

Introduction to Finance (PCP)

Exercise 3

In what follows, an (N, M) -model represents a one-period ($t = 0, T$) model with N assets (including one riskless asset) and M scenarios ($\#\Omega = M$).

1.

We consider a $(3, 3)$ -model with the initial price vector \mathbf{S}_0 and the price matrix D given as

$$\mathbf{S}_0 = \begin{pmatrix} 1 \\ 30 \\ 40 \end{pmatrix}, \quad D = \begin{pmatrix} 1 & 1 & 1 \\ 10 & 10 & 70 \\ 100 & 10 & 10 \end{pmatrix}.$$

1. Show that the market is arbitrage-free.
2. Find the martingale probability.
3. Show the market completeness.
4. Find the replicating portfolio of the option $\mathbf{C} = (10, 0, 0)$, and its price.

2.

We consider a $(3, 3)$ -model with

$$\mathbf{S}_0 = \begin{pmatrix} 1 \\ 50 \\ 40 \end{pmatrix}, \quad D = \begin{pmatrix} 1.1 & 1.1 & 1.1 \\ 44 & 55 & 77 \\ 55 & 44 & 22 \end{pmatrix}.$$

1. Show that the market is arbitrage-free.
2. Show that the market is not complete.

3.

We consider a $(3, 4)$ -model with

$$\mathbf{S}_0 = \begin{pmatrix} 1 \\ 10 \\ 12 \end{pmatrix}, \quad D = \begin{pmatrix} 1 & 1 & 1 & 1 \\ 8 & 9 & 9 & X \\ 10 & 10 & 14 & 14 \end{pmatrix}.$$

1. Let $X = 12$. Show that the market is arbitrage-free, but not complete.
2. Let $X = 11$. Show that the market is not arbitrage-free, and find an arbitrage opportunity.
3. Show that the market is not complete for any $X > 0$.