経済の数理解析

京都大学数理解析研究所　研究集会

京都大学数理解析研究所の共同研究事業の一つとして、下記のように研究集会を催しますので、ご案内申し上げます。

研究代表者　丸山　徹 (慶應義塾大学)

記

□ 日　時　　平成17年11月18日 (金) －11月20日 (日)

□ 場　所　　京大会館　101号室
〒606－8305　京都市左京区吉田河原町15－9　TEL075－751－8311
市バス　京大正門前　下車/京都バス　荒神橋　下車
京阪電車鴨東線丸太町駅下車徒歩約10分

□ 講演時間　　60分 (うち質疑応答10分)

プログラム

11月18日 (金)　　　* 印：講演者

午前の部　　座長：丸山　徹 (慶應義塾大学)

9：00－10：00　佐藤　伸 (慶應義塾大学)
On Choice under Complete Uncertainty and its Application to Mechanism Design

10：00－11：00　花薗　誠 (京都大学)
Holdup, Complementary Action, and Option Contracts

11：10－12：10　*河合　伸 (名古屋大学)，皆川　正 (名古屋大学)
Macroeconomic Implications of Coordination Failure in a Multi-Sector Cournot-Nash Model

午後の部　　座長：平野　載倫 (横浜国立大学)

13：30－14：30　*佐柄　信純 (法政大学)、Milan Vlach (京都情報大学院大学)
Convex Preferences in a Measure Space: Pareto Optimality and Core in Cake Division.

14：30－15：30　前田　隆 (金沢大学)
ファジィ利得を持つ双行列ゲームの均衡解の性質

座長：武隈　愼一 (一橋大学)

16：00－17：00　*土居　潤子 (京都産業大学)，三野　和雄 (大阪大学)
A Variety Expansion Model of Growth with External Habit Formation

17：00－18：00　＊中村　保 (神戸大学)、室　和伸 (神戸大学)
Technological Change and Wage Inequality in the Increase of Service Economy

18：30－20：00　懇親会

11月19日(土)

午前の部　　　座長:高岡　浩一郎(一橋大学)

9:00−10:00　新井　拓児(慶応義塾大学)
An Approximate Approach to the Exponential Utility Indifference Valuation

10:00−11:00　*林　高樹(慶応義塾大学)、吉田　朋宏(東京大学)
Covariance Estimation of Nonsynchronously Observed Diffusion Processes

座長:山崎　昭(一橋大学)

11:10−12:10　吉田　博之(日本大学)
Monetary Policy and Economic Fluctuations in a Sticky-Price Model

午後の部

13:30−14:30　*小野崎　保(青森公立大学), Gernot Sieg (Inst. Wirtschaftswissen.)
横尾昌紀(岡山大学)
Market Behavior may Vary Depending on a Single Agent's Behavior

座長:菊池　紀夫(慶應義塾大学)

14:30−15:30　藤田　敏治(九州工業大学)
非決定性動的計画法について

16:00−17:00　岩本　誠一(九州大学)
黄金の三位一体 ―― 最適化、不等式、等式 ――

座長:川又　邦雄

17:00−18:00　*グレーヴァ香子(慶應義塾大学), 奥野-藤原　正寛(東京大学)
Efficiency and Diversity in Voluntarily Repeated Prisoner's Dilemma

18:00−19:00　中山　幹夫(慶應義塾大学)
ゲーム理論の誕生とその周辺

11月20日(日)

午前の部　　　座長:須田　伸一(慶應義塾大学)

9:00−10:00　*白瀧　絵理(明治安田生命), 石川　竜一郎(筑波大学)
Cheap-Talk Game における均衡の精緻化について

10:00−11:00　細矢　祐誉(慶應義塾大学)
Excess Information and Tragedy of the Commons

座長:小宮　英敏(慶應義塾大学)

11:10−12:10　坂根　宏一(関西大学)
Existence of Equilibria for an Abstract Society with N Social Groups

午後の部

13:30−14:30　浦井　憲(大阪大学)
経済学的均衡と不動点定理:ホモロジー的観点から

座長:藤本　喬雄(福岡大学)

