The consumption-real exchange rate anomaly with extensive margins[☆]

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Abstract

This paper revisits the consumption-real exchange rate anomaly known as the Backus-Smith puzzle. Fundamentally the BS puzzle cannot be tested fully. This is because the "welfare-based" real exchange rate which captures variations in the number of varieties, i.e. extensive margins and which is relevant for the risk sharing is only partially observed. An expansion in extensive margins brings the "empirical-based" real exchange rate in an appreciation via a terms of trade appreciation while domestic consumption rises. In our theoretical model, a realistic BS correlation possibly arises when the elasticity of substitution between domestic and imported goods is high, consistent to the value of micro founded estimations in the literature.

Keywords: *terms of trade, real exchange rate, Backus-Smith puzzle, firm entry*
JEL classification: F12, F41, F43

Perfect international risk sharing with complete asset markets predicts that consumption in one country should rise when the price of consumption in that country becomes relatively cheap. However, this is not observed in reality. Table 1 reports correlations between relative consumptions and real exchange rates (defined as the price of foreign in

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terms of domestic basket) among industrialized countries. They are close to zero or even negative indicating that countries consume more when their price of consumption become relatively expensive. These consumption-real exchange rate anomalies are known as the Backus-Smith (BS) puzzle in the literature (Backus and Smith (1993) and Kollmann (1995)).

Table 1: BS correlations

Country	U.S.	ROW	Country	U.S.	ROW
Austria	-0.11	0.05	Italy	-0.28	-0.52
Belgium/Luxembourg	-0.16	0.50	Japan	0.05	0.25
Canada	-0.52	-0.31	Netherlands	-0.45	-0.20
Denmark	-0.14	-0.10	Portugal	-0.61	-0.77
Finland	-0.30	-0.49	Spain	-0.63	-0.64
France	-0.20	0.43	Sweden	-0.56	-0.40
Germany	-0.51	-0.27	U.K.	-0.51	-0.21
Greece	-0.45	-0.35	U.S.	N/A	-0.71
Ireland	-0.39	0.72	Median	-0.42	-0.27

Source: Corsetti et al. (2008a)

Recent attempts to solve the BS puzzle question the existence of complete asset markets in the first place, a strong assumption on which the puzzle itself relies.[1] Without complete markets a tight link between relative consumption and the real exchange rate is broken. Consumption risk is insured only partially across countries raising the possibility of reproducing a realistic BS correlation in theoretical models.

With such incomplete asset markets, Corsetti et al. (2008a) (henceforth CDL) show how the puzzle is solved due to a wealth effect induced by a low trade elasticity or combination of a high trade elasticity and highly persistent productivity shock. When the trade elasticity is low, excessively supplied domestic goods due to a positive productivity shock must be consumed by domestic agents who can be afforded by a terms of trade

[1] See for example Obstfeld and Rogoff (2000).

appreciation. When output takes a hump-shaped pattern over time due to a persistent shock, anticipation of future income rise raises demand for domestic goods well in excess of supply creating a short-run terms of trade appreciation. In CDL the key is the terms of trade appreciation following a positive productivity shock, which is driven by either mechanism in reproducing a realistic BS correlation. Also building upon incomplete markets, Benigno and Thoenissen (2008) discuss that a standard international real business cycle model which includes non-traded sector could successfully provide a realistic BS correlation through the well known Harrod-Balassa-Samuelson effect.[2]

This paper presents a complementary view of the BS puzzle. We argue that a realistic BS correlation can be reproduced in a theoretical model once we take into account firm entry, i.e. extensive margins. The theoretical model in this paper reflects recent developments in international macroeconomics. Corsetti et al. (2007) and Ghironi and Melitz (2005) provide a two-country general equilibrium model in which the number of firms is endogenously determined. In the first part of the paper we built a very simple static general equilibrium model in the spirt of Corsetti et al. (2007). The mechanism which provides a realistic BS correlation is shown analytically in this part. In the second part of the paper dynamics are introduced following Ghironi and Melitz (2005) with some extensions. The purpose of the second part is to reproduce a realistic BS correlation quantitatively using the standard calibration method in the international real business cycle literature.

On top of the extensive margins, there are several arguments which found our results. On the one hand, the key is the interaction between market incompleteness and the role played by extensive margins. On the other hand, they are based on the observation that statistically relevant or empirically constructed price indices do not or only imperfectly reflect fluctuations in extensive margins.

Recent empirical studies in the international trade literature emphasize the discrepancy between such "empirical" and "welfare-based" price indices in short and even in

[2]Recently Kollmann (2009) and Devereux et al. (2009) argue a resolution of the puzzle relining on another mechanism, hand-to-mouth behavior of subset households.

long run. For instance, Broda and Weinstein (2004, 2006) point out that large welfare gains are stemming from the increased number of imported varieties and note equally that empirical-based import price indices have an inflation bias. They report a 1.2% upward bias per year for the U.S import price index. In Broda and Weinstein (2010) they also report that in the U.S. an upward bias in CPI is around 0.8% per year from 1994 to 2003. Such discrepancy is important in considering the BS puzzle as well. This is because international risk sharing takes place on welfare basis not on empirical basis. Our main argument is that fundamentally the BS correlation cannot be tested fully unless we know the exact variations in extensive margins which are relevant to welfare.

The intuition of the paper can be summarized as follows: in our model with extensive margins, home bias in consumption and monopolistic competition, a positive productivity shock leads to entry of new varieties. Because of love for variety, this results in a higher demand for them. The increase in demand leads to an appreciation of the terms of trade and the "empirical-based" real exchange rate. Concurrently relative consumption rises providing a realistic zero or even negative BS correlation. In this instance our result is similar to CDL. Although it is generated by extensive margins, the key is the terms of trade appreciation providing a positive wealth effect.

Systematically, a realistic BS correlation might not be the case in "welfare-based" measure. The reason is that a rise in local extensive margins consumed with home bias might be translated in a depreciation in welfare-based price index. Our model predicts that a positive BS correlation could rest on welfare base.

In generating a realistic BS correlation with extensive margins, we claim the importance of a relatively high elasticity of substitution between local and imported goods. Intuitively the higher the elasticity for a given preference for variety, the stronger the wage appreciation due to extensive margins becomes. Then it becomes more plausible that we observe a realistic BS correlation. This point is shown quantitatively as well as analytically. In the second part of the paper we show that, under a mild love for variety, the value of elasticity around 7, which is in the range of micro founded estimations, can quantitatively provide a realistic BS correlation.

We also explore the role played by market incompleteness to obtain our results. Even under complete markets, the introduction of extensive margins is able to reproduce a realistic BS correlation. However, such a realistic BS correlation requires unrealistic dynamics, in particular for relative consumption. Due to perfect risk sharing, it is not usually optimal when a positive productivity shock occurs to experience a terms of trade appreciation. When a realistic BS correlation is reproduced under complete markets, this is only when foreign consumption rises relative to domestic consumption due to a strong positive transmission via a terms of trade depreciation following a positive productivity shock. We show that only the combination of market incompleteness and extensive margins can reproduce a realistic BS correlation in a plausible way. Therefore, the main focus in this paper builds upon incomplete asset markets while we place detailed discussion about complete markets in the appendix.

The remaining paper is organized as follows. In the next section a static general equilibrium model with endogenous entry is presented. The implication for the BS puzzle is discussed analytically in the following two sections. In section 4 dynamics are introduced. The BS puzzle is explored quantitatively using the standard calibration method in the literature in section 5. The final section offers brief concluding remarks.

1. The model

We build a simple static general equilibrium model. There are two countries, "Home" and "Foreign". Each of which is populated by a unit mass of atomic households. Foreign variables are denoted with asterisks. The number of firms in each country is endogenously determined. Each firm is assumed to represent one product variety. In this static model, there is no international borrowing and lending, trade is balanced. The main objective here is to analytically show how extensive margins and market incompleteness interact and might provide a realistic BS correlation.

1.1. Households

The Home representative household supplies inelastically one unit of labor. Her utility U is specified as

$$U = \frac{C^{1-\gamma}}{1-\gamma}, \tag{1}$$

where $\gamma\,(\geq 1)$ denotes relative risk aversion. C represents her consumption which is defined as

$$C = \left[\alpha^{\frac{1}{\omega}} C_H^{1-\frac{1}{\omega}} + (1-\alpha)^{\frac{1}{\omega}} C_F^{1-\frac{1}{\omega}}\right]^{\frac{1}{1-\frac{1}{\omega}}}, \tag{2}$$

where α $(\geq 1/2)$ captures home bias in consumption. The parameter ω (> 0) is the elasticity of substitution between locally produced (C_H) and imported goods from Foreign (C_F). Each goods are composed by subset of N and N^* number of varieties respectively as

$$C_H = V_H \left[\int_0^N c_t\,(h)^{1-\frac{1}{\sigma}}\, dh\right]^{\frac{1}{1-\frac{1}{\sigma}}}, \qquad C_F = V_F^* \left[\int_0^{N^*} c\,(f)^{1-\frac{1}{\sigma}}\, df\right]^{\frac{1}{1-\frac{1}{\sigma}}}, \tag{3}$$

where $V_H \equiv N^{\psi-\frac{1}{\sigma-1}}$ and $V_F^* \equiv N^{*\psi-\frac{1}{\sigma-1}}$. The parameter σ (> 1) denotes the elasticity of substitution among varieties. We assume $\sigma \geq \omega$. $c\,(h)$ $(c\,(f))$ is the demand for individual Home (Foreign) variety indexed by $h \in [0, N]$ $(f \in [0, N^*])$. The parameter ψ (≥ 0) represents the marginal utility stemming from one additional increase in the number of varieties. This specification follows Benassy (1996). In such a way firms' markup become distinct from love for variety. Specifically, the preference is Dixit and Stiglitz (1977) when $\psi = \frac{1}{\sigma-1}$. When $\psi = 0$, there is no utility gains in consuming a higher number of varieties.

