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Empirical studies \[1,2\] on trade by trade data in stock markets reveal that there exists a long memory in trade signs, specifically the correlation \(\rho(t)\) between trade signs \(\varepsilon_s\) and \(\varepsilon_{s+t}\) at time \(s\) and \(s+t\) decays very slowly as an inverse power law of time lag,

\[
\rho(t) \sim \frac{c_0}{t^\gamma} \quad (t \to \infty)
\]

with \(0 < \gamma < 1\), where \(c_0 > 0\) is a constant.

We present a discrete time model to describe a long memory in trade signs in \([1, n]\) and consider a stochastic processes for signed trade volumes \(W_t\) and stock price \(S_t\). As a scaling limits of the processes we derive

\[
X_t = c_1 B^1_t + c_2 B^H_t \\
Y_t = c_3 B^2_t + c_4 B^H_t,
\]

where \(X_t, Y_t\) are scaling limits of stock price proess and signed volume process respectively, \((B^1_t, B^2_t)\) are correlated Brownian motions, and \(B^H_t\) is a fractional Brownian motion with Hurst index. We apply a method of abstract polymer expansion developed in the mathematical theory of phase transitions to obtain our results.

References