14:30−15:30　*田中　敬一(京都大学), 山田　健(日本銀行), 渡部　敏明(日本銀行)
Approximation of Interest Rate Derivatives Prices by Gram-Charlier Expansion and Bond Moments

15:40−16:40　室井　芳史(日本銀行)
社債アメリカ型オプションの評価について

17:00−18:00　楠岡　成雄(東京大学)
拡散過程の期待値の近似について

Abstract

Strategy-proof Social Choice Correspondences: Possibility and Impossibility Results

Shin Sato

October 30, 2005

This paper investigates the possibility of resolve the impossibility result of the Gibbard-Satterthwaite theorem by admitting multiple social choices, that is, considering social choice correspondences instead of social choice functions. The Gibbard-Satterthwaite theorem shows there is no social choice function satisfying strategy-proofness (lying is not profitable), nondictatorship (there is no agent having superpower), range condition (the range of a SCF consists of at least three elements), and domain condition (the domain of a SCF is large enough). The standard approach to avoid this impossibility is to restrict the domain of preference, that is, weaken the domain condition. But in this paper, we take another approach; We does not require that a social choice is always decisive. In other words, we admit a multiple social choice. We give possibility and impossibility results. The possibility result is that we can construct a strategy-proof and efficient social choice correspondence satisfying anonymity and neutrality. The impossibility result is that such a strategy-proof social choice correspondence should lose decisiveness to a considerable degree if the society consists of three agents.

A Simple Holdup Model with Two-sided Investments: the Case of Common-Purpose Investments

Makoto Hanazono*
October 31, 2005

Abstract

This paper theoretically studies a simple, discrete holdup model with two-sided investments, in which each party's investment enhances the value of transaction by improving a common factor (the buyer's valuation, in text). We investigate whether contractual remedies to the holdup problem are available by adopting noncontingent or option contracts. The main result of this paper is that an option contract can completely remedy the holdup problem in several cases, whereas noncontingent contracts can only partly remedy it. Remedies by option contracts are more likely as the relative importance of the purely cooperative investment to the purely selfish investment becomes larger, and as the cost of investment becomes smaller.

Key words: Holdup, two-sided investments, common-purpose investments, option contracts
JEL classification numbers: D23, L14

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Macroeconomic Implications of Coordination Failure in a Multisector Cournot-Nash Model

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October 31, 2005

Abstract

It is known that the multisector Cournot-Nash models may exhibit multiple, Pareto-ranked equilibria. The main condition is that the price elasticity of demand is sufficiently low at the survival level of consumption. This paper investigates the macroeconomic implications of the model by analyzing the intra-industrial interaction and the circulative structure of multisector economy. It is shown that the subjective aggregate supply curve is downward sloping as well as the objective demand curve. It is also found that there is strategic complimentarity across firms within each industry as well as across sectors. Furthermore, there exhibits not only a form of strategic complimentarity but also strategic substitutability across sectors.

JEL classifications: E1, D43, D58, L16,
Keywords: Coordination Failure, Strategic Substitution, Strategic Complimentarity,

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Convex Preferences in a Measure Space: Pareto Optimality and Core in Cake Division

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Abstract

In this paper, we propose a convex-like structure in a non-atomic finite measure space, and use it for analysing problems of cake division. Our attention is focused on a non-atomic finite measure space whose field is the Borel σ-field of a topological space. However, the proposed structure and its basic properties can easily be extended to an arbitrary finite measure space whose measure has a convex range.

The structure makes it possible to introduce various types of convexity of preference relations and functions defined on the field under consideration. Having concepts and basic results analogous to those of standard convex analysis, we apply them, together with our previous results from Sagara and Vlach (2005), to the problems of cake division.

The main purpose is to achieve useful results that do not require the additivity of preference relations. In particular, we are concerned with the existence of Pareto optimal partitions, and the existence of core partitions in TU and NTU games arising in a pure exchange economy in which each participant is endowed with an initial "piece" of the cake.

