^* does not depend on the amount of net worth. This property has to do with the fact that the net worth does not appear in (18). This property implies that the same threshold applies to all entrepreneurs. Even an entrepreneur with great amount of net worth cannot get loans if its project's probability of success exceeds the threshold: $p > p^*$.

Second, both loan schedule $B_n(p)$, given by (17), and payment schedule $X_n(p)$, given by (16), are linear in net worth. This implies that I do not have to pay attention to the distribution of net worth in aggregating loan and payment.

As a consequence of the above two properties, the aggregate loan has a simple expression. From loan schedule (17) I can express the aggregate loan, B, as

$$\begin{aligned} B &= \int_n \int_{\underline{p}}^{p^*} B_n(p) dF(p) dH(n), \\ &= \frac{1-\phi}{\phi} \left[\frac{1}{1-\phi} \int_{\underline{p}}^{p^*} \left(\frac{p^*}{p} \right)^{\frac{1-\phi}{\phi}} dF(p) - \frac{\phi}{1-\phi} F(p^*) \right] N, \end{aligned} \tag{20}$$

where $F(\cdot)$ denotes the distribution of p and $H(\cdot)$ denotes the distribution of net worth indexed by n. Using the intermediary's zero profit condition, I can also express B as

$$\begin{aligned} B &= \frac{1}{R^f} \int_n \int_{\underline{p}}^{p^*} p X_n(p) dF(p) dH(n), \\ &= \frac{1-\phi}{\phi} \frac{R^e}{R^f} \left[\int_{\underline{p}}^{p^*} \left(\frac{p^*}{p} \right)^{\frac{1-\phi}{\phi}} dF(p) \right] N, \end{aligned} \tag{21}$$

From (20) and (21) I obtain a simple expression for the aggregate loan:

$$B = \frac{(1-\phi)(R^e/R^f)}{1-(1-\phi)(R^e/R^f)} F(p^*) N. \tag{22}$$

The aggregate loan is increasing in the premium, R^e/R^f, decreasing in the degree of moral hazard, ϕ, and increasing in threshold p^*. From (22) the aggregate loan is bounded from above as

$$B < \frac{(1-\phi)(R^e/R^f)}{1-(1-\phi)(R^e/R^f)} N,$$

because $F(p^*) < 1$. The upper bound denotes the aggregate loan without asymmetric information, given by (5).

The expression for the aggregate loan, (22), summarizes the impact of asymmetric information. With asymmetric information entrepreneurs with $p > p^*$ do not get loans because a borrowing interest rate is too high to earn non-negative profits, reflecting the high default rates of the other entrepreneurs. Asymmetric information causes a lemons problem and decreases the aggregate loan by $[1 - F(p^*)] \times 100$ percent relative to the aggregate loan without asymmetric information.

2.5. Equivalence Result

In order to isolate the pure role of asymmetric information I establish an equivalence result between two economies: one with asymmetric information and another with symmetric information and with taxes on the entrepreneur's net worth.

As shown in Section 2.4, a loan-net-worth ratio, B_t/N_t, under asymmetric information is lower than that under symmetric information. A wedge between the two ratio is given by $1 - F(p^*)$. I add the same wedge in the model with symmetric information by introducing taxes on the entrepreneur's net worth.

I consider an economy with symmetric information, analyzed in Section 2.2. A government imposes taxes on the entrepreneur's net worth. The government injects the tax revenue to entrepreneurs and makes them invest the tax revenues in a project on behalf of the government. The government rebates the return to entrepreneurs in the beginning of next period.

Let τ denotes the tax rate. The after-taxed net worth becomes $(1-\tau)N$ in aggregate. Following the same argument in Section 2.2, I obtain a loan-net-worth ratio, which coincides with that under asymmetric information, if the tax rate is set to

$$\tau = 1 - F(p^*).$$

After receiving the tax revenue from the government, entrepreneurs have an asset in aggregate, amounting to $(1-\tau)N + B + \tau N = N + B$, where B coincides with that under asymmetric information, given by (22). Therefore, both the aggregate loan, B, and the aggregate asset, coincide with those under asymmetric information, showing the equivalence result.

The equivalence result implies that adverse selection plays a similar role to a shock to the net worth explored quantitatively by Christiano, Motto and Rostagno (2010) and Gilchrist, Ortiz and Zakrajsek (2009) among others. The net worth shock transfers resources from entrepreneurs to households, reducing the amount of loan. While the tax I consider does not transfer resources from one to another, it eventually reduces the amount of loan and has a similar effect to the net worth shock.

3. General Equilibrium Models

Now I embed adverse selection in financial markets, analyzed in the previous section, into a dynamic general equilibrium model. In doing so I restrict my attention to one-period financing contract so that I can apply the results in the previous section directly. Set aside the adverse selection, the basic framework is the standard real business cycle model with exogenous countercyclical markups and endogenous capital utilization. As I show later those additional features serve as critical mechanisms amplifying uncertainty shocks.

I study analytically the effect of uncertainty shocks which change the distribution of entrepreneurs riskiness, in two real business cycle models. In the first model, I embed the adverse selection into the demand side of capital as in BGG (1999). In the second model, I embed the adverse selection into the supply side of investment as in Carlstrom and Fuerst (1997).

In the following I first describe technologies, preferences and shocks which are common between the two models. Then, I describe the two real business cycle models separately. In doing so I make clear the role of uncertainty shocks in the two models. I show analytically that the uncertainty shocks emerge as financial shocks in the first model, while the uncertainty shocks emerge as investment shocks in the second model.

3.1. Common Building Blocks

Preferences: There is a continuum of household, with preferences given by

$$E_t \sum_{s=0}^{\infty} \beta^s U(C_t, L_t), \quad 0 < \beta < 1, \tag{23}$$

where C_t denotes consumption, L_t denotes labor supply and $U(\cdot)$ satisfies $U_1 > 0$, $U_2 < 0$, $U_{11}, U_{22} < 0$ and $U_{11}U_{22} - U_{12}^2 > 0$.

A household, with measure unity, consists of the large number of family members who are either workers or entrepreneurs with their population f and $1-f$ respectively where $0 < f < 1$. Family members switch their job occupation randomly. Specifically, entrepreneurs become workers randomly with probability $1-\gamma$ where $0 < \gamma < 1$. I call γ as the surviving probability of entrepreneurs. The same number of workers become entrepreneurs randomly so that the proportion of workers or entrepreneurs stays constant over time.

The household, as a representative agent of the family members, consumes and saves. The household provides the perfect consumption insurance among its family members. Workers within the household supply labor and earn wage income. Entrepreneurs within the household specialize in investing in a project and accumulate their net worth.

Entrepreneurs can transfer their net worth to the household to which they belong to at the beginning of period. The entrepreneur chooses to accumulate the net worth over time and transfers the net worth only when it becomes to a worker, because the average return from a unit of goods invested in its project is strictly greater than the risk-free return earned by the household who makes deposits in intermediaries. While the entrepreneur is subject to a risk associated with its project, the household cares only about the average return because there are many family members of entrepreneurs within the household. This modeling device justifies the assumption of risk-neutral entrepreneurs analyzed in Section 2. This setting, adopted from Gertler and Karadi (2010), allows me to embed entrepreneurs into the standard real business cycle model, keeping the representative agent framework in a reasonable manner.

A household can save only through deposits in intermediaries with risk-free rate R_{t+1}. Then, the flow budget constraint is given by

$$C_t + B_t = R_t B_{t-1} + w_t L_t + \Theta_t, \tag{24}$$

where B_t denotes the amount of deposit at the end of time t, w_t denotes wages, and Θ_t includes the sum of the net transfer from entrepreneurs who belong to the household and the net payment to the state-contingent securities on the return to capital. The net payment matters only in Model-II.

I introduce an exogenous countercyclical markup in wages to make clear its role as the amplification mechanisms of uncertainty shocks. I model the markup in the simplest but an ad-hoc manner for the sake of exposition.[8] Specifically, I assume that workers have a monopolistic power over labor supply and set real wages equal to the markup over the marginal rate of substitution between consumption and labor. I assume the markups are exogenous and countercyclical, given by

$$\lambda_{w,t} = \lambda_w \left(\frac{Y_t}{Y}\right)^{-\omega}, \quad \lambda_w > 1, \quad \omega > 0, \tag{25}$$

where Y_t denotes output and Y denotes output in steady state. The time-varying markup, $\lambda_{w,t}$, is decreasing in output and countercyclical by construction.

Maximizing utility (23) subject to budget constraint (24) yields the first order conditions:

$$\frac{1}{R_{t+1}} = E_t \beta \left[\frac{U_1(C_{t+1}, L_{t+1})}{U_1(C_t, L_t)}\right], \tag{26}$$

$$w_t = -\lambda_{w,t} U_2(C_t, L_t)/U_1(C_t, L_t), \tag{27}$$

where $\lambda_{w,t}$ denotes an exogenous countercyclical markup in wages. Equation (26) implies that the gross interest rate on deposit, R_{t+1}, is risk-free. Equation (27) implies that wage is equated to the markup over the marginal rate of substitution between consumption and labor.[9]

[8]One can introduce a countercyclical markup in wages in a rigorous manner by introducing nominal rigidities in wages as in Erceg, Henderson and Levin (2000). While the introduction of nominal rigidities in wages provides a solid micro-foundation for the countercyclical markup, it complicates the model. Because I focus on the counter-cyclical markup as amplification mechanisms, but not on the micro-foundation of the countercyclical markup, I introduce the countercyclical markup in the simplest manner for the sake of exposition.

[9]Here I simply assume that workers set real wages equal to the markup over the marginal rate of substitution between consumption and labor. One can derive exactly the same condition under a more rigorous set up considered by Erceg, et al (2000) with flexible wages and an exogenous countercyclical markup in wages.

Technologies: There are competitive representative consumption goods producers, with production technologies given by

$$Y_t = (u_t K_t)^\alpha L_t^{1-\alpha}, \quad 0 < \alpha < 1, \tag{28}$$

where K_t denotes the aggregate capital, L_t denotes the aggregate labor and u_t denotes the capital utilization rate. Factor prices are equated to marginal products:

$$r_t^k = \alpha(u_t K_t)^{\alpha-1} L_t^{1-\alpha}, \tag{29}$$
$$w_t = (1-\alpha)(u_t K_t)^\alpha L_t^{-\alpha}, \tag{30}$$

where r_t^k denotes the rental rate of capital services, $u_t K_t$.

Capital providers set the capital utilization rate, u_t. As I explain later the capital providers are entrepreneurs in Model-I while they are entrepreneurs or intermediaries in Model-II. The capital providers earn the rental rate, $r_t^k u_t$, per unit of capital, with the associated cost of capital utilization, $a(u_t)$, in consumption goods unit. As in CEE (2005), the cost satisfies $a'(u_t), a''(u_t) > 0$ and $a(1) = 0$. Maximizing the rental rate minus the cost with respect to u_t results in

$$r_t^k = a'(u_t). \tag{31}$$

As is clear from equation (31), the capital utilization rate depends only on the net return on capital, r_t^k, and is increasing in r_t^k.

The goods market clearing requires that

$$Y_t = C_t + I_t + a(u_t)K_t, \tag{32}$$

where I_t denotes the aggregate investment. Term $a(u_t)K_t$ denotes the total cost associated with capital utilization rate u_t. The law of motion for capital satisfies:

$$K_{t+1} = (1-\delta)K_t + \bar{I}_t, \quad 0 < \delta < 1, \tag{33}$$

where $\bar{I}_t$ denotes the newly produced capital goods, which is different from investment I_t in terms of consumption goods, and δ denotes the capital depreciation rate.

Uncertainty Shocks: As in the partial equilibrium model in Section 2, an entrepreneur has a project with success probability p, which follows distribution $F_t(p)$. Here I introduce an exogenous disturbance to the distribution of the riskiness of project and discuss the implications to aggregate variables. In order to get a simple analytical expression and conduct simulations later I assume that the distribution of the riskiness of project is uniform over interval $[\underline{p}_t, 1]$:

$$F_t(p) = \frac{p - \underline{p}_t}{1 - \underline{p}_t}, \quad \underline{p}_t \equiv \underline{p} e^{\upsilon_t}, \quad 0 < \underline{p} < 1, \tag{34}$$

with

$$\upsilon_t = \rho_\upsilon \upsilon_{t-1} + \epsilon_{\upsilon,t}, \quad 0 < \rho_\upsilon < 1,$$

where $\epsilon_{\upsilon,t}$ denotes a disturbance i.i.d. with mean zero. When the negative uncertainty shock, $\epsilon_{\upsilon,t} < 0$, hits the economy, the distribution becomes more dispersed and the entrepreneur's project becomes more risky on average. Intermediaries face higher degree of asymmetric information about the riskiness of project, p, than before.

3.2. Model-I: Adverse Selection in the Demand Side of Capital

I embed adverse selection in financial markets, analyzed in Section 2, into the demand side of capital as in BGG (1999). In this model, entrepreneurs own, trade and rent out capital. In trading capital entrepreneurs purchase capital so that the entrepreneurs' activities determine the demand for capital. In the supply side of capital there are competitive capital goods producers subject to investment adjustment costs. If the demand for capital increases, the price of capital increases and so does investment. In this model uncertainty shocks emerge as financial shocks which change a wedge between a return to capital and a risk-free rate.

