On the Verification Theorem of Continuous-Time Optimal Portfolio Problems with Stochastic Market Price of Risk

Toshiki Honda * Shoji Kamimura †

In this paper, we study a continuous-time portfolio optimization problem in Kim and Omberg (1996). We show sufficient conditions to verify that a solution derived from the Hamilton-Jacobi-Bellman (HJB) equation is in fact an optimal solution to the portfolio selection problem.

Since Merton’s seminal work (Merton (1969, 1971)), many studies have been done on continuous-time portfolio optimization problems. In particular, there has been increasing interest in finding an optimal portfolio strategy when investment opportunities are stochastic, because many empirical works conclude that investment opportunities are time-varying. In general, it is difficult to derive an explicit solution for such problems. Kim and Omberg (1996) is an interesting exception. In their model, a market price of risk is driven by an Ornstein-Uhlenbeck process. Furthermore, their model is an incomplete market model. When a market is not complete, it is more difficult to find an analytical solution to the optimal portfolio problem.

There are two main approaches to solving the continuous-time portfolio optimization problem. One is the stochastic control approach and the other is the martingale approach. Following Kim and Omberg (1996), we employ the former approach in this paper. Since the market is incomplete in our model, the martingale approach is not applied directly. In the stochastic control approach, an optimal solution is conjectured by solving the HJB equation. It is necessary to verify that the conjectured solution is in fact a solution to the original problem. However, as Korn and Kraft (2004) pointed out, the verification is often skipped since it is very mathematically demanding. The conjectured solution from the HJB equation could be an incorrect solution to the original problem. Kim and Omberg (1996) examined the finiteness of the conjectured value function very carefully. However, they did not provide a verification theorem. Our aim in this paper is to derive sufficient conditions to confirm that the conjectured solution is in fact the solution to the original problem.

*Graduate School of International Corporate Strategy, Hitotsubashi University, 2-1-2 Hitotsubashi, Chiyoda-ku, Tokyo 101-8439, Japan; Email: thonda@ics.hit-u.ac.jp
†Graduate School of International Corporate Strategy, Hitotsubashi University, 2-1-2 Hitotsubashi, Chiyoda-ku, Tokyo 101-8439, Japan; Email: kamimura@ics.hit-u.ac.jp