On Characterization of Nash Equilibrium Strategy of Bi-matrix Games with Fuzzy Payoffs

金沢大学・経済学部　前田　隆 (Takashi Maeda)
Faculty of Economics
Kanazawa University

Abstract. In this paper, we consider fuzzy bi-matrix games, namely, two-person games with fuzzy payoff. Based on fuzzy max order, for such games, we define three kinds of concepts of Nash equilibrium strategies and investigate their properties.
Keywords. Bi-Matrix game; Fuzzy number; Fuzzy max order; Nash equilibrium strategy; Non-dominated Nash equilibrium strategy; Possibility measure; Necessity measures

1 Introduction

Since seminal works by Neumann-Morgenstern([13]) and Nash([11] and [12]), Game theory has played an important role in the fields of decision making theory such as economics, management, and operations research, etc. When we apply the game theory to model some practical problems which we encounter in real situations, we have to know the values of payoffs exactly. However, it is difficult to know the exact values of payoffs and we could only know the values of payoffs approximately, or with some imprecise degree. In such situations, it is useful to model the problems as games with fuzzy payoffs. In this case, since the expected payoffs of the game should be fuzzy-valued, there are no concepts of equilibrium strategies to be accepted widely. So, it is an important task to define the concepts of equilibrium strategies and investigate their properties. Compos([3]) has proposed a methods to solve fuzzy matrix games based on linear programming, but has not defined explicit concepts of equilibrium strategies. For matrix games with fuzzy payoffs, Maeda([9]) has defined minimax equilibrium strategies based on fuzzy max order and investigated their properties. For Bi-matrix games with fuzzy payoffs, Maeda([10]) has defined Nash equilibrium strategies based on possibility and necessity measures and investigated its properties. While, Aubin([2]) has considered fuzzy cooperative games.

In this paper, we consider fuzzy bi-matrix games. For such a game, we shall define three kinds of concepts of Nash equilibrium strategies and investigate their properties.

For that purpose, this paper is organized as follows. In Section 2, we shall give some basic definitions and notations on fuzzy numbers. In Section 3, we shall define fuzzy bi-matrix game with fuzzy payoffs and three kinds of concepts of Nash equilibrium strategies and investigate their properties. In Section 4, we investigate the properties of values of fuzzy matrix games by means of possibility and necessity measures.

A Variety Expansion Model of Growth with External Habit Formation*

Junko Doi† and Kazuo Mino‡

May 2005

Abstract

This paper introduces consumption externalities into one of the base line models of growth in which continuing expansion of product variety sustains long-term growth. We assume that consumers set a benchmark stock of consumption for each good so that there are commodity specific external effects. Each good is produced by a monopolistically competitive firm and the film exploits the presence of consumption external effects in determining its profit-maximizing price. Given those settings, we show that the introduction of consumption externalities may affect the balanced-growth characterization, transitional dynamics and policy effects in fundamental manners.

*We thank Hideyuki Adachi, Masahiro Ashiya, Elias Dinopoulos, Ryo Horii, Tamotsu Nakamura, Paul Segerstrom and seminar participants at various occasions for their helpful comments on earlier versions of this paper.

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Technological Change and Wage Inequality in the Increase of Service Economy

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Abstract

In some advanced countries wage inequalities between skilled and unskilled labor have increased. Especially, the college premium in the US has risen shapely with an increasing supply of college graduates in 1990's. This is called a "wage premium puzzle." To reconcile a rise in a factor price with increasing its supply, a huge increase in the demand is required. Although many theories have been put forward, the skill biased technical change or STBC has received considerable interests. However, the most European countries (and Japan) did not experience an increase in wage inequality but saw a substantial rise in unemployment. Although technology or supply side is crucial in determining the long-run wage inequalities, the demand-side plays an important role in the factor price dynamics in the short- or medium-run. Looking at the fact from a different point of view, this paper considers the skill premium as the sector premium. Increased income has expanded the service sector more than the manufacturing sector. The service sector, especially the financial service, needs more skilled labor than the manufacturing sector does. If the economic growth is driven by the technical progress in the manufacturing like in the early stage of economic development, then one does not see any skill premium puzzle. If, however, the unbiased or service-biased technological advance takes place, then the skill premium puzzle will occur. In addition, our analysis gives an explanation that the within group wage inequality is followed by the between group equality.