The consumer price index P which minimizes spending on consumption basket C is found as

$$P = \left[\alpha P_H^{1-\omega} + (1-\alpha)\, P_F^{1-\omega}\right]^{\frac{1}{1-\omega}}. \tag{4}$$

In the above expression P_H and P_F denote the price of C_H and C_F respectively and each of which is an index as well as follow

$$P_H = \frac{1}{V_H}\left[\int_0^N p(h)^{1-\sigma}\, dh\right]^{\frac{1}{1-\sigma}}, \qquad P_F = \frac{1}{V_F}\left[\int_0^{N^*} p(f)^{1-\sigma}\, df\right]^{\frac{1}{1-\sigma}}. \tag{5}$$

where $p(h)$ $(p(f))$ represents the domestic price of variety h produced in Home (Foreign). Observe that these price indices are defined in "welfare-based": they decrease (increase) with a rise (decrease) in the number of varieties. Specifically, deflation (inflation) is large when the love for variety ψ is high for a given change in extensive margins.

Finally, the optimal consumption for each basket are found as below:

$$C_H = \alpha\left(\frac{P_H}{P}\right)^{-\omega} C, \qquad C_F = (1-\alpha)\left(\frac{P_F}{P}\right)^{-\omega} C, \tag{6}$$

$$c(h) = V_H^{\sigma-1}\left(\frac{p(h)}{P_H}\right)^{-\sigma} C_H, \qquad c(f) = V_F^{\sigma-1}\left(\frac{p(f)}{P_F}\right)^{-\sigma} C_F. \tag{7}$$

We choose the welfare-based consumer price index P as a numéraire and define real prices as $\rho_H = \frac{P_H}{P}$, $\rho_F = \frac{P_F}{P}$, $\rho(h) = \frac{p(h)}{P}$, and $\rho(f) = \frac{p(f)}{P}$.

Similar expressions hold in Foreign.

1.2. Firms

Each firm representing one product variety competes monopolistically with others. It is assumed that upon entry a firm h must pay sunk entry costs in terms of f_E units of effective labor. The latter is defined as

$$f_E = z_E l_{EM}(h), \tag{8}$$

where z_E denotes a labor productivity on firm setting up efficiency. $l_{EM}(h)$ represents the amount of labor demanded for firm creation. With the above expression a rise in f_E is interpreted as an increasing regulation on firm entry.

After entry the firm produces output $y(h)$ by the following technology

$$y(h) = zl(h), \tag{9}$$

where z denotes a labor productivity in production which is symmetric across firms. $l(h)$ is the labor demand for the production of output $y(h)$.

We now specify the firm's pricing behavior in below. Operational real profits (dividends) of the firm are expressed by

$$d(h) = \left(\rho(h) - \frac{w}{z}\right) y(h), \tag{10}$$

where w denotes real wages. Goods market clearing condition implies that $y(h) = c(h) + c^*(h)$. Thus using the optimal demands addressed to the firm found in the previous section, $y(h)$ can be rewritten as

$$y(h) = N^{\psi(\sigma-1)-1} \rho(h)^{-\sigma} \rho_H^{\sigma-\omega} \left[\alpha C + (1-\alpha) Q^{\omega} C^*\right]. \tag{11}$$

where Q denotes the real exchange rate defined as $Q \equiv \frac{P^*}{P}$. Knowing the demand, the profit maximization behavior by the firm results in the following standard pricing:

$$\rho(h) = \frac{\sigma}{\sigma-1}\frac{w}{z}. \tag{12}$$

Real price $\rho(h)$ is set to be equal to real marginal costs over markup. We denote the price of exported goods by $\rho^*(h) = Q^{-1}\rho(h)$, denominated in Foreign consumption basket.

Without any heterogeneity across firms, they are symmetric in equilibrium. We denote the price which holds in such a symmetric equilibrium as $\rho_h \equiv \rho(h)$. The same type of notation holds for other variables.

Finally using the optimal pricing (12) and the fact that $\rho_H = N^{-\psi}\rho_h$ from (5) and symmetry, real dividends can be rewritten as

$$d_h = \frac{1}{\sigma}\rho_h^{1-\omega} N^{\psi(\omega-1)-1} \left[\alpha C + (1-\alpha) Q^{\omega} C^*\right]. \tag{13}$$

The above expression highlights some important aspects. First, it shows how dividends change along the price. The lower the price of individual variety (ρ_h), the higher the increase in dividends realizes when the elasticity of substitution between local and imported goods is higher than unity ($\omega > 1$). Second, in the above expression $N^{\psi(\omega-1)-1}$ captures an additional competing effect arising from domestic firms (N). When $\psi > \frac{1}{\omega-1}$, in the presence of a strong preference for the variety, an increase in N induces further dividends while it decreases with a relatively weak love for variety as $\psi < \frac{1}{\omega-1}$. In particular when $\omega = \sigma$ and $\psi = \frac{1}{\sigma-1}$, this term disappears.

1.3. Free entry, labor market clearing and balanced trade

In this subsection we fully characterize the general equilibrium by considering free entry, labor market clearing and the balanced traded conditions.

We start with the free entry condition. All dividends eared by each firm are assumed to finance her entry costs.[3] Thus each firm's dividends must be equal to her entry costs in equilibrium:

$$d_h = \frac{f_E w}{z_E}. \tag{14}$$

Next in the labor market one unit of total labor supplied is demanded in goods production and firm creation by N number of firms in equilibrium: $1 = Nl_h + Nl_{EM,h}$. Note that we have $y_h = (\sigma - 1)\frac{d_h}{w}z$ by (10) and (12) and $l_{EM} = \frac{d_h}{w}$ by (8) and (14). Using these expressions the above labor market clearing condition can be rewritten as[4]

$$1 = \sigma \frac{Nd_h}{w}. \tag{15}$$

Similar expressions hold in Foreign.

Finally the balanced trade implies that the value exported is equal to that imported: $N\rho_h c_h^* = QN^*\rho_f^* c_f$. Rewriting with the optimal demands found in the previous section, the balanced trade condition becomes

$$Q^{2\omega-1}N^{\psi(\omega-1)}\rho_h^{1-\omega}C^* = N^{*\psi(\omega-1)}\rho_f^{*1-\omega}C. \tag{16}$$

[3] In this static version of model there is no investment choice by households, i.e. no arbitrage between current and future consumption by investment. This is a distinct feature from a full dynamic model that we will discuss in the following section.

[4] It is easily shown that the labor market clearing condition is basically identical to the aggregated identity which can be obtained from aggregating budget constraints among households: $C = w$.

The equilibrium has characterized by these equations. Because of the non-linearity of the model, in what follows we solve its log-linearized version in order to explore the role played by extensive margins in the BS puzzle.

2. Extensive margins, relative wages and the terms of trade

We let express percentage deviations from the steady state level with sans-serif fonts. Relative wages, relative number of varieties in log-deviations are defined as $\mathsf{w}^R \equiv \mathsf{w} - (\mathsf{Q} + \mathsf{w}^*)$ and $\mathsf{N}^R \equiv \mathsf{N} - \mathsf{N}^*$. By log-linearizing, the model is reduced to the system of two equations, labor market clearing and free entry conditions, and two unknowns, w^R and N^R. We assume no regulation shock as $\mathsf{f}_E = \mathsf{f}_E^* = 0$ without loss of generality. By solving the system, w^R and N^R are expressed with relative exogenous shocks in log-deviations as $\mathsf{z}^R \equiv \mathsf{z} - \mathsf{z}^*$ and $\mathsf{z}_E^R \equiv \mathsf{z}_E - \mathsf{z}_E^*$:[5]

$$\mathsf{N}^R = \mathsf{z}_E^R. \tag{20}$$

$$\mathsf{w}^R = \frac{2\alpha(\omega-1)}{1+2\alpha(\omega-1)}\mathsf{z}^R + \psi\frac{2\alpha(\omega-1)}{1+2\alpha(\omega-1)}\mathsf{z}_E^R, \tag{21}$$

The number of varieties N^R changes one for one with a labor productivity shock on firm setting up efficiency z_E^R while it remains unchanged with a productivity shock on

[5] The log-linearized version of labor market clearing condition is

$$\mathsf{w}^R = \mathsf{N}^R + \mathsf{d}^R. \tag{17}$$

Free entry condition is expressed in log-deviations by

$$\mathsf{d}^R = \mathsf{w}^R - \mathsf{z}_E^R. \tag{18}$$

And using the log-linearized balanced trade condition, we can write log-linearized dividends as

$$\mathsf{d}^R = -2\alpha(\omega-1)\left(\mathsf{w}^R - \mathsf{z}^R\right) + \left[2\alpha\psi(\omega-1) - 1\right]\mathsf{N}^R. \tag{19}$$

Plugging this expression in the above two equations, the system becomes such that with two equations and two unknowns.

marginal costs z^R.

