Approximation of Interest Rate Derivatives’ Prices by Gram–Charlier Expansion and Bond Moments

Keiichi Tanaka*, Takeshi Yamada†, Toshiaki Watanabe‡

October 25, 2005

Abstract

In this paper, we develop easily implemented approximations of the prices of several interest rate derivatives. We study swaptions, constant maturity swaps (“CMS”), and CMS options. We approximate swaption prices under one forward measure by using a Gram–Charlier expansion which is an orthogonal decomposition of a density function in additive form. Higher-order approximations yield very accurate prices enough to price each transaction, and lower-order approximations are suitable for portfolio evaluation and risk management. In addition, we approximate CMS rates and CMS options by using bond moments.

Key Words: Gram–Charlier expansion, bond moment, swaption, constant maturity swap, convexity adjustment

JEL Classification Numbers: G13

*Corresponding author, Graduate School of Economics, Kyoto University, Yoshida-honmachi, Sakyo, Kyoto 606-8501, Japan (Tel 81-75-753-3411, E-mail: tanakak@econ.kyoto-u.ac.jp)

†Institute for Monetary and Economic Studies, Bank of Japan (E-mail: takeshi.yamada@boj.or.jp)

‡Institute for Monetary and Economic Studies, Bank of Japan (E-mail: toshiaki.watanabe@boj.or.jp)