Entrepreneurs and Intermediaries: There exist many entrepreneurs. At time t an entrepreneur starts its business with some amount of net worth in unit of consumption goods. The entrepreneur makes a one-period contract with an intermediary and receives a loan from the intermediary. Combining the net worth with the loan the entrepreneur purchases capital goods from capital goods producers with price q_t. In aggregate the balance sheet of entrepreneurs satisfies:

$$q_t K_{t+1} = N_t + B_t. \tag{35}$$

The left hand side of equation (35) denotes the value of capital purchased by entrepreneurs and the right hand side of equation (35) denotes the liability side of balance sheet, consisting of the aggregate net worth, N_t, and the aggregate loan, B_t.

At the end of time t the entrepreneur invests the capital goods in its project with success probability p and transforms the capital goods into specialized capital goods readily use for production. The riskiness of project, p, is private information to the entrepreneur. I assume that on average one unit of capital goods generates one unit of specialized capital goods for all projects. If the project fails the entrepreneur has nothing at hand. If the project succeeds the entrepreneur rents out the specialized capital goods to firms and earns rental rate $r^k_{t+1}u_{t+1}$ per unit of effective capital goods (specialized capital goods times capital utilization rates) at the beginning of time $t+1$. The entrepreneur incurs the cost of capital utilization rates, $a(u_{t+1})$, per unit of specialized capital goods. After renting out capital goods, the entrepreneur sells the depreciated capital goods to capital goods producers with price q_{t+1}. Consequently, on average the return from investing one unit of consumption goods, R^k_{t+1}, is given by,

$$R^k_{t+1} = \frac{r^k_{t+1} u_{t+1} + q_{t+1}(1-\delta) - a(u_{t+1})}{q_t}. \tag{36}$$

The return consists of the net return from renting out capital goods, $r^k_{t+1} u_{t+1}$, plus the capital gain from the depreciated capital goods, $q_{t+1}(1-\delta)$, minus the cost associated with capital utilization rate, $a(u_{t+1})$, per unit of capital purchased at price q_t.

After earning returns, the entrepreneur makes a loan payment to the intermediary. The remaining amount of consumption goods constitutes the entrepreneur's net worth at time $t+1$. At the beginning of time $t+1$, an idiosyncratic occupation shock hits the entrepreneur and it switches its job to a worker randomly with probability $1-\gamma$, and it stays an entrepreneur with probability γ, where $0 < \gamma < 1$. If the entrepreneur becomes a worker, it brings the net worth to the household to which it belongs. If the entrepreneur remains to be an entrepreneur, it starts its business with its net worth at time $t+1$ again. Those who just have become entrepreneurs from workers and those who do not have net worth receive a small amount of goods from the household to which they belong, so that they run their projects.

The financing problem between entrepreneurs and intermediaries proceeds as in the partial equilibrium model in Section 2. The returns, R^e and R^f in the model in Section 2, corresponds to $E_t R^k_{t+1}$ and R_{t+1} in this general equilibrium model respectively. In applying the solution in Section 2, I assume that the payment, $X_n(p)$, does not depend on states at time $t+1$. This assumption is innocuous because entrepreneurs behave as if they were risk neutral.

Now I derive equilibrium conditions by applying the solution in Section 2. First, I derive an equation for the demand for capital. Substituting (20) into (35) and using the expression for $F_t(\cdot)$, (34), I can express the value of purchased capital as

$$q_t K_{t+1} = \left[1 + \frac{(1-\phi)s_t}{1-(1-\phi)s_t} \frac{p^*_t - \underline{p}_t}{1-\underline{p}_t}\right] N_t, \tag{37}$$

where $s_t \equiv E_t R^k_{t+1}/R_{t+1}$ denotes the discounted return to capital and $\underline{p}_t = \underline{p} e^{\upsilon_t}$ denotes the lower bound of the support of the distribution of riskiness. Equation (37) defines the demand for capital. If the right-hand-side of equation (37) increases, for example, due to a change in the net worth, the demand curve of capital shifts upward. The price of capital, q_t, increases, which, in turn, increases investment and output.

Next, I consider an equation that determines threshold p^*_t above which entrepreneurs do not get loans. According to Proposition 1 the threshold is given by the zero profit condition, (18). Given uniform distribution $F_t(\cdot)$ I can express zero profit condition (18) as

$$v_{t+1,n}(p^*_t) = \int_{\underline{p}_t}^{p^*_t} \left[\left(1 - \frac{1}{s_t}\right) \frac{p}{1-\underline{p}_t} - \frac{1}{s_t} \frac{p - \underline{p}_t}{1-\underline{p}_t}\right] p^{-\frac{1}{\phi}} dp = 0, \tag{38}$$

Under an assumption that $v_{t+1,n}(1) < 0$, equation (38) has a unique interior solution.[10] Expanding equation (38), I obtain

$$0 = \frac{\phi}{2\phi-1}\left(1-\frac{2}{s_t}\right)\left[(p_t^*)^{\frac{2\phi-1}{\phi}} - \underline{p}_t^{\frac{2\phi-1}{\phi}}\right] - \frac{\underline{p}_t}{s_t}\frac{\phi}{1-\phi}\left[(p_t^*)^{-\frac{1-\phi}{\phi}} - \underline{p}_t^{-\frac{1-\phi}{\phi}}\right], \tag{39}$$

where I have assumed $\phi \neq 1/2$. Equation (39) defines p_t^* given s_t and $\underline{p}_t$.

Next, I derive an equation for an external finance premium, which constitutes an important financial variable in the model. I define an external finance premium, EFP_t, as a ratio of the interest rate of external finance to the opportunity cost of internal funds (or risk-free interest rates), R_{t+1}. In this model the interest rate of external finance corresponds to a loan interest rate. Because a loan interest rate differs among entrepreneurs with different level of riskiness, as shown in Lemma 2, I use the average loan interest rate as the interest rate of external finance.

I derive an expression for the average loan interest rate first and derive an expression for an external finance premium. Conditional on the success of project, an entrepreneur pays back to an intermediary following the payment schedule, (16). Reminding that R^e in the partial equilibrium model corresponds to $E_t R_{t+1}^k$ in this model and aggregating (16) over riskiness p and net worth N_j, I obtain the aggregate payment conditional on the success of project,

$$X_{t+1} = \int_n \int_{\underline{p}_t}^{p_t^*} X_n(p) dF_t(p) dH_t(n) = \frac{E_t R_{t+1}^k}{1-\underline{p}_t}\left[\left(\frac{p_t^*}{\underline{p}_t}\right)^{\frac{1-\phi}{\phi}} - 1\right] N_t \tag{40}$$

I define the average loan interest rate as a ratio of the aggregate payment to the aggregate loan: $R_{t+1}^b = X_{t+1}/B_t$, where $B_t = q_t K_{t+1} - N_t$ is given by (37). Then I can express the external finance premium, EFP_t, as follows:

$$\mathrm{EFP}_t = R_{t+1}^b / R_{t+1} = \frac{1-(1-\phi)s_t}{(1-\phi)(p_t^* - \underline{p}_t)}\left[\left(\frac{p_t^*}{\underline{p}_t}\right)^{\frac{1-\phi}{\phi}} - 1\right]. \tag{41}$$

The external finance premium must be positive or $EFP_t > 1$.[11]

Finally I derive the law of motion for aggregate net worth. In aggregate entrepreneurs earn return $R_{t+1}^k(N_t + B_t)$ and pay $R_{t+1}B_t$ to intermediaries at the beginning of period $t+1$. The aggregate payment amounts to $R_{t+1}B_t$ because the intermediary's zero profit

[10] The assumption, $v_{t+1,n}(1) < 0$, holds for a reasonable range of parameter values of s_t and ϕ.

[11] To see this note that the conditional repayment, given by (40), is strictly greater than the average repayment, $\int_n \int_p p X_n dF_t(p) dH_t(n)$, and the average repayment over B_t is R_{t+1} from the zero profit condition, (18). Without asymmetric information the external finance premium would become $\mathrm{EFP}_t = (1-\underline{p}_t)^{-1}\log(1/\underline{p}_t)$, which purely reflects the default risk of entrepreneurs.

condition holds.[12] A fraction, $1-\gamma$, of entrepreneurs become workers and bring their net worth to households. The same number of workers become new entrepreneurs. The new entrepreneurs and those who do not have any net worth receive the small amount of start up funds from households. I assume that the transfer from households at time t is proportional to output, given by ξY_t, where $0<\xi<1$. Then I can write the law of motion for aggregate net worth as

$$N_{t+1}=\gamma[(R^k_{t+1}-R_{t+1})B_t+R^k_{t+1}N_t]+\xi Y_{t+1}, \tag{42}$$

where $B_t=q_tK_{t+1}-N_t$ is given by (37)

In sum, four equations (37), (39), (41) and (42) describe the aggregate consequences of the financing problem between entrepreneurs and intermediaries.

Capital goods Producers: Competitive capital goods producers run two types of business. First, they produce new capital goods. Second, they purchase depreciated capital goods from entrepreneurs and sell new capital goods to entrepreneurs.

A capital goods producer purchases consumption goods, I_t, and transforms it into new capital goods, $\bar{I}_t$, using following technologies,

$$\bar{I}_t=\left[1-S\left(\frac{I_t}{I_{t-1}}\right)\right]I_t, \tag{43}$$

where function $S(\cdot)$ denotes investment adjustment costs satisfying $S(1)=S'(1)=0$ and $S''(1)>0$ in steady state. This functional form, introduced by CEE (2005), allows the model to generate a hump-shaped response of investment and output to various shocks, consistent with VAR-based evidence.

The capital goods producer purchases depreciate capital goods from entrepreneurs with price q_t, combines them with newly produced capital goods and produce capital goods, using the linear capital accumulation technology, (33). Then the capital goods producer sells the capital goods with price q_t to entrepreneurs. Given the price of capital, q_t, the capital goods producer chooses the amount of investment to maximize the expected profit:

$$\max_{\{I_t\}}E_t\sum_{s=0}^{\infty}\frac{\beta^sU_1(C_{t+s},L_{t+s})}{U_1(C_t,L_t)}\left\{q_{t+s}K_{t+s+1}-[I_{t+s}+q_{t+s}(1-\delta)K_{t+s}]\right\}, \tag{44}$$

subject to production technologies, (33) and (43). Because a household owns capital goods producers, the capital goods producer discounts future profits using the household's discount factor. Because of linearity in producing capital goods in (33), perfect competition results in the same price between new capital goods and old capital goods.

[12] Intermediaries take in deposits B_t from households with risk-free interest rate R_{t+1} and lend to entrepreneurs. The intermediary's zero profit condition results in the aggregate payment equal to $R_{t+1}B_t$, as shown in the first line of equation (21), where the right-hand-side denotes the aggregate payment divided by R_{t+1}.

Equilibrium of Model-I: In defining a competitive equilibrium I suppose that the economy starts from period 0. Given the set of initial condition, $\{K_0,\ N_0,\ I_{-1},\ A_{-1}\}$ and the processes of shocks, a *recursive competitive equilibrium* consists of the decision rules for the allocation, $\{Y_t,\, C_t,\, I_t,\, K_{t+1},\, L_t,\, N_{t+1},\, u_t,\, p_t^*\}$, and the pricing rules for the set of prices $\{r_t^k$, $w_t,\, q_t,\, R_{t+1}\}$, where the both rules are the functions of the states of the economy, satisfying:

- Given the pricing rules, the decision rules satisfy the intermediate goods producers' first order conditions, (29) and (30), the household's first order conditions, (26) and (27), the condition for capital utilization rate, (31), and solve capital goods producer's problem (44). Also, the decision rules satisfy the optimality conditions of entrepreneurs and intermediaries, (37) and (39), and the law of motion for capital, (42).

- All markets clear. That is, (32) and (33) hold, where Y_t in (32) is given by (28) and $\bar{I}_t$ in (33) is given by (43).

Uncertainty Shocks as Financial Shocks: Now I study the role of uncertainty shocks. In order to make clear the role of uncertainty shocks I log linearize the model around its steady state. Let $\hat{x}_t$ denote the deviation of variable x_t from its steady state at time t. Log-linearizing equations (37) and (39) and substituting out p_t^* I obtain:

$$\hat{s}_t = -\chi_1 \left(\hat{N}_t - \hat{q}_t - \hat{K}_{t+1} \right) - \chi_2 \upsilon_t, \qquad \chi_1, \chi_2 > 0. \tag{45}$$

For the derivation of equation (45), see Appendix.

Equation (45) summarizes both the effect of the uncertainty shocks and the role of credit frictions. Without the uncertainty shocks, $\upsilon_t = 0$, equation (45) coincides with the equation of the demand for capital in BGG (1999) after log-linearizion. The discounted return to capital, s_t, rises when the net worth decreases. The negative relationship between the discounted return to capital and the net worth serves as a financial accelerator or a balance sheet channel: a decrease in net worth increases the discounted return, which, in turn, decreases the net worth, and so on.