Contrary, relative wages w^R change with both shocks, z^R and z_E^R. A z_E^R shock, thus induced extensive margins, has impact on them because of love for variety ($\psi > 0$).

Whether wages appreciate or depreciate depends on the elasticity of substitution between local and imported goods ω and home bias α. Specifically, it shows a non-linearity in terms of ω provided a size of home bias α. Under $\mathsf{z}^R > 0$ and $\mathsf{z}_E^R > 0$, wages appreciate for Home when $0 < \omega < 1 - \frac{1}{2\alpha}$ while depreciate when $1 - \frac{1}{2\alpha} < \omega < 1$ and again appreciate when $1 < \omega$. What is important to observe is the fact that extensive margins add a further wage appreciation when the elasticity is high ($\omega > 1$) under love for variety ($\psi > 0$).[6]

We define the terms of trade as $TOT \equiv \frac{p_f^*}{p_h}$, the relative price of Foreign goods in terms of Home goods. Their first order variations are defined as $\mathsf{TOT} = \mathsf{Q} + \rho_f^* - \rho_h$. Provided the above expression of relative wages, TOT is expressed as

$$\mathsf{TOT} = \frac{1}{1 + 2\alpha(\omega - 1)}\mathsf{z}^R - \psi\frac{2\alpha(\omega - 1)}{1 + 2\alpha(\omega - 1)}\mathsf{z}_E^R. \tag{23}$$

The first term is the same one discussed in CDL with an endowment economy. CDL argue a possibility of the terms of trade appreciation ($\mathsf{TOT} < 0$) following a positive productivity shock on marginal costs ($\mathsf{z}^R > 0$). The terms of trade appreciate when the elasticity of substitution is very low ranging $0 < \omega < 1 - \frac{1}{2\alpha}$. With a relatively high elasticity, $\omega > 1 - \frac{1}{2\alpha}$, the terms of trade depreciate from the first term.[7]

[6] There is a restriction on parameters' values so that the number of varieties in one country does not increase infinitely, i.e. full agglomeration in one country. This kind of thing happens when dividends increase in one country more proportionally than the cost (wage) appreciation following an increase in the number of firms. To avoid it must be

$$1 > \psi\frac{4\alpha^2(\omega - 1)^2}{1 + 2\alpha(\omega - 1)}. \tag{22}$$

The above condition may not be respected under a very high love for variety ψ and elasticity of substitution ω. Figure 1 and Figure 2 satisfy this parameter restriction.

[7] In addition to the low trade elasticity, CDL analytically show that a short-run terms of trade appreciation hence a negative BS correlation takes place when a persistence of productivity shock and the

Different from CDL, however, under love for variety ($\psi > 0$) and a sufficiently high elasticity of substitution ($\omega > 1$), the second term prevents the terms of trade from depreciating when $\mathsf{z}_E^R > 0$. The higher the love for variety and the elasticity of substitution, the higher the appreciation from the second term becomes. Intuitively when the love for variety and the elasticity of substitution are high, a given number of varieties is further demanded and makes wages further appreciated. This cost appreciation works to counteract the first term depreciation due to more efficient production technology ($\mathsf{z}^R > 0$). The mechanism is in essence exactly the point discussed in Krugman (1989) where he argues how it is possible for a country to prevent from a terms of trade depreciation by providing higher extensive margins along her economic growth.[8]

In Figure 1 we provide a numerical example about percentage deviations in the terms of trade for different values of ω. The figure produces both with and without love for variety cases ($\psi = 0.2$ and $\psi = 0$). In calibrating, the two shocks, z^R and z_E^R, are supposed to be perfectly correlated as $\mathsf{z}^R = \mathsf{z}_E^R$. Home bias in consumption α is set to 0.72. The figure well captures how a high elasticity of substitution might contribute to a terms of trade appreciation under love for variety.

Based on the result found here we next discuss the implication for the BS puzzle.

3. The BS puzzle with extensive margins

Using the balanced trade condition, we can find the first-order relationship between the real exchange rate and relative consumption in our model. This is

elasticity of substitution are high in a bond economy. Such a short-run appreciation due to a high persistence of productivity shock and a high elasticity is also true in our model but it provides a mitigate result on the behavior of the terms of trade with extensive margins. As we will see in a full DSGE model, what is typically hump-shaped in the model are extensive margins. When the persistence of shock rises, the current wealth induced by extensive margins appears only gradually over time reducing the possibility of an impact terms of trade appreciation.

[8]For example, Hummels and Klenow (2005) and Galstyan and Lane (2008) document the empirical validity of the terms of trade appreciation due to higher extensive margins.

$$\mathsf{Q} = \frac{2\alpha - 1}{2\alpha\omega - 1} \left(\mathsf{C} - \mathsf{C}^* \right). \tag{24}$$

The above expression is exactly identical to the one found in CDL. Under the balanced trade (incomplete markets), it is difficult to reproduce a realistic BS correlation for a broad range of the elasticity of substitution. Households in Home consume more when their basket becomes relatively cheap under a sufficiently high elasticity, $\omega > \frac{1}{2\alpha}$. With home bias in consumption ($\alpha > 1/2$), this is the case roughly with $\omega > 1$. A way to generate a negative BS correlation is to suppose a very low elasticity as CDL.[9]

The above relationship holds at the "welfare-based" real exchange rate and consumption which fully capture variations in extensive margins. However, the BS puzzle is the puzzle about the empirically observed real exchange rate and consumption. In contrast to the welfare-based, such empirical-based real exchange rate does not contain (or poorly measures) variations in extensive margins. Let we suppose that the price indices do not reflect at all such variations for simplicity. Denoting the variables without fluctuations in extensive margins with$\widetilde{\ }$, the welfare based-real exchange rate is broken in two parts:

$$\mathsf{Q} = \widetilde{\mathsf{Q}} + \psi \left(2\alpha - 1 \right) \mathsf{N}^R. \tag{25}$$

where

$$\widetilde{\mathsf{Q}} = \left(2\alpha - 1 \right) \mathsf{TOT}. \tag{26}$$

Observe in welfare basis the real exchange rate depreciates for relatively higher extensive margins ($\mathsf{N}^R > 0$) under home bias ($\alpha > 1/2$) and love for variety ($\psi > 0$).

Consumption is also poorly measured using such price indices. For instance, total nominal consumption spending is given by PC. Statistical agencies divide this amount by $\widetilde{P}$ to measure "consumption" ($\widetilde{C}$). As a result, fluctuations in empirical-based consumption are given by

[9] It is worth noting that when $\alpha = 1/2$ and $\omega = 1$ it is possible to achieve the complete markets allocation without any financial assets. This is the case discussed in Cole and Obstfeld (1991). The movement of the terms of trade transmits the productivity gains so that the relative consumption (empirical as well as welfare-based) remains unchanged.

$$\widetilde{\mathsf{C}} \equiv \mathsf{P} + \mathsf{C} - \widetilde{\mathsf{P}} = \mathsf{C} - \psi \left[\alpha \mathsf{N} + (1-\alpha)\,\mathsf{N}^*\right]. \tag{27}$$

Provided the above definition, the welfare-based relative consumption is also broken in two parts as

$$\mathsf{C} - \mathsf{C}^* = \widetilde{\mathsf{C}} - \widetilde{\mathsf{C}}^* + \psi\,(2\alpha - 1)\,\mathsf{N}^R. \tag{28}$$

Plugging (25) and (28) in (24), we finally find the BS correlation with extensive margins as follows

$$\widetilde{\mathsf{Q}} = \frac{2\alpha - 1}{2\alpha\omega - 1}\left(\widetilde{\mathsf{C}} - \widetilde{\mathsf{C}}^*\right) - \psi \frac{2\alpha\,(2\alpha-1)\,(\omega-1)}{2\alpha\omega - 1}\mathsf{N}^R. \tag{29}$$

The tight relationship which holds together the welfare-based real exchange rate and consumptions is now broken. And it is possible to have a negative BS correlation between $\widetilde{\mathsf{Q}}$ and $\widetilde{\mathsf{C}} - \widetilde{\mathsf{C}}^*$ without relying on a low elasticity.

We see the above point more in detail. Supposing a perfect correlation between two types of shocks as $\mathsf{z}^R = \mathsf{z}_E^R$, we can rewrite (29) as

$$\widetilde{\mathsf{Q}} = \frac{(2\alpha-1)\left[1 - 2\psi\alpha\,(\omega-1)\right]}{2\alpha\,(\omega-1)\left[1 + 2\psi\,(1-\alpha)\right] + 2\alpha - 1}\left(\widetilde{\mathsf{C}} - \widetilde{\mathsf{C}}^*\right). \tag{30}$$

In the above expression not only for a low range of elasticity of substitution ($0 < \omega < 1 - \frac{2\alpha-1}{2\alpha[1+2\psi(1-\alpha)]}$), but also for a high range of elasticity ($1 + \frac{1}{2\psi\alpha} < \omega$) the BS correlation becomes negative.

