Stock Price Process and Long memory in Trade Signs.

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Empirical studies [1],[2] on trade by trade data in stock markets reveal that there exists a long memory in trade signs, specifically the correlation $\rho(t)$ between trade signs ε_s and ε_{s+t} at time s and $s+t$ decays very slowly as an inverse power law of time lag,

$$\rho(t) \sim \frac{c_0}{t^\gamma} \qquad (t \to \infty)$$

with $0 < \gamma < 1$, where $c_0 > 0$ is a constant.

We present a discrete time model to describe a long memory in trade signs in $[1, n]$ and consider a stochastic processes for signed trade volumes W_t and stock price S_t. As a scaling limits of the processes we derive

$$\begin{aligned} X_t &= c_1 B_t^1 + c_2 B_t^H \\ Y_t &= c_3 B_t^2 + c_4 B_t^H, \end{aligned}$$

where X_t, Y_t are scaling limits of stock price proess and signed volume process respectively, (B_t^1, B_t^2) are correlated Brownian motions, and B_t^H is a fractional Brownian motion with Hurst index. We apply a method of abstract polymer expansion developed in the mathematical theory of phase transtions to obtain our results.

References

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