Keywords: Stone-Gary utility, Baumol's Hypothesis, Wage Inequality, Technical Change

An approximate approach to the exponential utility indifference valuation

Takuji Arai

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Abstract

We propose, in this paper, a new valuation method for a contingent claim, which approximates to the exponential utility indifference valuation. In particular, we treat both ask and bid valuations. In the definition of the exponential utility indifference valuation, we require a strong condition related to the underlying contingent claim. The new valuation in this paper succeeds in reducing this condition by using a kind of power functions instead of the exponential function. Furthermore, we shall investigate some basic properties and an asymptotic behavior of our new valuation.

Keywords: Incomplete markets, Indifference value, p-optimal martingale measure, Reverse Hölder inequality.

Nonsynchronously observed diffusions and covariance estimation

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and

Nakahiro Yoshida
The University of Tokyo, Graduate School of Mathematical Sciences
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November 8, 2005

Abstract: We consider the problem of estimating the covariance/correlation of two diffusion-type processes when they are observed only at discrete times in a nonsynchronous manner. We first overview a new estimation procedure that we have been proposing, which is free of any 'synchronization' processing of original data and yields consistent estimators for the true covariance/correlation of the processes as the observation interval shrinks to zero. We then present our new results obtained recently by Hayashi and Yoshida (05a, b). In particular, we discuss (i) (joint) *asymptotic normality* of the covariance estimator with the realized volatilities and, as its direct application, (ii) asymptotic normality of the *correlation estimators* constructed by the same principle when the true volatilities and correlation are constant. Examples are also shown.

Key words: diffusions; discrete-time sampling; high-frequency data; mathematical finance; nonsynchronicity; quadratic variation; realized volatility

Monetary policy and economic fluctuations in a sticky-price model

Hiroyuki Yoshida*
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Abstract

This paper examines a simple monetary optimizing model with sticky prices. Two types of monetary policy rules are considered: constant money growth rules and interest-rate feedback (Taylor-type) rules. In the case of constant money growth rules, we show the existence of limit cycles through the Hopf bifurcation theorem. On the other hand, in the case of the interest-rate feedback rules, we show that active monetary policy leads to the determinacy of equilibrium path, while passive monetary policy induces economic fluctuations.

Keywords: Indeterminacy; Hopf bifurcation; Monetary policy

JEL classification: E32; E42, E52

*An earlier version of this paper is presented at the Fourth International Conference on Nonlinear Economic Dynamics, Chuo University, July 29-31, 2004. I wish to thank the participants for their valuable comments. Moreover, I wish to express my gratitude to two anonymous referees for their helpful and constructive comments. Special thanks are due to Junichi Minagawa for his assistance with numerical simulations. E-mail address: yoshida@eco.nihon-u.ac.jp

Stability, Chaos and Multiple Attractors: A Single Agent Makes a Difference

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Gernot SIEG
Institut für Wirtschaftswissenschaften, TU Braunschweig, Germany

Masanori YOKOO
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Abstract

This paper provides an example in which a slight behavioral heterogeneity may fundamentally change the dynamical properties of a nonlinear cobweb market with a quadratic cost function and an isoelastic demand function. We consider two types of producers; cautious adapters and naive optimizers. In a market of naive optimizers a single cautious adapter stabilizes the otherwise exploding market. In a market of cautious adapters a single naive optimizer may destabilize the market; without him there exists at most one periodic attractor in the market but with him there may appear many (and even infinitely many) coexisting periodic attractors.

非決定性動的計画について

九州工業大学・工学部　　藤田 敏治 (Toshiharu Fujita)
Faculty of Engineering,
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概要

動的計画法の理論は R. Bellman により創出され，非常に幅広い分野において研究・応用がなされてきた．その応用範囲は，理学，工学，経済学と多岐にわたる．動的計画法は，最適性の原理をその基本原理とし，様々な問題に対する再帰的アプローチを与える理論的枠組みである．非決定性動的計画は，この動的計画法をさらに多くの状況下の問題に対応させるべく登場した．その本質は，確定的でもなく確率的でもない推移法則にある．確定的推移法則の下，動的計画法における状態は，期の進行とともに常に単一のものが確定的に生じる．また，確率的推移法則の下では，次期の状態は確率的となる．一方，非決定性推移法則下においては，次期の状態は唯ひとつに定まるわけでもなく，確率的に生じるわけでもない．次の期には，一般に複数の状態が同時に現れるのである．この非決定性推移という概念の導入は，動的計画法に新たな展開をもたらすはずである．

