Introduction to Finance (PCP)

Solutions for Exercise 1

NOTE:

1) S_T denotes the price of the risky asset at the maturity T.
2) $\mathbb{Q}$ denotes the martingale probability.
3) a and b represent the number of shares of the riskless and the risky assets in the replicating portfolio, respectively.

1. 1. $\mathbb{Q}(S_T = 80) = \frac{1}{3}$, $\mathbb{Q}(S_T = 50) = \frac{2}{3}$. 2. $a = -\frac{125}{3}$, $b = \frac{5}{6}$. 3. $\frac{25}{3}$. 4. $\frac{20}{3}$.

2. 1. From the 1st FTAP, it suffices to show the existence of martingale probabilities. In other words, for any $S_U > 1000$, we have only to find $q \in (0,1)$ satisfying $qS_U + (1-q) \times 900 = 1000$. Actually, this equation holds when $q = \frac{100}{S_U - 900}$, which is positive and less than $\frac{100}{1000-900} = 1$.
2. Selling one share of the risky asset short at $t = 0$, and lending 1000 to someone, the investor's cashflow at the maturity T is either 0 or 100.
3. $\mathbb{Q}(S_T = 1050) = \frac{2}{3}$, $\mathbb{Q}(S_T = 900) = \frac{1}{3}$. 4. $a = -420$, $b = \frac{7}{15}$. 5. $\frac{140}{3}$.
6. 10.

3. 1. $\mathbb{Q}(S_T = uS) = \frac{1+r-d}{u-d}$, $\mathbb{Q}(S_T = dS) = \frac{u-1-r}{u-d}$.
2. $a = -\frac{d}{1+r}\frac{uS-K}{u-d}$, $b = \frac{uS-K}{(u-d)S}$. 3. $\frac{(1+r-d)(uS-K)}{(1+r)(u-d)}$.

4. 1. $\mathbb{Q}(S_T = 110) = \frac{3}{10}$, $\mathbb{Q}(S_T = 100) = \frac{7}{10}$. 2. $a = -\frac{1000}{103}$, $b = \frac{1}{10}$. 3. $\frac{30}{103}$.
4. $\frac{70}{103}$.

5. 1. $0.05 < r < 0.2$. 2. $\mathbb{Q}(S_T = 108) = \frac{1}{3}$, $\mathbb{Q}(S_T = 94.5) = \frac{2}{3}$.
3. $a = -\frac{560}{11}$, $b = \frac{16}{27}$. 4. $\frac{80}{33}$. 5. $\frac{50}{11}$.