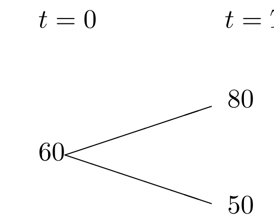
Introduction to Finance (PCP)

Exercise 1

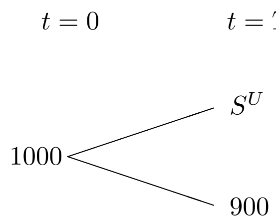
1.

Consider a one-period binomial model being composed of one risky asset and one riskless asset with 0 interest rate. Suppose that the fluctuation of the risky asset is described as follows:



1. Show the martingale probability.
2. Show the replicating portfolio of a call option with strike price 55.
3. Find its price.
4. What is the price of a put option with strike price 60?

2. Consider a one-period binomial model being composed of one risky asset and one riskless asset with 0 interest rate. Suppose that the fluctuation of the risky asset is described as follows:



1. Prove that our market is arbitrage-free whenever $S^U > 1000$.
   (Hint: The first fundamental theorem of asset pricing is useful.)
2. Construct an arbitrage portfolio when $S^U = 1000$.

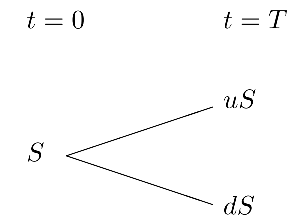
Let $S^U = 1050$.

3. Show the martingale probability.
4. Show the replicating portfolio of a call option with strike price 980.

5. Find its price.
6. What is the price of a put option with strike price 930?

3.

Consider a one-period binomial financial market model being composed of one risky asset and one riskless asset whose interest rate is given by $r > 0$. The risky asset price process is assumed to be given by:

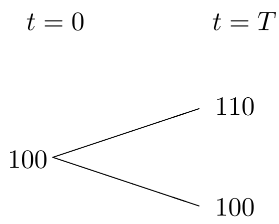


Suppose that S, u and d are positive real numbers satisfying $d < 1 + r < u$.

1. Show the martingale probability.
2. Show the replicating portfolio of a call option with strike price K, where $dS < K < uS$.
3. Find the price of the call option.

4.

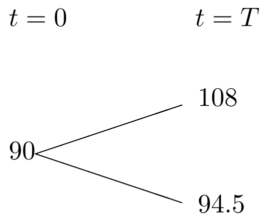
Consider the following one-period binomial model with interest rate 0.03.



1. Show the martingale probability.
2. Show the replicating portfolio of a call option whose strike price is 109.
3. Find its price.
4. What is the price of a put option with strike price 101?

5.

Consider the following one-period binomial model with interest rate $r > 0$.



1. What is an equivalent condition on $r > 0$ to the arbitrage-free condition?

Let $r = 0.1$.

2. Show the martingale probability.
3. Show the replicating portfolio of a call option whose strike price is 100.
4. Find its price.
5. What is the price of a put option with strike price 102?