The reason why a negative BS correlation appears for a low range of elasticity is almost the same as discussed in CDL: when Home is hit by a positive productivity shock, Home provides more goods with intensive as well as extensive margins. However, with a very low elasticity of substitution the goods market clearing requires an appreciation of the terms of trade (expensive Home goods), therefore providing a positive wealth effect for Home agents in order to absorb their own production with home bias.

Following a positive shock, extensive margins appear in Home ($\mathsf{N}^R > 0$). When the elasticity of substitution is high as $1 + \frac{1}{2\psi\alpha} < \omega$, because of the demand addressed to these new varieties, wages appreciate further. As we have seen in the previous section, the higher the elasticity of substitution, the stronger such a wage appreciation is. Counteracting the depreciation due to a higher productivity on marginal costs, the terms of trade tend to be appreciated with extensive margins which brings also the observed real exchange rate $\widetilde{\mathsf{Q}}$ into an appreciation. Concurrently, relative consumption $\widetilde{\mathsf{C}} - \widetilde{\mathsf{C}}^{*}$ rise providing a realistic BS correlation.

Note that without love for variety ($\psi = 0$) the expression collapses to (24) which is again identical to CDL. In Figure 2 we give a numerical example with the same parameters as in Figure 1. It is shown that a high elasticity of substitution can contribute to the resolution of the puzzle under love for variety.

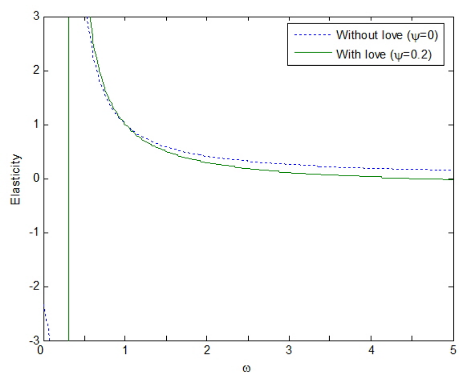

Figure 1: the terms of trade

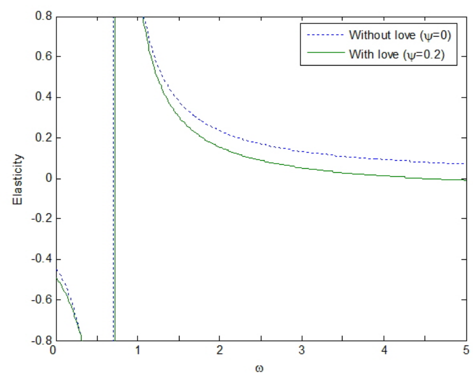

Figure 2: the BS correlation

4. Quantitative investigation

In what follows, we investigate whether the intuition described analytically in the previous sections quantitatively holds. For that purpose, we construct a two-country DSGE model in which the number of firms is endogenously determined and the international borrowing and lending is allowed using non-contingent bonds. Our model is considered as a simplified version of Ghironi and Melitz (2005) without firm heterogeneity.

All variables now have time index t. A productivity shock on marginal costs of production z_t and firm creation $z_{E,t}$ are assumed to be perfectly correlated for the sake of simplicity. Investment takes place in terms of new firm creation whose number is represented by $N_{E,t}$. Other than the investment dynamics we add three more realistic extensions compared to the static model: endogenous labor supply, internationally held non-contingent bonds and entry cost paid in terms of capital goods as well as labor. Only these modified points are discussed below.

4.1. Households

The Home representative household maximizes $E_t \sum_{s=t}^{\infty} \beta^{s-t} U_t$ at time period t. Her utility now depends on labor supply as well as consumption as follows:

$$U_t = \frac{C_t^{1-\gamma}}{1-\gamma} - \chi \frac{L_t^{1+\frac{1}{\varphi}}}{1+\frac{1}{\varphi}}. \tag{31}$$

The parameter χ (> 0) represents the degree of non satisfaction stemming from supplying labor service L_t. φ is the Frisch elasticity of labor supply.[10] With this specification the marginal disutility in providing one additional labor service is increasing.

4.2. Budget constraint with non-contingent bonds

In this dynamic version of the model, the budget constraint is given by

$$\begin{aligned} B_{t+1} + Q_t B_{*,t+1} + \frac{\vartheta}{2} B_{t+1}^2 + \frac{\vartheta}{2} Q_t B_{*,t+1}^2 + s_{h,t+1} \left(N_t + N_{E,t}\right) x_{h,t} + C_t \\ = \left(1 + r_t\right) B_t + Q_t \left(1 + r_t^*\right) B_{*,t} + s_{h,t} N_t \left(d_{h,t} + x_{h,t}\right) + T_t^f + w_t L_t, \end{aligned} \tag{32}$$

where B_{t+1} ($B_{*,t+1}$) is real holdings of Home (Foreign) bonds into $t+1$. To ensure zero bond holdings at the steady state, quadratic adjusting costs of bond holdings ϑ are introduced. T_t^f is the free rebate of adjusting costs which is exogenous for households. r_t (r_t^*) denotes real interest rate. It is assumed that investment takes place only domestically by purchasing mutual funds among domestic existing firms N_t and new entrants $N_{E,t}$. $s_{h,t+1}$ denotes real holdings of shares into $t+1$. $x_{h,t}$ represents real share price of Home mutual fund.

[10] With $\varphi = \infty$ the marginal disutility in supplying one additional unit of labor becomes constant, χ. When $\varphi = 0$ the marginal disutility becomes infinite and the labor supply becomes inelastic.

Euler equations for share and bond holdings are given by respectively

$$x_{h,t} = \beta (1-\delta) E_t \left(\frac{C_{t+1}}{C_t} \right)^{-\gamma} (x_{h,t+1} + d_{h,t+1}), \tag{33}$$

$$C_t^{-\gamma} (1 + \vartheta B_{t+1}) = \beta (1 + r_{t+1}) E_t C_{t+1}^{-\gamma}, \tag{34}$$

and

$$C_t^{-\gamma} (1 + \vartheta B_{*,t+1}) = \beta \left(1 + r_{t+1}^*\right) E_t \frac{Q_{t+1}}{Q_t} C_{t+1}^{-\gamma}.$$

Finally, the optimal labor supply decision gives

$$\chi (L_t)^{\frac{1}{\psi}} = w_t C_t^{-\gamma}. \tag{35}$$

Similar conditions hold in Foreign.

4.3. Firms

4.3.1. Entry

Different from the static model and original specification in Ghironi and Melitz (2005), we assume that new entrants need capital goods as well as labor in order to set up their production site. This type of specification is proposed in Bilbiie et al. (2007a) as a realistic extension. One firm/variety creation is assumed to need an amount of firms setting up goods f_E. The production of such goods is supposed to be done by the following Cobb-Douglas technology using capital goods K_t as well as labor $l_{EM,t}$ as inputs:

$$f_E = \left(\frac{z_t l_{EM,t}}{\theta} \right)^{\theta} \left(\frac{K_t}{1-\theta} \right)^{1-\theta}, \tag{36}$$

where θ $(1-\theta)$ is the share of labor (capital) in total costs. For simplicity we assume that capital goods K_t has the same composition as consumption goods C_t. The cost minimization problem by firms yields following factor demands

$$l_{EM,t} = \frac{\theta}{w_t} \mu_t f_E, \qquad K_t = (1-\theta) \mu_t f_E, \tag{37}$$

where $\mu_t = \left(\frac{w_t}{z_t}\right)^{\theta}$ is the real cost of firm creation. In particular when $\theta = 1$ only labor is used for entry.

It is assumed that production takes place only one period after the entry. The motion of firms is defined as

$$N_{t+1} = (1 - \delta)(N_t + N_{E,t}), \tag{38}$$

where δ denotes the "death shock" which takes place at the very end of each period after investment has been completed. As a result, a fraction of new entrants die without producing.

Similar conditions hold in Foreign.

4.3.2. Production

The production of intensive margins is almost identical to the previous static model. However, because capital goods are required for entry, the demand addressed for each firm now includes these terms:

$$y_{h,t} = c_{h,t} + c^*_{h,t} + N_{E,t} k_{h,t} + N^*_{E,t} k^*_{h,t}, \tag{39}$$

where $k_{h,t}$ ($k^*_{h,t}$) denotes the capital demand from Home (Foreign) new entrants. The expression of dividends also changes provided the above specification as follows

$$d_{h,t} = \frac{1}{\sigma} \rho_{h,t}^{1-\omega} N_t^{\psi(\omega-1)-1} \left[\alpha M_t + (1 - \alpha) Q_t^{\omega} M^*_t\right], \tag{40}$$

where M_t (M^*_t) is consumption and investment goods demand in each country defined as

$$M_t = C_t + N_{E,t} K_t, \qquad M^*_t = C^*_t + N^*_{E,t} K^*_t. \tag{41}$$

Note especially, using factor demand (37), when $\theta = 1$ (only labor is used as input) the expression of dividends is similar to the previous one.