Equation (45) includes the uncertainty shocks, υ_t, which do not appear in the original BGG (1999) model. In equation (45) the negative uncertainty shocks, $\upsilon_t < 0$, raise the discounted return to capital, s_t, or a wedge between a return to capital and risk free rates. That is, the uncertainty shocks increase the cost of borrowing of consumption goods firms relative to risk free rates. The negative shocks decrease the net worth and decrease the demand for capital from equation (37). It results in a fall in the price of capital, investment and output, generating business fluctuations consistent with business cycle facts as I explore quantitatively later.

The uncertainty shocks play exactly the same role as financial shocks considered by Hall (2010), Gilchrist, Ortiz and Zakrajsek (2009) and Gilchrist and Zakrajsek (2010). They introduce financial shocks which change a wedge between the return to capital and the risk free rate in a reduced form manner. In my model the residual terms in equation (45) have a solid micro-foundation and a clear interpretation. The uncertainty shocks affect the distribution of the riskiness of entrepreneurial project. The uncertainty shocks change the degree of asymmetric information exogenously and affect the severity of adverse selection endogenously, generating business fluctuations.

3.3. Model-II: Adverse Selection in the Supply Side of Investment

I embed adverse selection in financial markets, analyzed in Section 2, into the supply side of investment as in Carlstrom and Fuerst (1997). In this model, entrepreneurs own net worth and produce new capital goods. Entrepreneurs combine their own net worth and loans from intermediaries, purchase consumption goods and transform them into newly produced capital goods. In contrast to Model-I, the entrepreneurs' activities determine the supply of investment. The intermediaries not only provide loans to entrepreneurs but also purchase depreciated capital goods from entrepreneurs. In this model the uncertainty shocks emerge nearly as shocks to the marginal efficiency of investment (MEI shocks).

Entrepreneurs and Intermediaries: Intermediaries take in deposits from households with risk free rate R_{t+1}. The intermediaries can earn return either by purchasing capital or by investing in entrepreneurs and receiving newly produced capital from entrepreneurs. If the intermediaries purchase capital with one unit of consumption goods, they earn the rental rate of capital and sell the depreciated capital in the next period , so that return R^k_{t+1} is given by (36). Because the intermediaries promise to pay R_{t+1} to households, which may exceed risky return R^k_{t+1}, the intermediaries sell contingent claims with return $R^k_{t+1} - R_{t+1}$ to households. Consequently, the following arbitrage condition holds:

$$1 = E_t \beta \left[\frac{U_1(C_{t+1}, L_{t+1})}{U_1(C_t, L_t)} R^k_{t+1} \right]. \tag{46}$$

Intermediaries pay the risk free rate to households by combining a return to capital, R^k_{t+1}, and a net return from contingent claims, $R_{t+1} - R^k_{t+1}$.

Intermediaries are willing to provide loans to entrepreneurs as long as they receive risk-free rate R_{t+1} tomorrow in return for one unit of consumption goods today. If an entrepreneur provides $1/q_t$ units of newly produced capital goods today in return for one unit of consumption goods, tomorrow it yields return R^k_{t+1}, or risk free rate R_{t+1} if combined with a net payment from contingent claims. A financing problem between entrepreneurs and intermediaries features an intra-period arrangement. The return, R^f, in the partial equilibrium model in Section 2 corresponds to return $1/q_t$ in this general equilibrium model.

An entrepreneur specializes in producing new capital goods from consumption goods. The entrepreneur has linear technologies (projects) transforming one unit of consumption goods into μ_t unit of new capital goods on average, where μ_t denotes investment shocks, following a stochastic process:

$$\log(\mu_t) = \rho_\mu \log(\mu_{t-1}) + \epsilon_{\mu,t}, \quad 0 \leq \rho_\mu < 1. \tag{47}$$

As in the partial equilibrium model in Section 2, if the entrepreneur invests one unit of consumption goods in its project, the entrepreneur successfully produces $\theta(p)\mu_t$ units of new capital goods with probability p, where $\theta(p) \equiv 1/p$. Riskiness p is private information to the entrepreneur. In this model, an expected return on project is μ_t, which corresponds to R^e in the model in Section 2.

The financing problem between entrepreneurs and intermediaries proceeds as in the model in Section 2. At the beginning of time t, an entrepreneurs combines its own net worth and the loan from intermediaries to finance the purchase of consumption goods. In aggregate, the entrepreneur's balance sheet is given by

$$I_t = N_t + B_t^e, \tag{48}$$

where I_t denotes the investment in terms of consumption goods and B_t^e denotes the loan from intermediaries. Applying the result in Section 2, in equilibrium I can express the aggregate loan, B_t^e, as the right-hand-side of equation (22) with R^e/R^f in (22) replaced by $\mu_t/(1/q_t) = q_t\mu_t$. Then, I can rewrite (48) as

$$I_t = \left[1 + \frac{(1-\phi)q_t\mu_t}{1-(1-\phi)q_t\mu_t}\frac{p_t^* - \underline{p}_t}{1-\underline{p}_t}\right] N_t, \tag{49}$$

where I have used the uniform distribution of the riskiness of project, given by (34). Threshold, p_t^*, is determined by the zero profit condition, (39) with s_t replaced by $q_t\mu_t$, which is essentially the same as in Model-I. The aggregate newly produced capital satisfies:

$$\bar{I}_t = \mu_t I_t. \tag{50}$$

In contrast to Model-I, there is no investment adjustment cost in this model. Actually, the financial frictions embedded in the supply side of investment play a role similar to investment adjustment costs, as indicated by Carlstrom and Fuerst (1997).

In aggregate entrepreneurs produce new capital goods $\bar{I}_t$, given by (50), and pay $(1/q_t)B_t^e$ units of capital goods to intermediaries. Entrepreneurs rent remaining capital goods to consumption goods firms and sell the depreciated capital to intermediaries in the beginning of next period. Then, the law of motion for aggregate net worth is given by

$$N_{t+1} = \gamma[r_{t+1}^k + (1-\delta)q_{t+1}][\bar{I}_t - (1/q_t)B_t^e] + \xi Y_{t+1}, \tag{51}$$

where γ denotes the survival probability of entrepreneurs and ξY_{t+1} denote the lump-sum transfers at time $t+1$ from households for entrepreneurs who have failed in their projects or for newly born entrepreneurs.

Equilibrium of Model-II: Given the set of initial condition, $\{K_0, N_0, A_{-1}\}$ and the processes of shocks, a *recursive competitive equilibrium* consists of decision rules for the allocation, $\{Y_t, C_t, I_t, K_{t+1}, L_t, N_{t+1}, u_t, p_t^*\}$, and pricing rules for the set of prices $\{r_t^k, w_t, q_t, R_{t+1}\}$, where the both rules are the functions of the states of the economy, satisfying:

- Given the pricing rules, the decision rules satisfy the consumption goods producer's first order conditions, (29) and (30), the household's first order conditions, (26) and (27), the condition for capital utilization rate, (31), and the arbitrage condition, (46). Also, the decision rules satisfy the optimality conditions of entrepreneurs and intermediaries, (49) and (39), and the law of motion for aggregate net worth, (51).

- All markets clear. That is, (32) and (33) hold, where Y_t in (32) is given by (28) and $\bar{I}_t$ in (33) is given by (50).

Uncertainty Shocks as Investment Shocks: Now I study the role of uncertainty shocks in the model with adverse selection in the supply side of investment. I show that uncertainty shocks emerge nearly as the MEI shocks which change the marginal efficiency of investment.

Two equations, (39) and (49) with s_t replaced by $q_t\mu_t$, determine the investment, I_t. Log-linearizing those two equations I obtain:

$$\hat{I}_t = \left(\frac{1}{\chi_3}\right)(\hat{q}_t + \hat{\mu}_t) + \hat{N}_t + \left(\frac{\chi_4}{\chi_3}\right)\upsilon_t, \qquad \chi_3, \chi_4 > 0, \tag{52}$$

where exact expressions for coefficients χ_3 and χ_4 are given in Appendix. Then, the newly produced capital, $\bar{I}_t = \mu_t I_t$, after log-linearization, is given by

$$\hat{\bar{I}}_t = \left(\frac{1}{\chi_3}\right)\hat{q}_t + \hat{N}_t + \left[\left(\frac{1+\chi_3}{\chi_3}\right)\hat{\mu}_t + \left(\frac{\chi_4}{\chi_3}\right)\upsilon_t\right]. \tag{53}$$

Equation (53) makes clear that the uncertainty shocks, υ_t, emerge nearly as the MEI shocks, $\hat{\mu}_t$. Given $\hat{q}_t$ and $\hat{N}_t$ both the uncertainty shocks and the MEI shocks change the newly produced capital. On the one hand, the uncertainty shocks change the severeness of adverse selection and affect the amount of loans and investment. On the other hand, the MEI shocks affect not only the amount of loans but also the efficiency of investment. After log-linearization the two shocks affect the newly produced capital in the same manner.

In this model both the uncertainty shocks and the MEI shocks appear in the two equilibrium conditions: the condition for $\bar{I}_t$, (50) with I_t substituted out using (49), and the law of motion for aggregate net worth, (51). Adjusted the magnitude of the two shocks, the two shocks have the same effect on the newly produced capital in equation (50). A slight difference appears in the law of motion for aggregate net worth, (51). In (51), the effects on $\bar{I}_t$ of the two shocks are the same while the effects on B_t^e of the two shocks are different. I will explore the difference between the uncertainty shocks and the MEI shocks quantitatively in the next section.

4

4. Simulation

I log-linearize the two general equilibrium models, presented in Section 3, around steady state and conduct simulations to explore the quantitative effect of uncertainty shocks. In the following I first parameterize the two models. Next, I study impulse responses to uncertainty shocks for the two models. Then, I study mechanisms which amplify uncertainty shocks and help uncertainty shocks generate business cycles. Finally, I conduct stochastic simulations and examine how much volatility of key macroeconomic variables can be explained by uncertainty shocks.

4.1. Model Parameterization

Before setting parameters I specify the utility functional as follows:

$$U(C,L) = \log(C) - \psi \frac{L^{1+1/\nu}}{1+1/\nu}, \qquad \psi, \nu > 0.$$

I list the choice of parameters values in Table 1. Out of thirteen parameters, five $\{\phi, \underline{p}, \gamma, \xi, \rho_u\}$ are specific to the models with credit frictions and the others are conventional except the elasticity of markups.

I begin with conventional parameters. The period of time is quarterly. I set a preference discount factor as $\beta = 0.993$ so that the net real risk-free rate becomes 3 percent annual rate in steady state. I set the coefficient of the disutility of labor, ψ, in a way that the average hours worked becomes unity in steady state. I choose conventional values for a labor supply elasticity ($\nu = 1$), a capital income share ($\alpha = 0.36$) and a capital depreciation rate ($\delta = 0.025$). I set the curvature of investment adjustment costs to $S''(1) = 1$, which locates at the lower range of estimated values in the DSGE literature, though there is little guidance in the empirical literature about appropriate values. I choose this value because the implied curvature is enough to generate the reasonable hump-shaped responses of output and investment in Model-I. I set the parameter of capital utilization costs as $\chi \equiv a''(1)/a'(1) = 5$, which is consistent with estimated values in the DSGE literature including Justiniano, et al (2009a,b). I set a markup in wages in steady state as $\lambda_w = 1.2$ and the elasticity of markup in wages as $\omega = 2$. The elasticity is consistent with Gali, Gertler and Lopez-Salido (2007) who report that a wage markup is slightly more than twice as volatile as output. Also, they report that a contemporaneous correlation between a wage markup and output is -0.83, consistent with a countercyclical markup in the models.

Next I set parameters specific to the models with credit frictions, $\{\phi, \underline{p}, \gamma, \xi, \rho_u\}$. I set $\xi = 0.001$ which determines the amount of lump-sum transfers from households to entrepreneurs. As in Carlstrom and Fuerst (1997) I make the transfers small so that the transfers do not add additional dynamics. I set the AR(1) coefficient of uncertainty shocks as $\rho_u = 0.75$, following Gilchrist and Zakrajsek (2010) who use VARs and estimate the

Table 1: Parameters Values

Parameter	Description	Value	Parameter	Description	Value
β	discount factor	0.993	ν	labor supply elasticity	1
α	capital income share	0.36	δ	depreciation rate	0.025
$S''(1)$	adjustment costs	1	χ	capital utilization costs	5
λ_w	wage markup	1.2	ω	elasticity of markup	2
ϕ	moral hazard	0.55	$\underline{p}$	parameter of $F(\cdot)$	0.993
γ	survival rates	0.99	ξ	transfers	0.001
ρ_u	AR(1), uncertainty shocks	0.75			

auto-correlation of financial shocks equal to 0.75. As shown in Section 3 the uncertainty shocks in Model-I turn out to be the same as financial shocks after log-linearization.

