Fixed Points and Economic Equilibria

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Abstract
This work attempts to aggregate my recent researches on economic equilibrium and fixed point theorems. Since economic equilibrium arguments are based on the commodity/price (fact/value) duality (in an abstract sense), fixed point arguments should be constructed on a certain kind of mathematical structure representing the dual system of base sets. Moreover, since the concept of economic equilibrium always has moral or epistemic features hard to describe, equilibrium arguments should also be founded on minimum mathematical requirements such as set-theoretic or algebraic ones. In the following, new methods for economic equilibrium and fixed point arguments are introduced and developed toward unified homological (algebraic) treatment for the general equilibrium analysis. Especially, we relate equilibrium and fixed point results based on a generalized duality structure to arguments under Čech type homology theories and an extension of Lefschetz’s fixed point theorem.

Keywords: Fixed point theorem, Nash equilibrium, Abstract economy, Gale-Nikaido-Debreu theorem, General equilibrium, Čech homology theory, Vietoris homology theory, Browder’s fixed point theorem, Kakutani’s fixed point theorem, Lefschetz’s fixed point theorem.

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