Similar expressions hold in Foreign.

4.4. Characterizing the general equilibrium

As it is the case for the static model, we characterize the general equilibrium by free entry and labor market clearing and net foreign asset dynamics which replaced the balanced trade condition.

In the dynamic model, free entry condition equates real share price to real entry costs as

$$x_{h,t} = f_E \left(\frac{w_t}{z_t}\right)^{\theta}. \tag{42}$$

The labor market clearing condition gives $L_t = N_t l_t + N_{E,t} l_{EM,t}$ because now the labor supply which is demanded for production by N_t number of firms and firm creation by $N_{E,t}$ number of firms is endogenous. Noting $y_{h,t} = (\sigma - 1) \frac{d_{h,t}}{w_t} z_t$ and $l_{EM,t} = \theta \frac{x_{h,t}}{wt}$, the above condition can be rewritten as

$$L_t = (\sigma - 1) \frac{N_t d_{h,t}}{w_t} + \theta \frac{N_{E,t} x_{h,t}}{w_t}. \tag{43}$$

Similar expressions hold in Foreign.

Because of international borrowing and lending the trade is no more balanced and the following net foreign asset dynamics hold:[11]

[11] Aggregation implies the following net foreign assets accumulation for each country:

$$B_{t+1} + Q_t B_{*,t+1} = (1 + r_t) B_t + Q_t (1 + r_t^*) B_{*,t} + L_t w_t + N_t d_{h,t} - N_{E,t} x_t - C_t. \tag{44}$$

$$\frac{B_{t+1}^*}{Q_t} + B_{*,t+1}^* = \frac{(1 + r_t) B_t^*}{Q_t} + (1 + r_t^*) B_{*,t}^* + L_t^* w_t^* + N_t^* d_{f,t}^* - N_{E,t}^* x_t^* - C_t^*. \tag{45}$$

The above two equations (eliminating bonds position by Foreign using bond market clearings (47)) yield (46).

$$
\begin{aligned}
B_{t+1} + Q_t B_{*,t+1} = (1 + r_t) B_t &+ Q_t (1 + r_t^*) B_{*,t} \\
&+ \frac{1}{2} \left[L_t w_t + N_t d_{h,t} - Q_t \left(L_t^* w_t^* + N_t^* d_{f,t}^* \right) \right] \\
&- \frac{1}{2} \left[N_{E,t} x_t + C_t - Q_t \left(N_{E,t}^* x_t^* + C_t^* \right) \right]. \quad (46)
\end{aligned}
$$

Finally in addition to the above three equations, bond markets should be clear in equilibrium:

$$
B_{t+1} + B_{t+1}^* = 0, \qquad B_{*,t+1} + B_{*,t+1}^* = 0. \quad (47)
$$

The dynamic model contains 31 equations and 31 variables among which 8 are endogenous state variables ($N_t, N_t^*, B_t, B_t^*, B_{*,t}, B_{*,t}^*, r_t$ and r_t^*) and 2 are exogenous shocks (z_t and z_t^*). Table 2 summarizes the system. Details about the steady state are in the appendix. In what follows we calibrate the linearized version of the model and quantitatively explore the mechanism in generating a realistic BS correlation.

5. Calibration

The dynamic model is calibrated with parameters in Table 3. The value of constant risk aversion (γ), discount factor (β), Frisch elasticity of labor supply (φ) come from Bilbiie et al. (2007b) who choose them based on the standard RBC literature. The value of death shock (δ) is selected such that it matches to the U.S. empirical level of 10 percent job destruction per year as in Ghironi and Melitz (2005). The cost of adjusting bond holdings (ϑ) is set also following them.[12]

The elasticity of substitution between Home and Foreign goods (ω) is set to 6 in the benchmark calibration. Given this value, the elasticity of substitution among varieties (σ) is set to 7. These values may be considered too high compared to the lower value used in the open macroeconomics literature, which typically range from 0.5 to 2. However, this

[12]% deviations of bond positions are defined relative to the steady state consumption C in the linearized version of the model.

Table 2: The model

Price indices	$\alpha\rho_{H,t}^{1-\omega}+(1-\alpha)\,\rho_{F,t}^{1-\omega}=1$
	$\rho_{H,t}=N_t^{-\psi}\rho_{h,t}, \qquad \rho_{F,t}=N_t^{*-\psi}\rho_{f,t}$
	$\alpha\rho_{F,t}^{*1-\omega}+(1-\alpha)\,\rho_{H,t}^{*1-\omega}=1$
	$\rho_{F,t}^{*}=N_t^{*-\psi}\rho_{f,t}^{*}, \qquad \rho_{H,t}^{*}=N_t^{-\psi}\rho_{h,t}^{*}$
Pricing	$\rho_{h,t}=\frac{\sigma}{\sigma-1}\frac{w_t}{z_t}$
	$\rho_{h,t}^{*}=Q_t^{-1}\rho_{h,t}$
	$\rho_{f,t}^{*}=\frac{\sigma}{\sigma-1}\frac{w_t^{*}}{z_t^{*}}$
	$\rho_{f,t}=Q_t\rho_{f,t}^{*}$
Profits	$d_{h,t}=\frac{1}{\sigma}N_t^{\psi(\omega-1)-1}\rho_{h,t}^{1-\omega}\left[\alpha M_t+(1-\alpha)\,Q_t^{\omega}M_t^{*}\right]$
	$d_{f,t}^{*}=\frac{1}{\sigma}N_t^{*\psi(\omega-1)-1}\rho_{f,t}^{*1-\omega}\left[\alpha M_t^{*}+(1-\alpha)\,Q_t^{-\omega}M_t\right]$
Definition of M	$M_t=C_t+(1-\theta)\,N_{E,t}x_{h,t}$
	$M_t^{*}=C_t^{*}+(1-\theta)\,N_{E,t}^{*}x_{f,t}^{*}$
Free entry	$x_{h,t}=f_E\left(\frac{w_t}{z_t}\right)^{\theta}$
	$x_{f,t}^{*}=f_E^{*}\left(\frac{w_t^{*}}{z_t^{*}}\right)^{\theta}$
Optimal labor supply	$\chi\,(L_t)^{\frac{1}{\psi}}=w_tC_t^{-\gamma}$
	$\chi\,(L_t^{*})^{\frac{1}{\psi}}=w_t^{*}C_t^{*-\gamma}$
Labor Market clearing	$L_t=(\sigma-1)\,\frac{N_td_{h,t}}{w_t}+\theta\frac{N_{E,t}x_{h,t}}{w_t^{*}}$
	$L_t^{*}=(\sigma-1)\,\frac{N_t^{*}d_{f,t}^{*}}{w_t^{*}}+\theta\frac{N_{E,t}^{*}x_{f,t}^{*}}{w_t^{*}}$
Number of firms	$N_t=(1-\delta)\,(N_{t-1}+N_{E,t-1})$
	$N_t^{*}=(1-\delta)\left(N_{t-1}^{*}+N_{E,t-1}^{*}\right)$
Euler equation (shares)	$x_{h,t}=\beta\,(1-\delta)\,E_t\left(\frac{C_{t+1}}{C_t}\right)^{-\gamma}(x_{h,t+1}+d_{h,t+1})$
	$x_{f,t}^{*}=\beta\,(1-\delta)\,E_t\left(\frac{C_{t+1}^{*}}{C_t^{*}}\right)^{-\gamma}\left(x_{f,t+1}^{*}+d_{f,t+1}^{*}\right)$
Euler equation (bonds)	$C_t^{-\gamma}\,(1+\vartheta B_{t+1})=\beta\,(1+r_{t+1})\,E_tC_{t+1}^{-\gamma}$
	$C_t^{-\gamma}\,(1+\vartheta B_{*,t+1})=\beta\left(1+r_{t+1}^{*}\right)E_t\frac{Q_{t+1}}{Q_t}C_{t+1}^{-\gamma}$
	$C_t^{*-\gamma}\left(1+\vartheta B_{*,t+1}^{*}\right)=\beta\left(1+r_{t+1}^{*}\right)E_tC_{t+1}^{*-\gamma}$
	$C_t^{*-\gamma}\left(1+\vartheta B_{t+1}^{*}\right)=\beta\,(1+r_{t+1})\,E_t\frac{Q_t}{Q_{t+1}}C_{t+1}^{*-\gamma}$
Net foreign Asset	$B_{t+1}-(1+r_t)\,B_t+Q_t\left[B_{*,t+1}-(1+r_t^{*})\,B_{*,t}\right]$
	$=\frac{1}{2}\left[L_tw_t+N_td_{h,t}-Q_t\left(L_t^{*}w_t^{*}+N_t^{*}d_{f,t}^{*}\right)\right]$
	$-\frac{1}{2}\left[N_{E,t}x_t+C_t-Q_t\left(N_{E,t}^{*}x_t^{*}+C_t^{*}\right)\right]$
Bond market clearing	$B_{t+1}+B_{t+1}^{*}=0$
	$B_{*,t+1}+B_{*,t+1}^{*}=0.$

Table 3: Baseline parametrization

γ	constant risk aversion	2
β	discount factor	0.99
φ	Frisch elasticity of labor supply	2
σ	elasticity of substitution among varieties	7
ω	between Home and Foreign goods	6
α	home bias in consumption	0.72
δ	death shock	0.025
ϑ	bond holding adjusting costs	0.0025
θ	share of labor in entry costs	0.64
ψ	love for variety	0.2

is well in the range of micro founded estimations in the trade literature. For instance, Romalis (2007) estimates elasticities which range from 4 to 13. For the purposes of comparison, we also consider the standard value of elasticity in open macroeconomics ($\omega = 2$) as it is in Benigno and Thoenissen (2008). Provided the above value of elasticity we set $\sigma = 3.8$ following Ghironi and Melitz (2005).[13]

We set ψ, love for variety, to 0.2 arbitrary. This is slightly higher than the implied value under the Dixit-Stiglitz preference for the benchmark elasticity (1/(7-1)=0.17). Because of the ambiguity which surrounds this parameter, the role played by the love for variety ψ in the BS puzzle is explored with a sensitivity analysis in the following section.