I set remaining three parameters to hit following three targets in steady state: the leverage ratio of 1.5, the external finance premium of 2 percent annual rate, and the expected equity premium of 1 percent annual rate. The target value of the leverage ratio is lower than the literature[13], but it is consistent with U.S. Flow of Funds Accounts according to which the debt net worth ratio of non-farm non-financial corporate business is around 0.5, implying a leverage ratio of 1.5.[14] The target value of the external finance premium is close to the average spread of corporate bonds with various credit quality relative to comparable maturity Treasury yield from 1990 to 2008 of 1.92 percent, analyzed by Gilchrist, Yankov and Zakrajsek (2009). The target value of the expected equity premium does not mean to match any numbers but suggests that there exists a discrepancy between a return of capital and a risk-free rate due to credit frictions.

4.2. Responses to Uncertainty Shocks

I study responses to the uncertainty shocks in the two models presented in Section 3. First, I draw the responses in Model-I and show that the uncertainty shocks generate the co-movement of variables consistent with business cycles. Second, I draw the responses in Model-II and show that the uncertainty shocks have similar quantitative implications for key variables to the MEI shocks. Though Model-II exhibits fluctuations consistent with business cycles for key variables, the model fails to generate a co-movement for the price of capital and the net worth.

[13]For example, the leverage ratio in steady state is 2 in BGG (1999) and 4 in Gertler and Karadi (2010).

[14]Ajello (2010) analyzes the U.S. public non-financial companies included in Compustat and reports that 33 percent of the capital expenditures of those firms is funded using financial markets. While a leverage ratio in general heavily depends on the type of borrowers and markets, Ajello (2010)'s finding implies that leverage ratio of 1.5 is not low.

Before proceeding to simulation result I would like to emphasize the unique nature of the uncertainty shocks. The shocks have real effects because of asymmetric information and its by-product of adverse selection. With symmetric information the shocks would not have any real effects as I showed in Section 2.2.

Model-I: Figure 1 plots impulse response functions to the negative uncertainty shocks. In period $t = 0$, an economy is in steady state. In period $t = 1$, the negative uncertainty shocks hit the economy. I set the magnitude of the shocks in such a way that an external finance premium rises 1 percent annual rate from its steady state level at the impact of the shocks. Except the external financial premium the vertical axis of figures shows the percent deviation of variable from its steady state.

Figure 1 shows that the uncertainty shocks in Model-I generate the co-movement of variables consistent with business cycles. In response to the negative uncertainty shocks which increase the external finance premium by 1 percent annual rate, all variables decrease. The output decreases about 0.4 percent with its bottom reached in four periods after the shocks. The output shows a hump-shaped response thanks to the CEE adjustment costs. While the auto-correlation of shocks is relatively low, $\rho_v = 0.75$, the output shows a persistent response: the output stays below a half of its bottom even after 20 periods.

The investment decreases about 1.2 percent with a hump-shaped response. The magnitude of the change is about three times as great as output. The consumption decreases slightly in the initial periods and continue decreasing until eighteen periods after the shocks. The hours show a response similar to the output. The capital utilization rate also decreases with a hump-shaped response.

The notable feature of Model-I appears in the responses of the price of capital and the net worth. The price of capital decreases about 0.3 percent at the impact of the shocks and quickly moves back to the steady state in seven periods. The net worth shows a response similar to the price of capital, because the current net worth is mainly determined by the gross return on capital, which, in turn, is mainly determined by the current price of capital, as is clear from equations (36) and (42). With the right co-movement of the price of capital and the net worth, the uncertainty shocks in Model-I generate fluctuations consistent with business cycles.

Model-II: Next I proceed to responses to the uncertainty shocks in Model-II. Figure 2 plots two impulse response functions: one to the negative uncertainty shocks (solid line) and the other to the negative MEI shocks (dashed line). I set the magnitude of the uncertainty shocks as same as before. I set the magnitude of the MEI shocks in such a way that the responses of output coincide with those to the uncertainty shocks at an impact. As a result, the magnitude of the MEI shocks is about one-third of the magnitude of the uncertainty shocks, as shown in the right bottom panel in Figure 2.

Figure 2 makes three important observations. First, the uncertainty shocks fail to

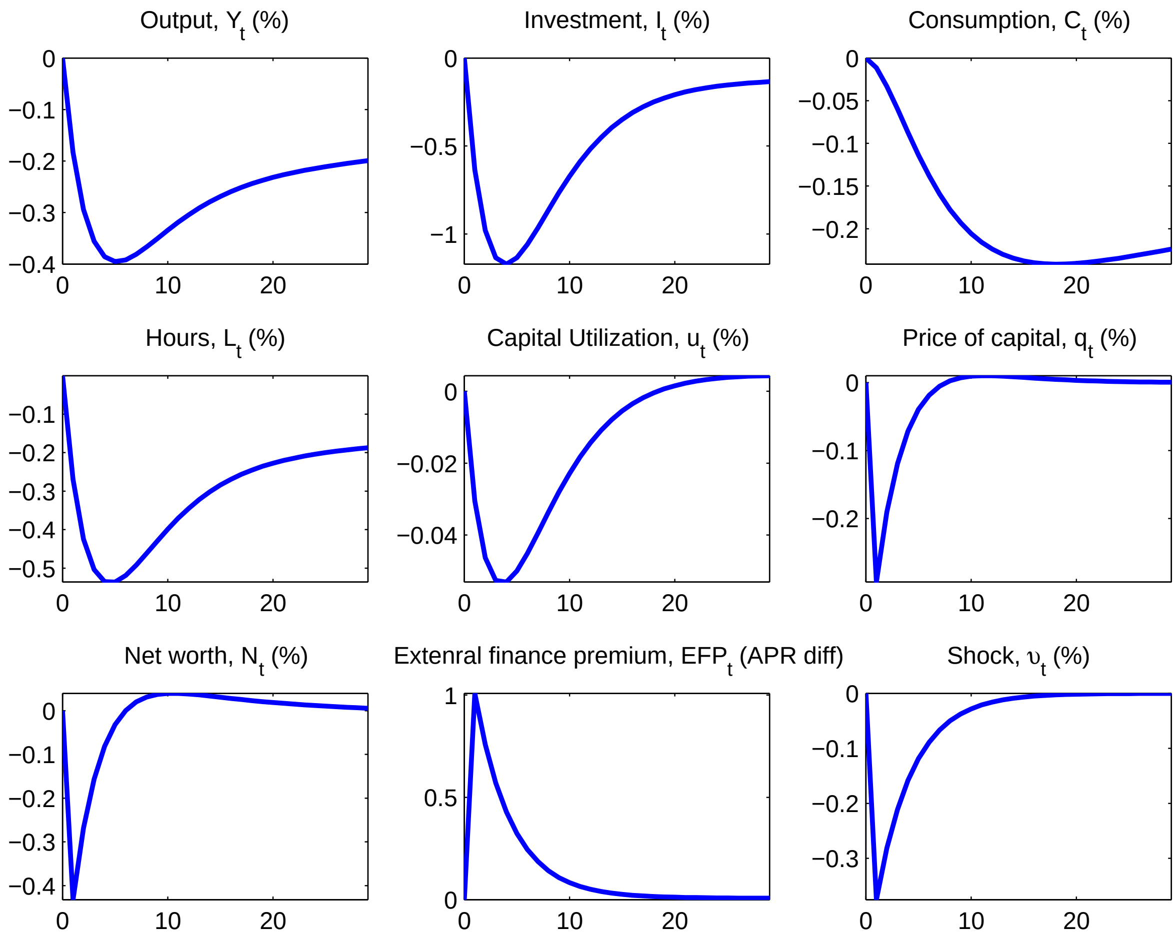

Figure 1: Impulse Responses to Uncertainty Shocks (Model I)

generate the co-movement of the price of capital and the net worth in Model-II. In this model the negative uncertainty shocks shift the supply curve of investment inward. As a result, the price of capital increases, while the investment decreases. As shown in the law of motion for net worth, (51), the current net worth is increasing in the price of capital. An increase in the price of capital causes an increase in the net worth.

Second, the uncertainty shocks succeed in generating the co-movement of variables except the price of capital and the net worth. In response to the negative uncertainty shocks, an external finance premium increases slightly more than 1 percent annual rate. The output decrease about 0.65 percent at the impact of the shocks and moves gradually back to the steady state. The investment decreases about 2.2 percent whose magnitude is slightly more than three times as great as the output. The consumption decreases slightly in the initial periods and continue decreasing, showing a very persistent response. The hours show responses similar to the output and the capital utilization rate decreases too.

Third, the uncertainty shocks generate observationally equivalent responses to those to

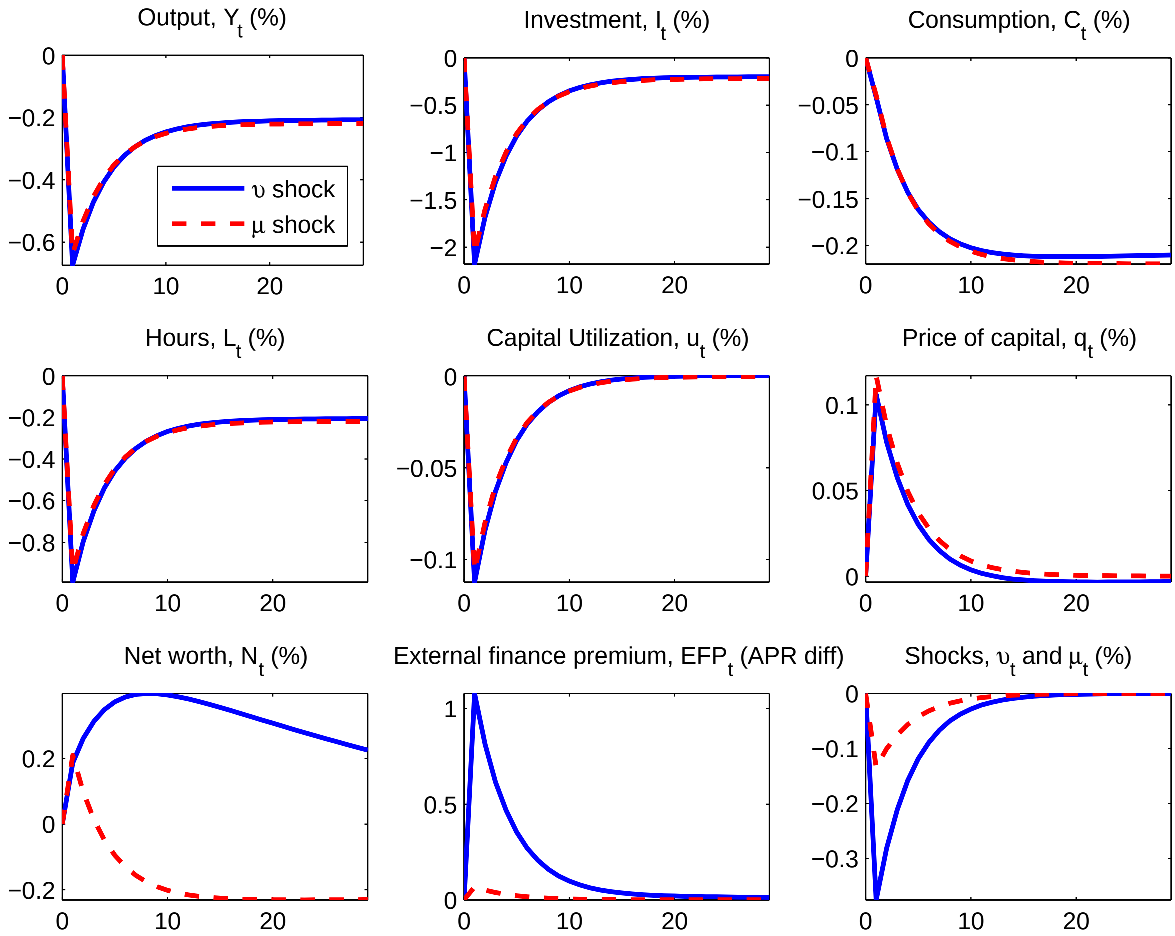

Figure 2: Impulse Responses to Uncertainty Shocks (Model II)

the MEI shocks for all variables except the net worth and the external finance premium. As argued in Section 3.3, the two shocks appear in two equations in Model II. On the one hand, the uncertainty shocks, after adjusted the magnitude of the shocks, have the same effect on newly produced capital, $\bar{I}_t$, as do the MEI shocks. On the other hand, the two shocks have the different effect on the aggregate loan, B_t^e, and so do on the net worth. Because the two shocks affect the newly produced capital in the same manner, the responses of all variables except the net worth and the external finance premium almost coincide with minor differences.

The minor differences reflect feedback effect from the net worth, which shows different responses between the two shocks. In response to the uncertainty shocks the net worth increases because the price of capital increases. In response to the MEI shocks the net worth does not increase very much because the aggregate loan, B_t^e, is greater than that in response to the uncertainty shocks, while the effect on newly produced capital, $\bar{I}_t$, is almost the same.[15] As a result, from the law of motion for net worth, (51), the net worth does not increase relative to that in response to the uncertainty shocks.