本論分では，非決定性推移法則下での動的計画を非決定性動的計画と呼ぶ．まず，有限段問題を定式化し再帰式を与え，その応用例として，行列積計算量最小化問題をその数値例とともに扱う．次いで，無限段問題を定式化し，最適方程式を導く．最後に，無限段問題に対して与える Howard の政策改良アルゴリズムにより，線形最大化方程式を解き得ることを示す.

The Golden Trinity
– Optimality, Inequality and Equality –

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Abstract

This paper considers a triplet of optimization, inequality and equality through the Golden ratio. When a conditional optimization problem which optimizes an objective function under a constraint function yields a pair of optimum value and optimum point, an inequality holds between the objective function and the constraint function. Conversely, if an inequality holds between two functions together with an equality condition, we can easily solve the optimization problem by applying the inequality and the equality condition. Thus an optimization is equivalent to an inequality. Further an inequality between two functions with an equality condition is stated as an equivalent equality form by adding a supplementary nonnegative term. Conversely, an equality between two functions with an additional nonnegative term is stated in an inequality form without the nonnegative term. Thus an inequality is also equivalent to an equality. Therefore, the triplet is equivalent each other. In this sense, we call an equivalent triplet trinity.

Furthermore, we introduce the Golden ratio into trinity – Golden optimum solution, Golden inequality, and Golden equality –. Such a trinity is called Golden. In a class of quadratic functions, we specify six Golden trinities on the basis of three quadratic forms : $x^2 + y^2$, $x^2 - y^2$, $x^2 + (x - y)^2$. The triplet is called the Golden triplet of quadratic forms.

Efficiency and Diversity in Voluntarily Repeated Prisoner's Dilemma

Abstract

We consider a large society of homogeneous players, in which players are randomly matched in each period to play prisoner's dilemma as well as to choose whether to play the game again with the same partner. Moreover, there is no information flow across matches because of private actions and random deaths. We extend the notion of Neutrally Stable Distribution (NSD) into our extensive-form model and characterize trust-building NSD. NSD consisting on a single strategy must have some periods of trust-building to play Defect but keep the partnership. For some parameters, NSD with diverse trust-building periods exists and is more efficient than any single-strategy NSD, since the former can start cooperating earlier. When cheap-talk is introduced, the most efficient NSD becomes the unique equilibrium evolutionarily stable set.

ゲーム理論の誕生とその周辺

慶應義塾大学経済学部
中山幹夫

フォン・ノイマンのミニマックス定理によって誕生したゲームの理論は，その後，フォン・ノイマンとモルゲンシュテルンの共著『ゲームの理論と経済行動』で展開された協力ゲームの理論と，さらにその後のナッシュによる非協力ゲームの理論によって，その土台が築かれた．ここでは，これらの創始者達が何をどのように考えたかを当時の科学的時代背景に留意して振り返る．とくに，エミール・ボレルこそゲーム理論の創始者だと主張するフレッシェとの論争や，シュタインハウスの孤独なゲーム理論研究などにも触れ，フォン・ノイマンのアプローチとの違いに言及したい．主な内容は以下のとおりである．

1. von Neumann のミニマックス定理とこれをとりまく話題

ミニマックス定理（1928）；混合戦略；モルゲンシュテルンの混合戦略 (1928)；ツェルメロの定理 (1913)；ゼロ和２人ゲームに関するボレルの研究 (1921); シュタインハウスの研究 (1925)；および，フレッシェとフォン・ノイマンの論争 (1953)

2. von Neumann-Morgenstern の著書と関連する話題

社会科学の形式化；ｎ人協力ゲームと安定集合；差別解；存在定理；公理的方法

3. von Neumann の均衡モデルと角谷の不動点低利

不等式制約と経済モデル；フォン・ノイマンのレンマ（1937）と角谷の不動点定理（1941);「数理計画経済学」

4. Nash のｎ人非協力ゲーム

非協力ゲームとナッシュ均衡 (1950)；均衡の仮想的プレイ解釈と進化ゲーム (1951)

5. Nash の２人協力ゲーム

2 人協力ゲームとナッシュ・プログラム (1953); 実験，合理性, トリガー戦略，および限定された合理性

Date: October 27, 2005.