The value of home bias in consumption (α) is taken from CDL. We set the share of labor (θ) in entry costs as 0.64 based on Heathcote and Perri (2002). This is the standard value in the DSGE model including capital and labor in its production function.

[13] A low elasticity in the open macroeconomics literature is controversial: recently Imbs and Mejean (2009) argue the conventional estimation in the open macroeconomics literature about the elasticity of substitution has a downward bias without considering the heterogeneity of these values among sectors. They propose the value of elasticity around 7 in calibrating DSGE models and support the view of the "elasticity optimism".

Productivity process is selected from Backus et al. (1992) such that $\mathsf{Z}_{t+1} = \Omega \mathsf{Z}_t + \xi_t$ where $\mathsf{Z}_t = \begin{bmatrix} z_t, & z_t^* \end{bmatrix}'$, $\xi_t = \begin{bmatrix} \xi_t, & \xi_t^* \end{bmatrix}'$ and where the correlation of shocks and error terms are given by

$$\Omega = \begin{bmatrix} 0.906 & 0.088 \\ 0.088 & 0.906 \end{bmatrix}, \text{ and } V(\xi) = \begin{bmatrix} 0.73 & 0.19 \\ 0.19 & 0.73 \end{bmatrix}. \tag{48}$$

5.1. Intuition by impulse responses

Impulse responses for the real exchange rate, the terms of trade and relative consumption are reported in Figure 3. For the real exchange rate and relative consumption both welfare and empirical-based are documented. The shock used is one percent increase in Home productivity. For simplicity we omit spillover terms in the correlation matrix (48) in this exercise.

On impact of the shock new entry takes place in Home. The relative number of varieties increases steadily showing a hump-shaped pattern. Over time the terms of trade appreciate for Home in spite of persistent efficiency gains due to a positive productivity shock. The reason is identical to the one explained in the first part of the paper. A combination of a relatively high elasticity of substitution and a mild love for variety are sufficient to change the direction of the terms of trade from a depreciation into an appreciation (dotted lines on upper and lower panels). Reflecting these terms of trade appreciation, the empirical-based real exchange rate appreciates as well (solid line on upper panel). However, the welfare-based real exchange rate remains depreciated reflecting a higher Home originated extensive margins (solid line on lower panel).

At the same time of the empirical-based real exchange rate appreciation, relative consumption rises (crossed line on upper panel). It is well known that with non-contingent bonds the condition which holds under complete markets between the real exchange rate and relative consumption is only verified in expected first difference. Home and Foreign households stabilize their consumption using such bonds only in the aftermath, not ex-anté of a shock. As a result, a tight link between the real exchange rate and relative consumption under complete markets is broken. However, in our model the market incompleteness alone is not sufficient to reproduce a realistic BS correlation as they remain positively correlated in welfare-based (crossed and solid line on lower panel). We need a wealth effect induced by higher extensive margins which brings the terms of trade into an appreciation and the fact that such fluctuations in extensive margins are not fully measured in observed real exchange rates as argued in Broda and Weinstein (2004, 2006, 2010).

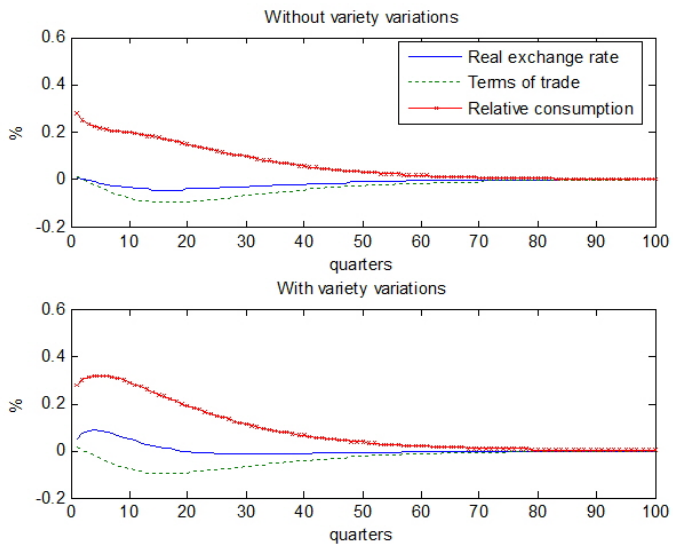

Figure 3: IRFs under incomplete markets.

5.2. Characteristics of the theoretical model

Table 4 reports second moments of the dynamic model. The US data comes from either Backus et al. (1992) or Heathcote and Perri (2002) except for the BS correlation (-0.27) drawn from CDL, the median among OECD countries relative to the ROW. All variables are empirical-based denoted with $\widetilde{}$ such that for simplicity they do not contain any variations in extensive margins. GDP in this model is $\widetilde{Y}_t = w_t L_t + N_t \widetilde{d}_{h,t}$ (labor + financial income). Investment value is $N_{E,t}\widetilde{x}_{h,t}$, share price multiplied by the number of new entrants. Net export (trade balance) is defined as $\widetilde{TB}_t = \left(\widetilde{X}_t - \widetilde{IM}_t\right) / \widetilde{Y}_t$ where $\widetilde{X}_t$ and $\widetilde{IM}_t$ denote export and import value respectively. For the purpose of comparison we report second moments obtained with the lower elasticity ($\sigma = 3.8$ and $\omega = 2$) and those under complete markets as well.[14]

Under incomplete markets with a high elasticity of substitution between Home and Foreign goods ($\omega = 6$), a high investment volatility (15.25) and a strong negative cross country correlation of investment (firm entry) (-0.91) appear. Because firm creation requires labor services, employment is internationally correlated negatively (-0.91) and output as well (-0.52). This high elasticity is what is needed for providing a realistic BS correlation (0.17) along the mechanism discussed in the first part of the paper. With an alternative elasticity ($\omega = 2$) investment volatility declines (8.38) and Home and Foreign investment becomes less correlated (-0.77) while the BS correlation remains in the puzzle (0.95).

Second moments under complete markets are quite similar to those under incomplete markets. The result is reminiscent of Heathcote and Perri (2002) who discuss that only the balanced trade case is very different and close to the reality. Although they are similar, under complete markets, investment volatility increases (18.93) and it becomes correlated more negatively (-0.94). This can be interpreted based on the result of Corsetti et al. (2007). Although they argue with a static model, it is shown that under complete markets there is more entry compared to incomplete markets in a more efficient country. Intuitively because consumption is perfectly insured under complete markets, the equilibrium allocation of firms becomes such that the world has the maximum number of varieties.

The BS correlation becomes negative (-0.19) under complete markets but the reason is quite different from the one discussed for incomplete markets. Under complete markets when it becomes negative there is a very strong positive transmission via the terms of trade *depreciation* following a positive shock. As a result empirical-based consumption in Foreign rises more than Home while the empirical-based real exchange rate is depreciated. However, as it is reported in Corsetti et al. (2008b) using VAR model, empirical-based

[14]Second moments of the theoretical model are calculated using the frequency domain technique presented in Uhlig (1998) for HP filtered series. The smoothing parameter is set to 1600.

consumption should rise in a country receiving a positive shock relative to the rest of the world. Hence a realistic BS correlation which appears under complete markets would not be considered as plausible because it bases on unrealistic pattern of consumption. We place details about complete markets in the appendix.

Obviously the model shares principle (bad) characteristics of the standard two-country real business cycle model, such discussed in Heathcote and Perri (2002): lower volatility in the terms of trade and the real exchange rate, higher cross country correlation of consumption than output and negative cross country correlation of investment and employment. In summary we can say that adding extensive margins with a high elasticity of substitution brings the BS correlation into a realistic range. On the other hand, other puzzles in international real business cycle remain without significant quantitative improvement.

5.3. Sensitivity analysis

Figure 4 and Figure 5 examine the BS correlation with different values of the elasticity of substitution (ω) and love for variety (ψ) with the baseline parameters. It becomes steadily weaker and finally becomes negative as the elasticity increases. For $\omega = 7$, a proposed value in Imbs and Mejean (2009), the BS correlation is -0.23. For love for variety, almost the same pattern is observed. The BS correlation becomes weaker and arrives in negative range as love for variety increases. Again the intuition is shed by the analytical part of the paper where the interaction between the elasticity of substitution and the love for variety has been emphasized.