The third observation of the equivalence between the uncertainty shocks and the MEI shocks provides a rationale for the finding by Justiniano, et al (2009b). They estimate a DSGE model and find that the MEI shocks serve as the most important shocks driving the U.S. business cycles. They interpret the MEI shocks as something related with financial factors. They show empirically that their estimated MEI shocks are negatively correlated with an external finance premium, though their estimated model abstracts from financial factors and does not have an external finance premium. In Model-II, imperfect financial markets materialize the uncertainty shocks and the negative uncertainty shocks increase an external finance premium, consistent with the finding by Justiniano, et al (2009b). The result obtained here suggests that the uncertainty shocks combined with imperfect financial markets can be a candidate for the source of the MEI shocks.

4.3. Amplification Mechanisms

The previous analysis on responses to the uncertainty shocks shows that both Model-I and Model-II generate the co-movement of key variables: output, investment, consumption and hours. Here I study mechanisms behind the success in generating the co-movement. Set aside the adverse selection in financial markets, the two models have additional features relative to the standard real business cycle model: a countercyclical markup in wages and variable capital utilization rates. The two features amplify the uncertainty shocks and help generate the co-movement of key variables.

I show analytically that both a countercyclical markup in wages and variable capital utilization rates are essential in generating the co-movement of key variables. The equation governing a co-movement between hours and consumption is the intra-temporal optimality condition of households, (27), which equates wages to a markup over the marginal rate of substitution between consumption and hours. After substituting out wages w_t, capital utilization rates u_t, markup $\lambda_{w,t}$ and output Y_t using (30), (31), (25) and (28) respectively, I log-linearize equation (27) and obtain a relationship between consumption and hours:

$$\hat{C}_t = \left[(\omega+1)(1-\alpha)\left(1+\frac{\alpha}{\chi+1-\alpha}\right)-(1+1/\nu)\right]\hat{L}_t + \frac{(\omega+1)\alpha\chi}{\chi+1-\alpha}\hat{K}_t, \tag{54}$$

where $\omega \geq 0$ denotes the elasticity of markup and $\chi > 0$ denotes the elasticity of capital

[15] From equations (48)-(50), on the one hand, the uncertainty shocks affect the newly produced capital, $\bar{I}_t$, only through an effect on the aggregate loan, B_t^e. On the other hand, the MEI shocks affect the newly produced capital not only through an effect on the aggregate loan but also through the change in the marginal efficiency of investment. Because I scale the magnitude of the investment shocks in such a way that the effect on the newly produced capital becomes the same, the effect on the aggregate loan to the MEI shocks becomes smaller than that of the uncertainty shocks.

utilization costs. I put the derivation of equation (54) in Appendix. If $\omega = 0$ a counter-cyclical markup in wages vanishes. If $\chi = \infty$ variable capital utilization rates vanish.

In order to understand the role of a counter-cyclical markup in wages and variable capital utilization rates, I consider the case of the standard business cycle model in which the two are shut off: $\omega = 0$ and $\chi = \infty$. In this case, a coefficient on hours becomes negative, $-(\alpha + 1/\nu) < 0$, because the two parameters, α and ν, have to satisfy $0 < \alpha < 1$ and $\nu > 0$. A co-movement between consumption and hours depends mainly on a coefficient on hours, because the capital moves very slowly. Therefore, without a counter-cyclical markup in wages and variable capital utilization rates, both Model-I and Model-II would fail to generate a co-movement between the two variables. Intuitively, in recessions hours decrease and wages increase without neutral technological shocks and variable capital utilization rates. As a result of substitution from hours to consumption, consumption increases without a counter-cyclical markup in wages. This result reflects a famous co-movement problem: only neutral technology shock can easily generate the co-movement among key variables in a real business cycle framework, first pointed out by Barro and King (1984).

Equation (54) shows that a coefficient on hours in equation (54) is increasing in the degree of a counter-cyclical markup in wages (ω) and is increasing in the degree of variable capital utilization rates ($1/\chi$). Intuitively, in recessions a labor market becomes less competitive due to an increase in markup. Also capital utilization rates decrease, mitigating an increase in wages due to a decrease in hours. If the effect of a counter-cyclical markup supported by variable capital utilization rates dominates the substitution effect, consumption decreases. Under the baseline parameters values, a coefficient on hours is positive and that's why both Model-I and Model-II succeeds in generating a co-movement between hours and consumption.

The mechanisms generating a co-movement between hours and consumption also serve as amplification mechanisms. In response to the negative uncertainty shocks, an decrease in the aggregate demand is enhanced by a decrease in consumption as well as a decrease in investment, resulting in an amplified response of output.

To see the effect of the amplification mechanisms, Figure 3 plots impulse response functions to the negative uncertainty shocks with various degree of a counter-cyclical markup: from baseline $\omega = 2$ to no variable markup $\omega = 0$. The first raw of Figure 3 plots the responses of output and consumption in Model-I and the second raw of Figure 3 plots those in Model-II.

The responses in the two models exhibit high sensitivity to the value of ω around baseline $\omega = 2$. When ω drops to $\omega = 1.8$, the output decreases less by about one-fourth and the consumption increases initially in Model-I, while in Model II the output decreases less by about a half and consumption increases initially too.

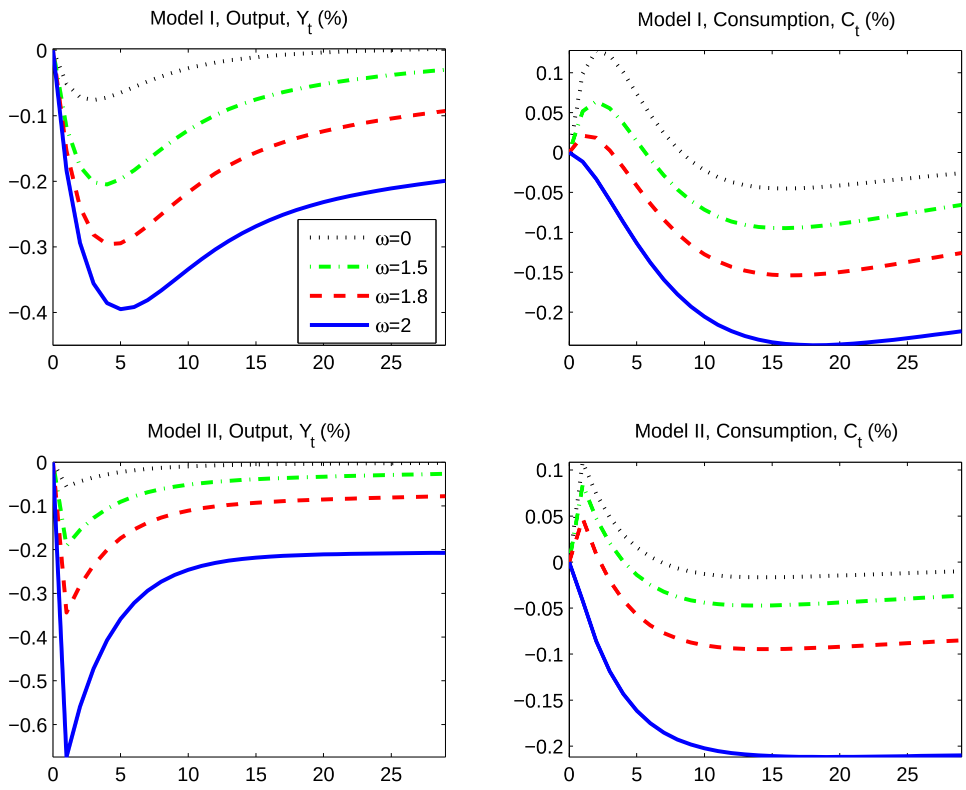

Figure 3: Impulse Responses to Uncertainty Shocks with Various Values of ω

4.4. Business Fluctuations

I have shown that the uncertainty shocks drive business cycles for key variables in both Model-I and Model-II. Still, the result is qualitative rather than quantitative, focusing on a co-movement among key variables. Now I take the uncertainty shocks slightly more seriously and ask the following question: how much volatility can the uncertainty shocks explain for key variables in the U.S.?

In order to answer the question I conduct a typical business cycle stochastic simulation.I assume that the uncertainty shocks, $v_{u,t}$, follow an AR(1) process with its disturbance following a normal distribution with mean zero. I simulate the time series of the disturbances and generate the time series of key variables from a log-linearized model. Using the artificial data with the same size as sample data, I calculate the key statistics in the same manner as I do for the sample data. I repeat this process for one-thousand times and report the average values of the statistics.[16]

A central issue in conducting the simulation lies in the value of the standard deviation of

[16]For some series of the disturbances, threshold p_t^* exceeds unity, which violates condition $p_t^* < 1$. I exclude those series and repeat the process until I obtain one-thousand series of artificial data satisfying $p_t^* < 1$ for all t.

the uncertainty shocks. Because the uncertainty shocks are not observable I should guess a reasonable value for the standard deviation. I set the value in such a way that the simulated data hits the target value of 0.77 percent for the standard deviation of an external finance premium. The value coincides with the premium of sample data, defined by a difference between Moody's BAA corporate bond yield versus the U.S. 10-year Treasury yield.

I use the above procedure to set the standard deviation of the uncertainty shocks for the following reason. According to Gilchrist, Yankov and Zakrajsek (2009) and Gilchrist and Zakrajsek (2010) who analyze various external finance premiums between the U.S. corporate bonds yield and the U.S. Treasury yield of comparable maturity, the average of the standard deviation of the premiums is above 2 percent. The value of 0.77 percent used in the simulation is less than a half of the standard deviation implied by the data analyzed by them. Also, Gilchrist and Zakrajsek (2010) use VARs and show that more than 40 percent of the volatility of excess return on capital, which is closely related with an external finance premium, is explained by financial shocks which turn out to be the same as the uncertainty shocks in Model-I.[17] Albeit this is little better than a guess it is not unrealistic that about 40 percent ($\approx 0.77/2$) of the volatility of the average premium attributes to the uncertainty shocks.

Table 2 shows the result of the stochastic business cycle simulations. The upper left in Table 2 shows standard deviations and cross correlations for sample data from 1987 to 2010, detrended by Hodrick-Prescott filter with smoothing parameter 1600 except for premiums. For comparison I use another premium defined by a difference between high-yield B-rated corporate bonds from the Merrill Lynch's High Yield Master file versus AAA corporate bond yields of comparable maturity, in addition to a premium defined by a difference between BAA corporate bonds yield versus the U.S. 10-year Treasury yield. The upper right and the lower left in Table 2 show the corresponding average statistics calculated from one-thousand artificial data series with the same sample size in Model-I and Model-II respectively.

The simulation result, reported in Table 2, reveals three findings. First, both in Model-I and in Model-II the uncertainty shocks generate the significant fluctuations of output, consumption, investment and hours. In Model-I and in Model-II the uncertainty shocks explain about 20 percent and 30 percent of the volatility of the data respectively for output, investment and hours, while the shocks explain about 10 percent of the volatility for consumption, respectively.

Second, both in Model-I and in Model-II the uncertainty shocks generate a co-movement among output, consumption, investment and hours. For investment and hours both Model-I

[17]Gilchrist and Zakrajsek (2010) uses terminologies, an external finance premium and an excess bond premium (an excess return on capital), interchangeably. For clarification I stick to the following terminologies in this paper. Here, an excess return to capital refers $s_t \equiv E_t R^k_{t+1}/R_{t+1}$, in Model-I, and an external finance premium refers EFP_t defined by equation (41) in Model-I.

Table 2: Cyclical Behavior of the U.S. and the Model Economy

Variable	Data (1987-2010) SD (%)	Data: Cross-Correlation of Output with x(-1)	x	x(+1)	Model I SD (%) [ratio to Data]	Model I: Cross-Correlation of Output with x(-1)	x	x(+1)
Output	1.12	0.87	1.00	0.87	0.24 [0.22]	0.89	1.00	0.89
Consumption	0.80	0.68	0.87	0.85	0.08 [0.10]	0.12	0.38	0.60
Investment	4.67	0.86	0.93	0.80	0.82 [0.18]	0.91	0.97	0.79
Hours	1.79	0.74	0.88	0.90	0.35 [0.19]	0.90	0.99	0.85
Premium, BAA-Treasury	0.77	-0.50	-0.39	-0.20	0.77 [1.00]	-0.65	-0.50	-0.11
Premium, High-AAA	2.17	-0.48	-0.29	-0.03				

Variable	Model II SD (%) [ratio to Data]	Model II: Cross-Correlation of Output with x(-1)	x	x(+1)
Output	0.39 [0.34]	0.55	1.00	0.55
Consumption	0.08 [0.10]	-0.06	0.31	0.55
Investment	1.26 [0.27]	0.58	0.99	0.49
Hours	0.57 [0.32]	0.56	1.00	0.52
Premium	0.77 [1.00]	-0.55	-0.87	-0.44

Note: The data of output, consumption, investment, hours are taken logs and are detrended by Hodrick-Prescott filer with smoothing parameter value 1600. The simulated numbers are the average of the statistics calculated in the same manner as the data for the artificial data generated from the model for 1000 times.

and Model-II succeeds in reproducing high contemporaneous correlation of output. Model-I also succeeds in reproducing high cross correlation of output with investment and hours, while Model-II reproduces smaller cross correlation relative to sample data. The success of Model-I reflects its persistence mechanisms. The CEE adjustment costs generate a hump-shaped response of various variables and make responses persistent. For consumption both Model-I and Model-II reproduce mild contemporaneous correlation of output relative to sample data. While Model-I reproduces positive cross correlations, Model-II fails to reproduce positive cross-correlation of output with lagged consumption. Relatively poor performance on consumption reflects a co-movement problem in a real business framework, though the problem is mitigated by amplification mechanisms embedded in the two models.