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Suggestive Dominant Strategies in Cheap-Talk Games*

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October 24, 2005

Abstract: We reconsider the stable communication in cheap-talk games between an informed and an uninformed players by providing a new criterion of 'suggestive domination.' This suggestive domination eliminates some unstable factors that the informed player may change his action in an additional communication. Moreover the domination also achieves a Pareto improvement in the cheap-talk games. Our main result guarantees the existence of the *suggestive dominant equilibrium* based on the suggestive domination.

JEL Classification: C72; D80.
Keywords: Cheap talk; Communication proof; Trumping relation.

1. Introduction

This paper provides a new refinement concept in cheap-talk games. The refinement concept, *the suggestive domination*, is a relation over strategy profiles with reasonable beliefs and achieves a weakly Pareto improvement by the domination. We show the existence of the *suggestive dominant equilibrium* based on the suggestive domination. The equilibrium guarantees the stable communication between the players.

*This paper is based on a part of the M.A. thesis by the first author (Shirataki [4]).

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Excess Information and Tragedy of the Commons*

Yuhki Hosoya†
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October 29, 2005

Abstract

We consider the problem of information pollution, especially excess information provided by supplier, named a catalog. Thicker catalog should bore and discourage consumer to search and buy. So that, if supplier makes his catalog thicker, then he can sell more products, but other supplier can sell less products, as the case of "tragedy of the commons". We analyze this situation, and find a phenomenon that both price and total welfare decline when new entry occurs, named "counterfeit competition".

Keywords: Information pollution, thickness of catalog, tragedy of the commons, counterfeit competition.

*The author would like to thank Shinji Takamatsu and Toshiyuki Hirai for helpful comments and suggestions.
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EXISTENCE OF EQUILIBRIA FOR AN ABSTRACT SOCIETY WITH N SOCIAL GROUPS

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Abstract

We study an abstract model of a society with n social groups in which all members of each group behave cooperatively within their group, and act as a united body noncooperatively toward other groups. The model enables us to analyze a *strategic interaction among social groups.* The purpose of the present paper is to prove the existence of equilibria for this abstract society model.

Keywords : Cooperation, Noncooperation, Strategic interaction among social groups, Nash equilibria

Fixed Points and Economic Equilibria

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Abstract

This work attempts to aggregate my recent researches on economic equilibrium and fixed point theorems. Since economic equilibrium arguments are based on the commodity/price (fact/value) duality (in an abstract sense), fixed point arguments should be constructed on a certain kind of mathematical structure representing the dual system of base sets. Moreover, since the concept of economic equilibrium always has moral or epistemic features hard to describe, equilibrium arguments should also be founded on minimum mathematical requirements such as set-theoretic or algebraic ones. In the following, new methods for economic equilibrium and fixed point arguments are introduced and developed toward unified homological (algebraic) treatment for the general equilibrium analysis. Especially, we relate equilibrium and fixed point results based on a generalized duality structure to arguments under Čech type homology theories and an extension of Lefschetz's fixed point theorem.

Keywords : Fixed point theorem, Nash equilibrium, Abstract economy, Gale-Nikaido-Debreu theorem, General equilibrium, Čech homology theory, Vietoris homology theory, Browder's fixed point theorem, Kakutani's fixed point theorem, Lefschetz's fixed point theorem.

JEL classification: C60; C62; C70; D50

Approximation of Interest Rate Derivatives' Prices by Gram–Charlier Expansion and Bond Moments

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Abstract

In this paper, we develop easily implemented approximations of the prices of several interest rate derivatives. We study swaptions, constant maturity swaps ("CMS"), and CMS options. We approximate swaption prices under one forward measure by using a Gram–Charlier expansion which is an orthogonal decomposition of a density function in additive form. Higher-order approximations yield very accurate prices enough to price each transaction, and lower-order approximations are suitable for portfolio evaluation and risk management. In addition, we approximate CMS rates and CMS options by using bond moments.