Table 4: Second moments

% std.dev. relative to $\widetilde{Y}$	$\widetilde{Y}$	$\widetilde{C}$	$N_E\widetilde{x}_h$	L	$\widetilde{TB}/\widetilde{Y}$	TOT	$\widetilde{Q}$
US data	1.71	0.84	5.38	0.66*	0.45	2.99*	3.73*
	(1.00)	(0.49)	(3.15)	(0.34*)		(1.79*)	(2.23*)
Incomplete	1.46	0.48	15.25	0.83	0.46	0.09	0.04
	(1.00)	(0.33)	(10.42)	(0.56)		(0.06)	(0.03)
$\sigma = 3.8$ $\omega = 2$	1.40	0.45	8.38	0.82	0.19	0.33	0.15
	(1.00)	(0.32)	(5.98)	(0.58)		(0.24)	(0.10)
Complete	1.76	0.47	18.93	1.19	0.74	0.09	0.04
	(1.00)	(0.27)	(10.76)	(0.68)		(0.05)	(0.02)
$\sigma = 3.8$ $\omega = 2$	1.47	0.44	8.90	0.92	0.17	0.42	0.19
	(1.00)	(0.30)	(6.07)	(0.63)		(0.29)	(0.13)
Correlation with output	$\widetilde{C}$	$N_E\widetilde{x}_h$	L	$\widetilde{TB}/\widetilde{Y}$	TOT	$\widetilde{Q}$	
US data	0.76	0.90	0.87*	−0.28	−0.24*	0.13*	
Incomplete	0.64	0.89	0.78	−0.18	0.55	0.55	
$\sigma = 3.8$ $\omega = 2$	0.68	0.96	0.81	−0.67	0.67	0.67	
Complete	0.39	0.87	0.86	0.06	0.91	0.91	
$\sigma = 3.8$ $\omega = 2$	0.61	0.96	0.83	−0.65	0.65	0.65	
Cross country correlation	$\widetilde{Y}$	$\widetilde{C}$	$N_E\widetilde{x}_h$	L	$\frac{\widetilde{C}}{\widetilde{C^*}}, \widetilde{Q}$	$\frac{C}{C^*}, Q$	
US data	0.58*	0.36*	0.30*	0.42*	-0.27^{CDL}		
Incomplete	−0.52	0.90	−0.91	−0.91	0.17	0.85	
$\sigma = 3.8$ $\omega = 2$	−0.33	0.93	−0.77	−0.95	0.95	0.97	
Complete	−0.67	0.98	−0.94	−0.96	−0.19	1.00	
$\sigma = 3.8$ $\omega = 2$	−0.39	0.99	−0.80	−0.96	0.84	1.00	

Data with * from Heathcote and Perri (2002). Others from Backus et al. (1992).

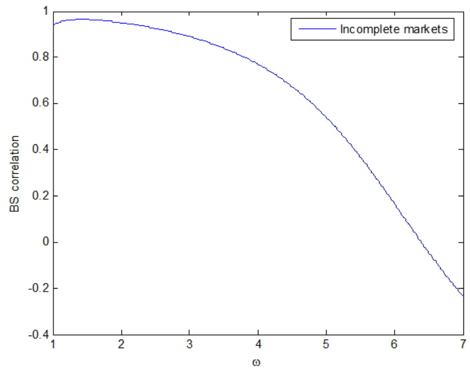

Figure 4: the BS correlation and the elasticity of substitution.

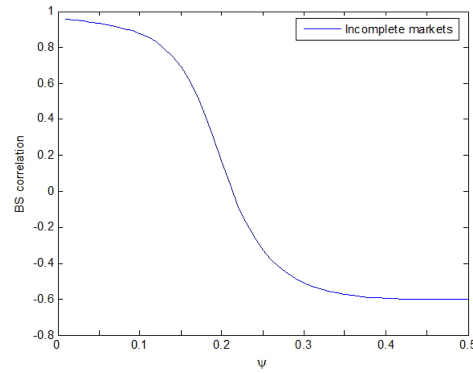

Figure 5: the BS correlation and love for variety.

6. Conclusion

This paper revisits the consumption-real exchange rate anomaly known as the Backus-Smith puzzle. We examine how extensive margins can contribute to the resolution of the puzzle. Our argument is based on the observation that the BS correlation cannot be tested fully because of unobservable fluctuations in extensive margins.

Higher extensive margins, following a positive productivity shock, work as a demand shock under love for variety and bring the terms of trade in an appreciation. Reflecting such terms of trade movement, the empirical-based real exchange rate appreciates as well while consumption rises resolving the puzzle. Along the story the asset market incompleteness and a relatively high trade elasticity which is consistent to micro founded estimations play important roles.

In contrast the welfare-based real exchange rate depreciates because of higher extensive margins consumed with home bias. Thus the puzzling positive correlation might remain in welfare basis. One of challenges in the future research would be to test this prediction.

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Appendix A. Complete markets

We discuss the implication of extensive margins under complete asset markets. As it is explained in the paper when a realistic BS correlation appears under complete markets it requires implausible pattern of consumption. Here we look this point more in depth analytically and quantitatively.

Appendix A.1. Static model

Under complete markets, the marginal utility which stems from an additional nominal wealth is the same across countries:

$$Q = \left(\frac{C^*}{C}\right)^{-\gamma}. \tag{A.1}$$

Using the above perfect risk sharing condition, we can write relative dividends in first order deviations as $\mathsf{d}^R = (\lambda - 1)\,\rho^R + [\psi(\lambda - 1) - 1]\,\mathsf{N}^R$. Plugging this expression in the labor market clearing and free entry conditions, we have the solution for w^R and N^R as follows

$$\mathsf{w}^R = \frac{\lambda - 1}{\lambda}\mathsf{z}^R + \frac{\psi(\lambda - 1)}{\lambda}\mathsf{z}_E^R, \tag{A.2}$$

$$\mathsf{N}^R = \mathsf{z}_E^R. \tag{A.3}$$

where

$$\lambda \equiv \omega\left[1-(2\alpha-1)^2\right]+(2\alpha-1)^2\frac{1}{\gamma}.$$

The parameter λ roughly represents the elasticity of substitution between local and imported goods. Relative wages w^R increase (decrease) with a positive productivity shock on marginal costs z^R when $\lambda > 1$ ($\lambda < 1$). Extensive margins N^R change one by one with an investment shock z_E^R as it is in the balanced trade case. With love for variety ($\psi > 0$) and a sufficiently high elasticity of substitution ($\lambda > 1$) wages appreciates with higher extensive margins. When the elasticity is low ($\lambda < 1$) they depreciate. As it is the case for incomplete markets, we can find a restriction on parameters as $1 > \psi\frac{(\lambda-1)^2}{\lambda}$ for complete markets.

Provided the above expression of relative wages, the terms of trade in first order deviations are given by

$$\mathsf{TOT} = \frac{1}{\lambda}\mathsf{z}^R - \frac{\psi(\lambda-1)}{\lambda}\mathsf{z}_E^R. \tag{A.4}$$

In contrast to the terms of trade fluctuations analyzed in the paper, under complete markets following a positive productivity shock on marginal costs z^R, they never appreciate from the first term. As it is under incomplete markets, however, following a positive shock on firm creation efficiency z_E^R, the second term adds a terms of trade appreciation when ($\lambda > 1$) with love for variety ($\psi > 0$). Because of the first term, with whatever correlation of the shocks, the terms of trade less likely appreciate compared to those under incomplete markets.

Appendix A.1.1. Implication for the BS puzzle

By constructing empirical-based measure, the BS correlation with extensive margins under complete markets become:

$$\widetilde{Q} = \gamma\left(\widetilde{C} - \widetilde{C}^*\right) + (\gamma-1)\psi(2\alpha-1)\mathsf{N}^R. \tag{A.5}$$

The tight link between the empirical-based relative consumption and real exchange rate that we find ordinary under complete markets is broken because of extensive margins. And it is possible even under complete markets to generate a negative BS correlation. This happens when there is a relatively strong positive transmission via the terms of trade depreciation ($\widetilde{Q} > 0$) (in spite of the higher number of varieties $\mathsf{N}^R > 0$ which adds them an appreciation) to the extent that the relative empirical-based consumption decreases ($\widetilde{C} - \widetilde{C}^* < 0$).

In particular, with perfect correlation between two shocks, $\mathsf{z}^R = \mathsf{z}_E^R$, the BS correlation is described as

$$\widetilde{Q} = \frac{1 - \psi (\lambda - 1)}{1 - \psi (\lambda\gamma - 1)} \gamma \left(\widetilde{C} - \widetilde{C}^* \right). \tag{A.6}$$

Without love for variety ($\psi = 0$) or with log utility case ($\gamma = 1$), we are in the original puzzle. But depending on the value of parameters the BS correlation would become negative or positive.

However, we consider a realistic BS correlation under complete markets is implausible because it requires unrealistic empirical-based consumption pattern such that $\widetilde{C} - \widetilde{C}^* < 0$ combined with a terms of trade depreciation following a positive shock.