Third, the uncertainty shocks succeed in reproducing the predictive power of an external finance premium in Model-I, while it is not the case in Model-II. According to the two data series of premiums, the lagged premium shows a higher correlation with the current output (-0.50 and -0.48 respectively) more than do the current premiums (-0.39 and -0.29 respectively) and the lead premiums (-0.20 and -0.03 respectively). On the one hand, Model-I shows a similar pattern; -0.65, -0.50 and -0.11 for the correlation of output with the lagged, the current and the lead premium respectively, while the degree of correlations is slightly higher relative to sample data. On the other hand, Model-II does not show such a pattern: a premium is highly contemporaneously correlated with output. The difference between Model-I and Model-II has to do with a difference in persistence mechanisms. In

Model-I the response of output is hump-shaped thanks to the CEE investment adjustment costs, while in Model-II the response of output is the same as that of the premium. This observation implies that a hump-shaped response, generated by persistence mechanisms, is crucial for the model to reproduce the predictive power of premiums.

5. Extensions

In the previous section I showed that the uncertainty shocks in Model-I generate a co-movement among variables including the price of capital and the net worth, consistent with the U.S. business cycles. In this section I focus on Model-I and extend the model in two ways.[18]

First, I compare the uncertainty shocks with risk shocks considered by CMR (2010) and clarify a distinction between the two shocks. Second, I introduce unrealized uncertainty shocks about the riskiness of project, which change only ex-ante uncertainty about the riskiness of project but do not change the true distribution of the riskiness of project ex-post. I interpret the unrealized uncertainty shocks as shocks affecting intermediaries' animal spirits or intermediaries' mood (pessimism and optimism). I show analytically that the unrealized uncertainty shocks are more powerful than the uncertainty shocks.

5.1. Comparison with Risk Shocks

The uncertainty shocks in Model-I share similar quantitative implications with risk shocks considered by CMR (2010). However, the two shocks are conceptually different. In the following I explain the risk shocks briefly and provide two examples to shed light on conceptual differences between the two shocks

CMR (2010) builds a DSGE model incorporating BGG (1999) and introduce shocks to the standard deviation of idiosyncratic productivity of entrepreneurs, which they call risk shocks. In their model entrepreneurs own, trade and rent out capital as in Model-I. All entrepreneurs are identical ex-ante. The entrepreneurs make a debt contract with intermediaries, borrow funds and purchase capital. At the beginning of the next period an idiosyncratic shock hits the entrepreneurs and a realized return constitutes private information to the entrepreneurs. The intermediaries observe the return if they use costly monitoring technologies. Under the debt contract an entrepreneur who cannot pay a promised return goes bankrupt. The bankrupt entrepreneur is monitored and taken all assets by an intermediary.

As presented in Appendix, three equations summarize credit frictions in CMR (2010): (i) a balance sheet equation, (35), (ii) an optimality condition relating excess return $s_t \equiv E_t R^k_{t+1}/R_{t+1}$ and the threshold of realized productivity under which entrepreneurs

[18] The extension for Model-II is straight forward and its implications are similar to Model-II.

go bankrupt, and (iii) an intermediary's zero profit condition. Let $u_{r,t}$ denote the risk shocks to the standard deviation of idiosyncratic productivity. Log-linearizing the three equations results in the following relationship:

$$\hat{s}_t = -\chi_{1,r}\left(\hat{N}_t - \hat{q}_t - \hat{K}_{t+1}\right) - \chi_{2,r}E_t u_{r,t+1}. \tag{55}$$

Under conventional parameters values, coefficients satisfy $\chi_{1,r}, \chi_{2,r} > 0$. Given that the risk shocks are persistent, for example $u_{r,t} = \rho_r u_{r,t-1} + \epsilon_{r,t}$ with $0 < \rho_r < 1$, the risk shocks emerge as financial shocks as do the uncertainty shocks. Equation (55) share the same structure with its counterpart of the uncertainty shocks, (45). This observation makes clear that the two shocks have similar quantitative implications.

Now I discuss conceptual distinctions between the uncertainty shocks and the risk shocks. The uncertainty shocks concern the degree of uncertainty about the riskiness of project, while the risk shocks concern the riskiness of project. In other words, the riskiness of project itself does not matter to the uncertainty shocks, while it does matter to the risk shocks. What matters to the uncertainty shocks is the degree of uncertainty (asymmetric information) about the riskiness, not the riskiness itself.

In order to make clear distinctions between the two shocks I provide two examples. As shown in CMR (2010) an increase in risk, caused by the risk shocks, decreases output and vice-versa. Contrary to the risk shocks, the first example shows that an increase in risk, caused by the uncertainty shocks, increases output. The second example shows that a decrease in risk, caused by the uncertainty shocks, decreases output. While those results sound counter-intuitive, the results clarify distinctions between the two shocks.

Instead of the distribution of riskiness, given by (34), suppose that the distribution stays uniform but the uncertainty shock appears in the upper bound of the support:

$$F_t(p) = \frac{p - \underline{p}}{\bar{p}_t - \underline{p}}, \quad \bar{p}_t \equiv \bar{p}e^{-\upsilon_t}, \quad 0 < \underline{p} < \bar{p} < 1. \tag{56}$$

As before the negative uncertainty shock, $\upsilon_t < 0$, increases the degree of uncertainty of riskiness. The difference from the baseline distribution, (34), appears in the riskiness. The negative uncertainty shocks decrease the overall riskiness of project, because an increase in $\bar{p}_t$ with $\underline{p}$ fixed implies that there are more projects with less riskiness. (Remember that the riskiness of project is measured by $1/p$).

Using new distribution (58), I derive a log-linearized equation summarizing the effect of uncertainty shocks, similar to equation (45), as follows:

$$\hat{s}_t = -\chi_1\left(\hat{N}_t - \hat{q}_t - \hat{K}_{t+1}\right) - \chi_3\upsilon_t, \quad \chi_1, \chi_3 > 0, \tag{57}$$

where coefficient χ_1 is the same as in equation (45). For the derivation of equation (57) see Appendix.

As the first example, consider the negative uncertainty shocks, $\upsilon_t < 0$. The negative uncertainty shocks in this case increase $\bar{p}_t$ and decrease the over all riskiness of project. However, equation (57) implies that the negative uncertainty shocks increase an excess return to capital and decrease output as we saw in the previous section. Next as the second example, consider the positive uncertainty shocks, $\upsilon_t > 0$. The positive uncertainty shocks decrease $\bar{p}_t$ and increase the over all riskiness of project. However, equation (57) implies that the positive uncertainty shocks decrease an excess return to capital and increase output. Unlike the risk shocks in CMR (2010), a decrease (increase) in riskiness does not necessarily result in an increase (decrease) in output.

The above two examples make a stark contrast between the uncertainty shocks and the risk shocks. Still, the negative uncertainty shocks have to be accompanied with an increase in riskiness in order to generate counter-cyclical external finance premiums as observed in the sample data.[19] In that sense, the uncertainty shocks and the risk shocks share a similar empirical implication: both the two shocks are in some extent captured by an increase in riskiness. Bloom (2009) and Bloom, et al (2010) document that uncertainty, measured by various second moments, is counter-cyclical and propose a model in which a change in uncertainty drives business cycles. In my context, uncertainty referred by them can be interpreted as riskiness. The models proposed in this paper provide another mechanisms through which a change in riskiness (accompanied by a change in uncertainty about riskiness) drives business cycles.

5.2. Unrealized Uncertainty Shocks

The previous subsection makes clear that the heart of the uncertainty shocks lies in ex-ante uncertainty about the riskiness of project. This observation suggests that shocks to ex-ante uncertainty, not necessarily realized ex-post, can have similar effect to the uncertainty shocks. I call the shocks as unrealized uncertainty shocks, which change only ex-ante uncertainty about the riskiness of project but do not change the true distribution of the riskiness of project ex-post. I interpret the unrealized uncertainty shocks as shocks affecting intermediaries' animal spirits or intermediaries' mood (pessimism and optimism), because a change in ex-ante uncertainty is not realized ex-post. It turns out that the unrealized uncertainty shocks are more powerful than the uncertainty shocks.

I extend Model-I by introducing the unrealized uncertainty shocks. As in Model-I, I assume that the distribution, *perceived* by intermediaries, of riskiness of project is uniform,

[19] In practice, an increase in riskiness, caused by the uncertainty shocks, has to accompany with a decrease in output as in Model-I, because an increase in riskiness is reflected to an increase in an external finance premium. As shown in the data in Table 2, an external finance premium is counter-cyclical. Pro-cyclical external finance premium in the above two examples is at odd with the data.

given by

$$F_t(p) = \frac{p - \underline{p}_t}{1 - \underline{p}_t}, \quad \underline{p}_t = \underline{p} e^{\upsilon_t + \eta_t}, \quad 0 < \underline{p} < 1, \tag{58}$$

where η_t denotes the unrealized uncertainty shocks while υ_t denotes the uncertainty shocks. Intermediaries perceive that the distribution is given by (58) in making a contract with entrepreneurs, but the true distribution turns out to be (58) with $\eta_t = 0$. Then, the intermediaries provide a menu of loan contracts based on distribution (58). Consequently, a threshold above which entrepreneurs do not get funded is given by zero profit condition (39) with $\underline{p}_t$ given by $\underline{p}_t = \underline{p} e^{\upsilon_t + \eta_t}$. The aggregate loan is given by individual loan (17) integrated over net worth and p according to distribution (58) with $\eta_t = 0$:

$$B_t = \left[\frac{1}{2\phi - 1} \frac{(p_t^*)^{\frac{1-\phi}{\phi}}}{1 - \underline{p} e^{\upsilon_t}} \left((p_t^*)^{\frac{2\phi-1}{\phi}} - (\underline{p} e^{\upsilon_t})^{\frac{2\phi-1}{\phi}} \right) - \frac{p_t^* - \underline{p} e^{\upsilon_t}}{1 - \underline{p} e^{\upsilon_t}} \right] N_t, \tag{59}$$

where I have assumed $\phi \neq 1/2$. The aggregate loan, given by equation (59), coincides with the aggregate loan, implied by equation (37) in Model-I, if and only if there is no unrealized uncertainty shocks: $\eta_t = 0$. With the unrealized uncertainty shocks, the zero profit condition does not hold ex-post. I assume that the intermediary's profits are transfered to households in a lump-sum manner.

As in Model-I, I log-linearize the zero profit condition, equation (59) and balance sheet equation (35), and rearrange those equations to obtain the following equation:

$$\hat{s}_t = -\chi_1 \left(\hat{N}_t - \hat{q}_t - \hat{K}_{t+1} \right) - \chi_2 \upsilon_t - \chi_4 \eta_t, \quad 0 < \chi_2 < \chi_4, \tag{60}$$

where coefficients χ_1 and χ_2 are the same as in equation (45) and the expression for coefficient χ_4 is given in Appendix. Equation (60) shows that the unrealized uncertainty shocks, η_t, and the uncertainty shocks, υ_t, affect the economy in the same manner. The negative (unrealized) uncertainty shocks raise an excess return to capital, $\hat{s}_t$, and cause recessions by aggravating the degree of adverse selection in financial markets. More importantly, equation (60) shows that the unrealized uncertainty shocks have greater effect on $\hat{s}_t$ than do the uncertainty shocks.

To understand why the unrealized uncertainty shocks have greater effect than do the uncertainty shocks, consider two economies where one is hit by the uncertainty shocks only, $\upsilon_t < 0$, and the other is hit by the unrealized uncertainty shocks only, $\eta_t = \upsilon_t$. Because the magnitude of the shocks is the same, intermediaries in the two economies provide the same menu of loan contracts, characterized by the same threshold, p_t^*. Given threshold p_t^*, the aggregate loan in the first economy with $\upsilon_t < 0$ is greater than in the second economy with $\upsilon_t = 0$, according to equation (59). The type-p entrepreneurs receive loans for $p = \underline{p}_t$ to p_t^* in the first economy, while they do so for $p = \underline{p} > \underline{p}_t$ to p_t^* in the second economy. Therefore, fewer entrepreneurs receive loans and a drop in the aggregate loan is greater in the second economy than in the first economy.

