Local indeterminacy in continuous-time models: the role of returns to scale*

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Abstract: The existence of local indeterminacy in infinite-horizon models with externalities is a well established fact. As initially shown by Benhabib and Farmer [2], this conclusion is generally a consequence of increasing returns at the social level, a large enough elasticity of intertemporal substitution in consumption and large enough elasticity of the labor supply (see also among others Benhabib and Farmer [3], Wen [9], Pintus [8], Lloyd-Braga, Nourry and Venditti [6]. It is worth noticing that all these contributions deal either with an aggregate model or with a two-sector model characterized by identical technologies at the private level.

As increasing returns do not clearly appear to be supported by empirical evidences (see Basu and Fernald [1]), Benhabib and Nishimura [4] show in a seminal contribution that local indeterminacy is also compatible with constant returns at the social level in a two-sector model with sector-specific externalities. Assuming a linear utility function with respect to consumption, they prove that this result is based on a capital intensity reversal, the consumption good being capital intensive at the private level but labor intensive at the social level. More interestingly, this result is established without any restriction on the elasticity of labor supply.

Generalizing the Benhabib-Nishimura formulation by the consideration of a non-linear utility function with respect to consumption, Nishimura and Venditti [7] and Garnier, Nishimura and Venditti [5] show that while local indeterminacy is fundamentally based on a large elasticity of intertemporal substitution in consumption, it requires a low enough elasticity of the labor supply. Moreover, they prove that saddle-point stability is ensured as soon as the elasticity of the labor supply is large enough. Clearly, this last conclusion appears to be in complete opposition with the results derived under increasing returns within aggregate models or two-sector models with identical private technologies.

Our goal in the present paper is to understand such a drastic difference between the two formulations. We consider a two-sector model with Cobb-Douglas technologies, sector-specific externalities, endogenous labor and a non-linear separable CES utility function. Contrary to the Benhabib and Farmer [3] formulation, we consider different technologies at the private level (the capital intensity difference at the private level is non zero). Moreover, we assume that one of the two sectors has constant social returns to scale (possibly without externalities) while the other sector is characterized by constant returns either at the private or at the social level. Proceeding that way, we get a framework compatible with both the Benhabib-Farmer [3] and the Benhabib-Nishimura [4] formulations.

We first show that when labor is inelastic, for any configuration of the returns to scale, local indeterminacy is obtained under the same condition exhibited by Benhabib and Nishimura [4], namely a capital intensity reversal between the private and the social levels. Secondly, we prove that when labor is infinitely elastic, saddle-point stability is obtained as soon as the consumption good sector has increasing social returns. On the contrary, if the investment good sector has increasing social returns, local indeterminacy may arise provided the consumption good is capital intensive at the social level and the elasticity of intertemporal substitution in consumption admits intermediary values. Considering these two results simultaneously we finally prove that local indeterminacy requires a low elasticity of labor when the consumption good has increasing social returns, but requires either low enough or large enough elasticity of labor when the investment good has increasing social returns. Indeed, in this last configuration, local indeterminacy is always ruled out when the elasticity of labor admits intermediary values. To summarize, we show that the drastic difference between the Benhabib-Farmer [3] and the Benhabib-Nishimura [4] formulations fundamentally relies on the returns to scale in the investment good sector.
References