Appendix A.2. Dynamic model

With complete markets the model becomes simpler. We discuss only these modified points compared to the dynamic model in the paper. The real budget constraint for the Home representative household now contains the state-contingents securities in the place of non-contingent bonds. This is

$$\begin{aligned} & C_t + s_{h,t+1} x_{h,t} \left(N_t + N_{E,t} \right) \\ & \qquad + \sum_{s_{t+1}} b_{t+1} \left(S_{t+1} \right) q_t \left(S_{t+1} \mid S_t \right) + Q_t \sum_{s_{t+1}} b_{t+1}^* \left(S_{t+1} \right) q_t^* \left(S_{t+1} \mid S_t \right) \\ & \qquad\qquad = w_t L_t + s_{h,t} N_t \left(x_{h,t} + d_{h,t} \right) + b_t \left(S_t \right) + Q_t b_t^* \left(S_t \right). \end{aligned} \tag{A.7}$$

The household has now access to the full set of Arrow-Debreu securities which give one unit of Home or Foreign goods in the next period. $b_{t+1}(S_{t+1})$ $(b^*_{t+1}(S_{t+1}))$ is holdings of such assets into $t+1$ indexed by the future state of nature S_{t+1}. $q_t(S_{t+1} \mid S_t)$ $(q^*_t(S_{t+1} \mid S_t))$ denotes its real price which is conditional on the current state of nature S_t.

First order conditions about these state-contingent securities yield the well known perfect risk sharing condition as (A.1) with time indices. Under complete markets, Euler equations about non-contingent bonds, bonds markets clearing conditions and the evolution of net foreign asset are no more needed. Other first order conditions remain the same as they are in the paper. Finally the model contains 25 equations and 25 variables among which 2 are endogenous state variables (N_t and N^*_t) and 2 are exogenous shocks (z_t and z^*_t).

Appendix A.3. (Unrealistic) characteristics under complete markets

Here we quantitatively show how it is implausible a realistic BS correlation obtained under complete markets. For this purpose we look impulse responses as in the paper with the same baseline parameters and shocks (Figure A-1). Following a rise in Home labor productivity the terms of trade always depreciate in spite of a relatively high number of Home originated firms. The welfare-based real exchange rate strongly depreciates (solid line on lower panel) mainly due to higher Home extensive margins consumed with home bias. Elimination of such fluctuations in extensive margins from consumption baskets makes the empirical-based relative consumption decreased (crossed line on upper panel) while the empirical-based real exchange rate remains depreciated (solid line on upper panel). In other words, households in Home consume less only with intensive margins whilst more with extensive margins in order to achieve a perfect risk sharing. Again, although it is possible to create a realistic BS correlation, it must be based on implausible consumption pattern.

Next we perform sensitivity analysis with benchmark parameters (Figure A-2 and Figure A-3). Under complete markets as the elasticity of substitution increases the BS correlation declines and afterwards starts to increase. The analytical solution again sheds light on this non-linearity. As it can be seen from (A.4), when the elasticity of substitution or love for variety approaches to a sufficiently high value, the terms of trade change their direction, from depreciation to appreciation. Because variations in the empirical-based relative consumption remain negative along such a rise in elasticity or love for variety, the non-linear pattern of the BS correlation as in Figure A-2 and A-3 appears.

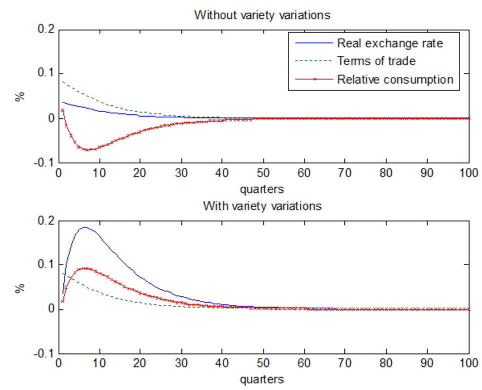

Figure A-1: IRFs under complete markets.

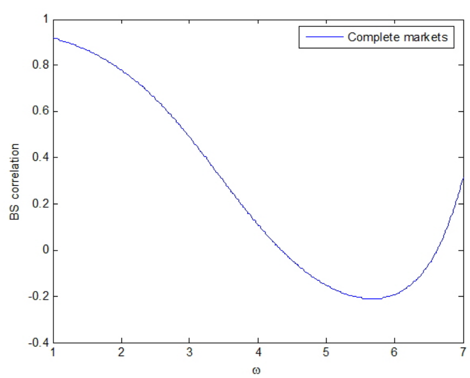

Figure A-2: the BS correlation and the elasticity of substitution.

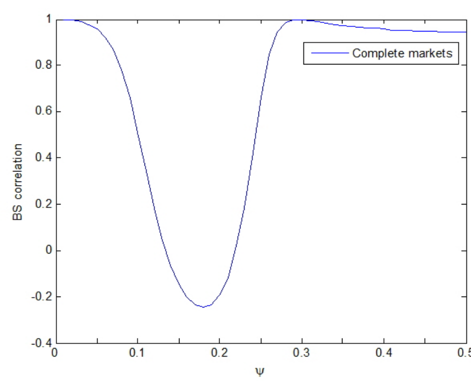

Figure A-3: the BS correlation and love for variety.

Appendix B. Steady state

Steady state values are expressed without time indices. At the symmetric steady state, it must be $Q = 1$, $p_h = p_h^* = p_f = p_f^*$, $N = N^*$ and $P_H = P_H^* = P_{F,} = P_F^*$. Then

$\rho_H = \rho_H^* = \rho_F = \rho_F^* = 1$ and $\rho_h = \rho_h^* = \rho_f = \rho_f^* = N^\psi$. Also $C = C^*$, $N_E = N_E^*$ and $K = K^*$. Then $M = M^*$. Also $d_h = d_f^*$ and $x_h = x_h^* = x_f = x_f^*$.

First we start to find steady state ratios relative to M denoted with S_i^M where $i = D$ (dividends), I (investment), W (wages) and C (consumption). The steady state share of real dividends relative to the demand addressed to each firm becomes

$$S_D^M \equiv \frac{Nd_h}{M} = \frac{1}{\sigma}.$$

With this condition, Euler equation about share holdings and the motion of firms the steady state share of investment becomes

$$S_I^M \equiv \frac{N_E x_h}{M} = \frac{\beta\delta}{1-\beta(1-\delta)}\frac{1}{\sigma}.$$

Using the above two steady state shares, from the labor market clearing condition the steady state share of labor income becomes

$$S_W^M \equiv \frac{Lw}{M} = S_D^M(\sigma-1) + \theta S_I^D.$$

Finally at the symmetric steady state, aggregated demand must be equal to aggregated income: $C + N_E x_h = Lw + Nd_h$, using this identity, we have

$$S_C^M \equiv \frac{C}{M} = S_W^M + S_D^M - S_I^M. \tag{B.1}$$

Noting $M = C + (1-\theta) N_E x_h$, with the above steady state ratios defined relative to M, the steady state ratios relative to the consumption C denoted with S_i are expressed as follows

$$S_I \equiv \frac{N_E x_h}{C} = \frac{S_I^M}{1-(1-\theta)S_I^M},$$

$$S_D \equiv \frac{Nd_h}{C} = S_D^M\left[1 + (1-\theta)S_I\right], \tag{B.2}$$

$$S_W \equiv \frac{Lw}{C} = S_D\left(\sigma - 1\right) + \theta S_I.$$

Based on the above ratios, we can find explicit solutions about variables at the steady state. From the definition of price index we have,

$$\rho_h = N^{\psi}. \tag{B.3}$$

From the optimal pricing, steady state wages are expressed as

$$w = \frac{\sigma - 1}{\sigma} N^{\psi}, \tag{B.4}$$

We choose χ so that the steady state labor supply becomes the unity: $L = 1$. From the optimal labor supply condition this implies

$$\chi = wC^{-\gamma}. \tag{B.5}$$

Also we have $\frac{N_E x}{C} = S_I$. Using the steady state low of motion, $\delta N = \left(1 - \delta\right) N_E$ and the free entry condition, $x = w^{\theta}$, S_I can be rewritten as

$$\frac{Nw^{\theta}}{C} = \frac{1 - \delta}{\delta} S_I. \tag{B.6}$$

Because $L = 1$, the steady state ratio of wage is given by

$$\frac{w}{C} = S_W. \tag{B.7}$$

In the above two equations eliminating C and plugging B.4 the unknown is N. Solving this equation, the steady state number of variety is given by

$$N^{1-\psi(1-\theta)} = \left(\frac{\sigma - 1}{\sigma}\right)^{1-\theta} \frac{1 - \delta}{\delta} \frac{S_I}{S_W}. \tag{B.8}$$

The remaining variables are easily found as follows

$$w = \frac{\sigma - 1}{\sigma} N^{\psi}, \tag{B.9}$$

$$C = \frac{w}{S_W}, \tag{B.10}$$

$$d_h = \frac{CS_D}{N}, \tag{B.11}$$

$$N_E = \frac{\delta}{1-\delta}N, \tag{B.12}$$

$$x_h = w^\theta. \tag{B.13}$$

Especially, from (B.5) the value of χ is found. With baseline parametrization it must be $\chi = 0.39$.