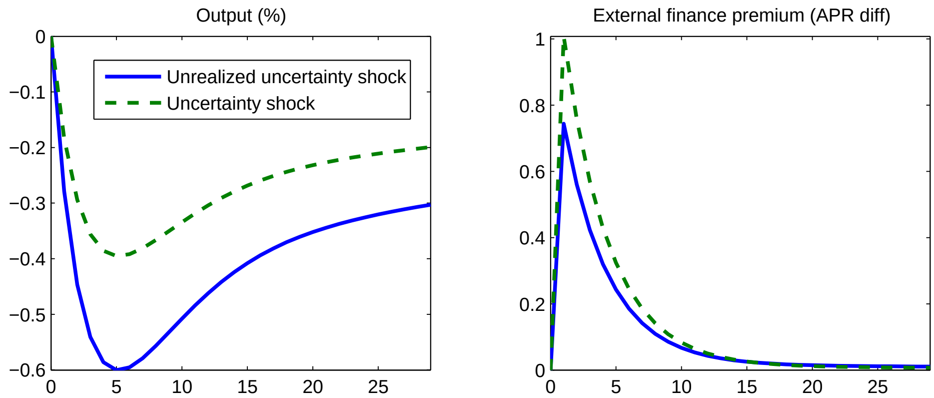

Figure 4: Impulse Responses to Unrealized Uncertainty Shocks (Model I)

To explore a quantitative difference between the unrealized uncertainty shocks and the uncertainty shocks, I plot impulse responses of the two shocks in Figure 4. In plotting the responses I assume that the unrealized uncertainty shocks follow the same stochastic process as the uncertainty shocks and I use the same magnitude of the initial shocks for the two shocks. The left panel in Figure 4 shows that the response of output to the unrealized uncertainty shocks is about 50 percent greater than the response to the uncertainty shocks. The right panel in Figure 4 shows that the response of an external finance premium to the unrealized uncertainty shocks is less than the response to the uncertainty shocks. The result suggests that given a rise in an external finance premium, the unrealized uncertainty shocks generate much larger drop in output. For example, given four percent rise in the premium, the unrealized uncertainty shocks generate 3.2 percent drop in output at bottom while the uncertainty shocks generate only 1.6 percent drop in output. The magnitude of 3.2 percent is slightly smaller than but comparable with a drop in output observed in the great recession in the U.S.

6. Conclusion

In this paper I build a dynamic model in which imperfect financial markets materialize uncertainty shocks. I model imperfect financial markets by introducing asymmetric information on the riskiness of project and an agency problem. Asymmetric information causes adverse selection in financial markets, while an agency problem limits the amount of borrowing. I solve a static optimal contracting problem between intermediaries and entrepreneurs and embed it into dynamic general equilibrium models.

In a dynamic general equilibrium framework I consider the effect of uncertainty shocks which change the degree of uncertainty about the riskiness of project. On the one hand, the uncertainty shocks emerge as financial shocks if I embed credit frictions in the demand side of capital (Model-I). On the other hand, the uncertainty shocks emerge as shocks to the marginal efficiency of investment (MEI shocks) if I embed credit frictions in the supply side of investment (Model-II). The result suggests that the uncertainty shocks and the mechanisms studied here serve as micro-foundations for financial shocks and investment shocks, which have received a lot of attention as a source of business cycles and the causes of a financial crisis (Hall ,2010, Gilchrist, et al, 2009, and Gilchrist and Zakrajsek, 2010 for financial shocks, and Justiniano, et al, 2009a, 2009b for investment shocks).

In a quantitative analysis, I show that the uncertainty shocks generate significant fluctuations consistent with business cycles for standard variables both in Model-I and Model-II. Amplification mechanisms embedded in a counter-cyclical markup in wages and variable capital utilization rates play a crucial role in generating business cycles. A difference between the two models appears in the response of the price of capital and net worth. For those two variables, while the uncertainty shocks generate right co-movement in Model-I, the shocks generate wrong co-movement in Model-II. The result suggests either that the uncertainty shocks in the supply side of investment is not significant or that Model-II needs other mechanisms which solve the co-movement problem of the price of capital and net worth.

In this paper I focused on mechanisms through which shocks to uncertainty in financial markets generate business cycles, and did not discuss policy issues. In order to derive policy implications it is essential to model a counter-cyclical markup in wages in a micro-founded manner. One candidate is to introduce nominal wage rigidities. Introducing nominal rigidities opens a door to discussing both conventional and unconventional monetary policy. My another paper, Ikeda (2011), introduces nominal rigidities and discusses the policy issues.

Appendix

Proof of Proposition 1: The proposition states that there exists a unique symmetric equilibrium if a solution to condition (18) uniquely exists. The proposition consist of an equilibrium part and a uniqueness part. First I show the equilibrium part. The candidate for an equilibrium satisfies (IC1), (IC2) and (PCe) so that entrepreneurs choose their best contract offered by intermediaries. The intermediary's zero profit condition holds so that intermediaries have no incentive to exit the market. What remains to be shown is that there is no profitable deviation from the candidate for an equilibrium in the first stage.

In Proposition 1, it is assumed that there exits a unique solution to condition (18). This implies that an intermediary's profits, $V_n(p)$, are positive for $p < p^*$, and negative for $p > p^*$. To see this, note that distribution $F(p)$ has full support on $[\underline{p}, \bar{p}]$. From the expression for $\omega(p)$, (19), I obtain $\omega(\underline{p}) > 0$. From the expression for $V_n(p)$, (18), an intermediary's profits are positive for p close to $\underline{p}$. The assumption of unique p^* implies that $V_n(p) > 0$ for $p < p^*$ and $V_n(p) < 0$ for $p > p^*$.

Suppose that there exists the i-th intermediary's profitable deviation, $\{X_n^i(p), B_n^i(p)\}$, with threshold p^i, which satisfy (IC1), (IC2) and (PCe). There are three cases: (i) $p^i < p^*$, (ii) $p^i = p^*$ and (iii) $p^i > p^*$. In case (i), Lemma 1 and $p^i < p^*$ implies that $X_n^i(p) < X_n(p)$ for all $p \le p^i$. An entrepreneur with $p \le p^i$ prefers schedule $\{X_n(p), B_n(p)\}$ to $\{X_n^i(p), B_n^i(p)\}$, because the type-p entrepreneurs profits are increasing in $X_n(p')$ for $p' \ge p$, as in (13). Therefore, no entrepreneur would choose $\{X_n^i(p), B_n^i(p)\}$. The deviated schedule results in zero profits and cannot be an profitable deviation.

In case (ii), an only difference between $\{X_n(p), B_n(p)\}$ and $\{X_n^i(p), B_n^i(p)\}$ lies in the inequality in (IC2). For $\{X_n(p), B_n(p)\}$, the inequality holds with equality for $p \le p^*$. For $\{X_n^i(p), B_n^i(p)\}$, there exist some p's such that the inequality does not hold with equality. Without loss of generality I assume that $\{X_n^i(p), B_n^i(p)\}$ satisfies a property that the inequality holds with equality for p close to $\underline{p}$. Because $\omega(p) > 0$ for p close to $\underline{p}$ and an intermediary's profits are given by (18), a deviated schedule which does not satisfy the property earns lower profits than another schedule whose only difference from the former is that it satisfies the property. In addition, without loss of generality, I assume that for a deviated schedule there exists $p' \in (\underline{p}, p^i)$ such that (IC2) holds with equality in an interval between p' and threshold p^i. Otherwise, the argument in (i) applies and no entrepreneur would choose the deviated schedule.

From the above argument, the deviated schedule satisfies that $X_n^i(p) = X_n(p)$ for p in an interval between p' and $p^i = p^*$, and $X_n^i(p) < X_n(p)$ for $p < p'$ (or $p \le p'$). If all intermediaries stayed in the market, the i-th intermediary would succeed in attracting only safe entrepreneurs with p in an interval between p' and p^i, because entrepreneurs with p in the interval choose intermediaries randomly. An intermediary's profits made from the type-p entrepreneur is given by

$$pX_n(p) - R^f B_n(p) = pX_n(p)\left(1 - \frac{R^f}{R^e}\right) - \frac{R^f}{R^e}\int_p^{p^*} X_n(p)dp,$$

where I have substituted out $B_n(p)$ using (14). Since $R^f < R^e$, the profits made from the type-p entrepreneur is positive for p close to p^*. This implies that the i-th intermediary would be able to earn positive profits by choosing p' close to p^* if the other intermediaries stayed in the market. However, after observing $\{X_n^i(p), B_n^i(p)\}$, the other intermediaries best response is to leave the market, otherwise they would end up with negative profits. Therefore, given that the i-th intermediary stays in the market, it faces distribution $F(p)$: it has to make an contract not only with safe entrepreneurs but also with risky entrepreneurs.

I show that the deviated schedule results in negative profits in case (ii). The above argument about $X_n^i(p)$ in case (ii) implies that for any small value $\delta > 0$ there exists an interval with measure δ such that the inequality in (IC2) holds with equality for p lower than the interval. I construct a new schedule $\tilde{X}_n(p)$ whose only difference from $X_n^i(p)$ is that there exists $\tilde{p}$ in the interval such that $\tilde{X}_n(\tilde{p}) - X_n^i(\tilde{p}) = \epsilon$ for small value $\epsilon > 0$, and that the inequality in (IC2) holds with equality for $p < \tilde{p}$. Also, I construct another schedule $\tilde{X}_n^i(p)$ whose only difference from $X_n^i(p)$ is that the inequality in (IC2) holds with equality for $p < \tilde{p}$ in the interval. Then, from (15) I have: for $p \leq \tilde{p}$

$$\tilde{X}_n(p) - \tilde{X}_n^i(p) = \frac{(1-\phi)}{\phi p} \left\{ \int_p^{\tilde{p}} \left[\tilde{X}_n(p) - \tilde{X}_n^i(p) \right] dp + \Delta(\delta) \right\}, \tag{61}$$

where $\Delta(\delta) \equiv \int_{\tilde{p}}^{p^*} [\tilde{X}_n(p) - \tilde{X}_n^i(p)] dp$. Since $\tilde{X}_n(p) = \tilde{X}_n^i(p)$ for p higher than the interval, $\Delta(\delta) \to 0$ as $\delta \to 0$. Solving (61) for $\tilde{X}_n(p) - \tilde{X}_n^i(p)$ with the terminal condition, $\tilde{X}_n(\tilde{p}) - \tilde{X}_n^i(\tilde{p}) = \epsilon$, I obtain[20]

$$\tilde{X}_n(p) = \tilde{X}_n^i(p) + \left(\frac{\tilde{p}}{p}\right)^{\frac{1}{\phi}} \epsilon,$$

for $p \leq \tilde{p}$. Then from the expression for the intermediary's profits, (18), I can express the difference of intermediary's profits as

$$\int_{\underline{p}}^{p^*} \omega(p)[\tilde{X}_n(p) - \tilde{X}_n^i(p)] dp = \int_{\underline{p}}^{\tilde{p}} \omega(p) \left(\frac{\tilde{p}}{p}\right)^{\frac{1}{\phi}} \epsilon dp + \tilde{\Delta}(\delta),$$

where $\tilde{\Delta}(\delta) \equiv \int_{\tilde{p}}^{p^*} \omega(p)[\tilde{X}_n(p) - \tilde{X}_n^i(p)] dp$. Similar to $\Delta(\delta)$, $\tilde{\Delta}(p) \to 0$ as $\delta \to 0$. Because δ is arbitrary, $\tilde{X}_n(p)$ earns more profits than $\tilde{X}_n^i(p)$ if

$$\int_{\underline{p}}^{\tilde{p}} \omega(p)(1/p)^{\frac{1}{\phi}} dp > 0. \tag{62}$$

This has to be the case since $\tilde{p} < p^*$ and the intermediary's profits, $V_n(p)$ given by (18), are positive for $p < p^*$.

Note that $X_n^i(p) - \tilde{X}_n^i(p) \to 0$ as $\delta \to 0$ for all p by construction. This and the above argument imply that new schedule $\tilde{X}_n(p)$ earns more profits than the deviated schedule, $X_n^i(p)$. Compared with $X_n^i(p)$ new schedule $\tilde{X}_n(p)$ has more p's such that constraint (IC2) holds with equality for p. Repeating this argument I reach a conclusion that the candidate for an equilibrium, $X_n(P)$, which satisfies (IC2) with equality for all $p \leq p^*$, earns more profits than the deviated schedule,

[20] Given $\epsilon > 0$ and $\delta > 0$ such that $\epsilon > [(1-\phi)/(\phi\tilde{p})]\Delta(\delta)$, there exists $\tilde{\epsilon}(\delta)$ such that $\epsilon = \tilde{\epsilon}(\delta) + [(1-\phi)/(\phi\tilde{p})]\Delta(\delta)$. As $\delta \to 0$, $\Delta(\delta) \to 0$ and $\tilde{\epsilon}(\delta) \to \epsilon$ so that $\tilde{X}_n(\tilde{p}) - \tilde{X}_n^i(\tilde{p}) = \epsilon$.

$X_n^i(p)$. Since $X_n(p)$ earns zero profits, $X_n^i(p)$ results in negative profits. Therefore, the deviated schedule cannot be a profitable deviation.