Key Words: Gram–Charlier expansion, bond moment, swaption, constant maturity swap, convexity adjustment

JEL Classification Numbers: G13

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社債アメリカ型オプションの評価について

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近年、信用派生商品の価格の評価法についての議論が活発に行われている。信用派生商品の価格計算の研究の多くは、構造モデル・アプローチと誘導モデル・アプローチのふたつの手法のどちらかを用いて研究されてきた。構造モデル・アプローチは、企業価値を確率過程としてモデリングする手法であり、企業価値があらかじめ決められた閾値を下回るときに企業はデフォルトしたとみなすモデルである。この手法は企業のデフォルトの経済的な意味合いがはっきりしているものの、企業価値の観測の難しさや、閾値の決め方などの問題をはらんでいる。その一方で、本研究で用いられた、誘導モデル・アプローチではデフォルト時刻をハザード過程で規定された停止時刻で表現するために、企業のデフォルトの経済的な意味合いが、構造モデル・アプローチと比較すると余りはっきりしていないモデルであるが、比較的に商品の価格の計算が簡便であり、かつ企業の格付けなどによりモデルに含まれるパラメータの推定が比較的容易に行えると言う利点を持っている。本研究では、後者のアプローチ、すなわち誘導モデル・アプローチにより社債オプションの価格の評価法について考えてみた。早期行使権のついた信用派生商品は、アメリカ型オプションのほかにもコーラブル債や転換社債など多数存在しているが、先行研究は必ずしも多くはなく、その意味では数多くの応用が存在している研究であると考えられる。

数学的には、アメリカ型オプションの価格は、最適停止問題を通じて定式化される事が知られている。ところが、企業のデフォルトをジャンプとして表現したために、早期行使権を持つ信用派生商品の解析のためには、ジャンプのついた確率過程に関する最適停止問題を考察する必要性がある。その一方で、この問題の解の存在は必ずしも自明とは言えず、最適停止時刻の存在証明を別に行なう必要性があった。そこで、その証明を与えた上で、偏微分方程式 (相補性問題) を用いたオプションの価格関数の定式化を行った。また、オプションの複製戦略の構築についても考察を行った。なお、数値計算については、3 項分岐木モデルを用いることで価格計算をしてみた。そこで、その数値計算結果についても併せて議論を行う。

拡散過程の期待値の近似：ディリクレ条件付きの場合

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V_i, $i = 0, 1, \ldots, d$, は $\mathbf{R}^N$ 上の滑らかなベクトル場で

$$V_i = \sum_{j=1}^{N} V_i^j(x) \frac{\partial}{\partial x^j}$$

としたとき、$v_i^j \in C_b^\infty(\mathbf{R}^N)$, $i = 0, \ldots, d$, $j = 1, \ldots, N$ とする。$B(t) = (B^1, \ldots, B^d(t))$ は d-次元ブラウン運動とする。簡単のために $B^0(t) = t$, $t \in [0, \infty)$, とおく。

Stratonovich 型の SDE

$$dX(t; x) = \sum_{i=0}^{d} V_i(X(t, x)) \circ dB^i(t),$$
$$X(0, x) = x$$

を考える。$f : \mathbf{R}^N \to \mathbf{R}$ があまり滑らかさを持たないときに $E[f(X(t, x))]$ の近似値を与える方法を考えることがファイナンスではしばしば重要である。

その近似計算方法として「楠岡近似」と呼ばれる方法があるが、そのバージョンの一つとして、使いやすい方法を二宮-Victor が見いだした。その方法と精度について講演する。

ファイナンスにおいてはノックアウト・ノックイン条件の付いたデリバティブが存在する。その価格の計算は大ざっぱに言って次の量の計算に帰着する。

$$E[f(X(t, x)), \ \min_{s \in [0,t]} X^1(s, x) \geq 0], \qquad x \in \mathbf{R}^N, \ x^1 \geq 0, \ t \geq 0$$

これは、Dirichlet 境界条件のついた拡散過程の期待値である。その近似の可能性についても述べる。