In case (iii), suppose that there exists a profitable deviation, $\{X_n^i(p), B_n^i(p)\}$, with $p^i > p^*$. Without loss of generality I assume that constraint (IC2) holds with equality for $p \le p^*$ in $X_n^i(p)$, because it is profitable to do so from the argument in case (ii). Then, the difference of profits between the two schedules is given by

$$\int_{\underline{p}}^{\bar{p}} \omega(p)[X_n(p) - X_n^i(p)]dp = \int_{p^*}^{\bar{p}} \omega(p)[-X_n^i(p)] > 0. \tag{63}$$

In the equality I have used the condition implied by the zero profit condition: $\int_{\underline{p}}^{p^*} \omega(p)\,(1/p)^{\frac{1}{\phi}} dp = 0$. In the strict inequality I have used $\omega(p) < 0$ for $p > p^*$.[21] The inequality in (63) implies that the deviated schedule, $X_n^i(p)$, cannot be a profitable deviation. This completes the proof that the candidate for an equilibrium is actually an equilibrium.

Now I shall show the uniqueness of the equilibrium. As before I assume that the solution to the zero profit condition, (18), uniquely exists. For any schedule $\{\tilde{B}_n(p), \tilde{X}_n(p)\}$ satisfying conditions (IC1), (IC2), (PCe) and (8) yet differing from equilibrium schedule $\{B_n(p), X_n(p)\}$, I can construct the i-th intermediary's profitable deviation. Let $\{B_n^i(p), X_n^i(p)\}$ be such that it satisfies (IC1), (IC2) with equality, (PCe) and such that threshold p^i satisfies $p^* - p^i = \epsilon$ for small value $\epsilon > 0$. Under this deviated schedule the i-th intermediary's profits become positive if the i-th intermediary faces distribution $F(\cdot)$ up to a constant scaling factor. For small enough ϵ the deviated schedule becomes close to the equilibrium schedule and becomes more attractive to entrepreneurs than $\{\tilde{B}_n(p), \tilde{X}_n(p)\}$. Then the deviated schedule succeeds in earning positive profits. This completes the proof of the uniqueness of the equilibrium.

Derivation of Equation (45): I derive equation (45) using equations (37) and (39) in two steps. First, log-linearizing equation (39) I obtain:

$$\hat{p}_t^* = \theta_{p^*,s}\hat{s}_t + \upsilon_t, \tag{64}$$

with

$$\theta_{p^*,s} \equiv \frac{-\frac{\phi}{2\phi-1}\left((p^*)^{\frac{2\phi-1}{\phi}} - \underline{p}^{\frac{2\phi-1}{phi}}\right)}{(1-2/s)(p^*)^{\frac{2\phi-1}{\phi}} + (\underline{p}/s)(p^*)^{-\frac{1-\phi}{\phi}}} > 0,$$

Second, log-linearizing equation (37) I obtain

$$\hat{s}_t = \frac{[1-(1-\phi)s]qK}{B}(\hat{q}_t + \hat{K}_{t+1} - \hat{N}_t) + [1-(1-\phi)s]\left(-\frac{p^*}{p^*-\underline{p}}\hat{p}_t^* + \frac{1-p^*}{1-\underline{p}}\frac{\underline{p}}{p^*-\underline{p}}\upsilon_t\right).$$

Substituting out for $\hat{p}_t^*$ using (64) I obtain:

$$\hat{s}_t = -\chi_1\left(\hat{N}_t - \hat{q}_t - \hat{K}_{t+1}\right) - \chi_2\upsilon_t,$$

[21] Otherwise the zero profit condition would not hold at p^*.

where

$$\chi_1 \equiv \left\{1 + \frac{[1-(1-\phi)s]p^*}{p^* - \underline{p}}\theta_{p^*,s}\right\}^{-1} \frac{[1-(1-\phi)s]qK}{B} > 0, \tag{65}$$

$$\chi_2 = \left\{1 + \frac{[1-(1-\phi)s]p^*}{p^* - \underline{p}}\theta_{p^*,s}\right\}^{-1} \left[\frac{1-(1-\phi)s}{p^* - \underline{p}}\left(p^* - \underline{p}\frac{1-p^*}{1-\underline{p}}\right)\right] > 0. \tag{66}$$

This completes the derivation of equation (45). ■

Derivation of Equation (52): I derive equation (52) using equations (49) and (39) with s_t replaced by $q_t\mu_t$. Note that the two equations are almost the same as those used in deriving equation (45). Then, χ_3 and χ_4 in equation (52) are given by χ_1, given by (65), and χ_2, given by (66), respectively, where s and qK are replaced by q and I respectively. ■

Derivation of Equation (54): Taking into account the functional form of utility function I combine equation (27) and equation (30) and obtain:

$$(1-\alpha)(u_t K_t)^\alpha L_t^{-\alpha} = w_t = \lambda_{w,t}\psi L_t^{1/\nu} C_t.$$

Substituting out for $\lambda_{w,t}$ using equation (25) I obtain:

$$(1-\alpha)(u_t K_t)^\alpha L_t^{-\alpha} = \lambda_w (Y_t/Y)^{-\omega}\psi L_t^{1/\nu} C_t.$$

Substituting out for Y_t using equation (28) I obtain:

$$(1-\alpha)Y^\omega/(\lambda_w\psi) = [(u_t K_t)^\alpha L_t^{1-\alpha}]^{-\omega-1} L_t^{1+1/\nu} C_t.$$

Log-linearizing this equation results in:

$$-(\omega+1)\alpha(\hat{u}_t + \hat{K}_t) + [-(\omega+1)(1-\alpha) + 1 + 1/\nu]\hat{L}_t + \hat{C}_t = 0.$$

Log-linearizing equation (31) gives a relationship between the return to capital and capital utilization rates: $\hat{r}_t^k = \chi\hat{u}_t$. Substituting out for $\hat{r}_t^k$ using the log-linearized equation of equation (29), I express capital utilization rates as:

$$\hat{u}_t = \frac{1-\alpha}{\chi + 1 - \alpha}(-\hat{K}_t + \hat{L}_t).$$

From the above two equations I obtain:

$$\hat{C}_t = \left[(\omega+1)(1-\alpha)\left(1 + \frac{\alpha}{\chi + 1 - \alpha}\right) - (1 + 1/\nu)\right]\hat{L}_t + \frac{(\omega+1)\alpha\chi}{\chi + 1 - \alpha}\hat{K}_t.$$

This completes the derivation of equation (54). ■

Equations Relating to Risk Shocks: Let ω_t denote an idiosyncratic productivity shock, following $\omega_t \sim F_t$, where F_t denotes a c.d.f. with mean unity and standard deviation $\sigma_t = \sigma e^{u_{r,t}}$. Let $\bar{\omega}_t$ denote a threshold under which entrepreneurs go bankrupt. Define $\Gamma_t(\bar{\omega}_t)$ and

$G_t(\bar{\omega}_t)$ as $\Gamma_t(\bar{\omega}_t) \equiv G_t(\bar{\omega}_t) + \bar{\omega}_t[1 - F_t(\bar{\omega}_t)]$ and $G_t(\bar{\omega}_t) \equiv \int_0^{\bar{\omega}_t} \bar{\omega}_t dF_t(\bar{\omega}_t)$ respectively. Except the introduction of the risk shock, the setup and the notation is exactly the same as BGG (1999) and Christiano and Ikeda (2010, Section 6) who provide a simple analysis on BGG (1999). The equations characterizing the demand for capital consists of three equations:

$$
\begin{aligned}
&q_t K_{t+1} = N_t + B_t, \\
&0 = E_t \left[[1 - \Gamma'_{t+1}(\bar{\omega}_{t+1})] s_{t+1} + \frac{\Gamma'_{t+1}(\bar{\omega}_{t+1}) \left\{ [\Gamma_{t+1}(\bar{\omega}_{t+1}) - \mu G_{t+1}(\bar{\omega}_{t+1})] s_{t+1} - 1 \right\}}{\Gamma'_{t+1}(\bar{\omega}_{t+1}) - \mu G'_{t+1}(\bar{\omega}_{t+1})} \right], \\
&0 = [\Gamma_{t+1}(\bar{\omega}_{t+1}) - \mu G_{t+1}(\bar{\omega}_{t+1})] s_{t+1}(N_t + B_t) - B_t,
\end{aligned}
$$

where q_t denotes the price of capital, K_{t+1} denotes the capital stock, N_t denotes the net worth, B_t denotes the loan, and $s_t \equiv E_t R^k_{t+1} / R_{t+1}$ as defined in Model-I, and parameter $0 < \mu < 1$ determines monitoring costs. Readers who are interested in the model's set up and in deriving those three equations may refer BGG (1999, Appendix A) or Christiano and Ikeda (2010, Section 6). I log-linearizing those three equations and substitute out for $\hat{B}_t$ from the first equation using the third equation, and substitute out for $\hat{\bar{\omega}}_{t+1}$ using the second equation. As a result, I obtain equation (55). In practice, log-linearization is done numerically because of the complexity of the equations.

Derivation of Equation (57): As I derived equation (45) I log-linearized two equations (37) and (39) with distribution (34) replaced by (58). With new distribution (58) those two equations become:

$$
\begin{aligned}
q_t K_{t+1} &= \left[1 + \frac{(1-\phi) s_t}{1 - (1-\phi) s_t} \frac{p^*_t - \underline{p}}{\bar{p}_t - \underline{p}_t} \right] N_t, \\
0 &= \frac{\phi}{2\phi - 1} \left(1 - \frac{2}{s_t} \right) \left[(p^*_t)^{\frac{2\phi-1}{\phi}} - \underline{p}^{\frac{2\phi-1}{\phi}} \right] - \frac{\underline{p}}{s_t} \frac{\phi}{1-\phi} \left[(p^*_t)^{-\frac{1-\phi}{\phi}} - \underline{p}^{-\frac{1-\phi}{\phi}} \right].
\end{aligned}
$$

Log-linearizing those two equations I obtain

$$
\begin{aligned}
\hat{s}_t &= \frac{[1 - (1-\phi)s] qK}{B} (\hat{q}_t + \hat{K}_{t+1} - \hat{N}_t) + [1 - (1-\phi)s] \left(-\frac{p^*}{p^* - \underline{p}} \hat{p}^*_t - \frac{\bar{p}}{\bar{p} - \underline{p}} \upsilon_t \right), \\
\hat{p}^*_t &= \theta_{p^*,s} \hat{s}_t,
\end{aligned}
$$

where coefficient $\theta_{p^*,s} > 0$ is the same as in equation (64). Substituting out for $\hat{p}^*_t$ I obtain:

$$
\hat{s}_t = -\chi_1 \left(\hat{N}_t - \hat{q}_t - \hat{K}_{t+1} \right) - \chi_3 \upsilon_t,
$$

where coefficient $\chi_1 > 0$ is given by (65) and coefficient χ_3 is given by

$$
\chi_3 = \left\{ 1 + \frac{[1 - (1-\phi)s] p^*}{p^* - \underline{p}} \theta_{p^*,s} \right\}^{-1} \frac{\bar{p}}{\bar{p} - \underline{p}} > 0.
$$

This completes the derivation of equation (57). ■

Derivation of Equation (60): Log-linearizing equations (35), (59) and (39) with $\underline{p}_t = \underline{p}e^{\upsilon_t+\eta_t}$ I obtain

$$\hat{q}_t + \hat{K}_{t+1} - \hat{N}_t = \theta_{q,p^*}\hat{p}^*_t - \theta_{q,\upsilon}\upsilon_t,$$
$$\hat{p}^*_t = \theta_{p^*,s}\hat{s}_t + \upsilon_t + \eta_t,$$

where

$$\theta_{q,p^*} \equiv \frac{1-\phi}{\phi}\left(\frac{B}{qK} + \frac{N}{qK}\frac{2p^* - \underline{p}}{1-\underline{p}}\right) > 0,$$
$$\theta_{q,\upsilon} \equiv \frac{\underline{p}}{1-\underline{p}}\left[\frac{N}{qK}\frac{1}{\phi}\left(\frac{p^*}{\underline{p}}\right)^{\frac{1-\phi}{\phi}} - 1\right) > 0,$$

and $\theta_{p^*,s}$ is given by (64). Solving those two equations for s_t, I obtain

$$\hat{s}_t = -\frac{1}{\theta_{q,p^*}\theta_{p^*,s}}\left(\hat{N}_t - \hat{q}_t - \hat{K}_{t+1}\right) - \left(1 - \frac{\theta_{q,\upsilon}}{\theta_{q,p^*}}\right)\frac{1}{\theta_{p^*,s}}\upsilon_t - \frac{1}{\theta_{p^*,s}}\eta_t.$$

Because this equation should coincide with equation (45) when $\eta_t = 0$, it must be the case that $1/(\theta_{q,p^*}\theta_{p^*,s}) = \chi_1$ and $(1 - \theta_{q,\upsilon}/\theta_{q,p^*})(1/\theta_{p^*,s}) = \chi_2$. Let $\chi_4 \equiv 1/\theta_{p^*,s}$. Then the above equation implies that $\chi_4 > \chi_2$. ■

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